An improved solution to estimate relative permeability in tight oil reservoirs

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Abstract The relative permeability is a critical property to predict field performance by reservoir numerical simulation. However, determining the relative permeability curve by experiments is time consuming and difficult for tight oil reservoirs due to the extremely low permeability. Therefore, many studies focus on the analytical solution to determine the relative permeability curve. However, no existing solutions take the effect of boundary layer, Jamin and wettability into account. This results in overestimated results because boundary layer is even up to 60 % of the throat and throat to pore ratio reaches up to 400 in tight oil reservoirs. Therefore, an improved analytical solution is desirable to solve these two problems. In this paper, based on the collected experimental results, the Purcell's solution is modified and an improved solution considering the effect of boundary layer, Jamin and wettability is developed. Compared to the results of the relative permeability experiments, the improved solution is more accurate and reliable. Because the throat distribution of the tight oil reservoir in Daqing Field is narrower than that in Changqing Field, the productivity of the tight oil reservoir in Daqing Field is less than that in Changqing Field at first. However, the breakthrough time of the tight oil reservoir in Daqing Field is longer than that in Changqing Field. This improved solution can only be applied to tight oil reservoirs, because boundary layer and Jamin have little effect on the flow behavior in conventional reservoirs.

Keywords Tight oil reservoir · Boundary layer · Jamin · Relative permeability

Introduction

There are generally two solutions to determine the relative permeability curve. One is the experimental solution and the other one is the analytical solution. Due to the low permeability in tight oil reservoirs, the displacement pressure is high and the flow rate is low in relative permeability experiments. As a result, the experimental solution is time consuming and difficult to be conducted. The analytical solution, however, has several advantages. First, because the analytical solution is based on the capillary pressure data and mercury injection experiments are widely used in tight oil reservoirs, it has abundant basic data. Second, it can reflect the effect of the micropore–throat structure on relative permeability. The most important advantage is, it is easy to obtain the relative permeability. Therefore, many scholars focus on the analytical solution and have got many achievements. Although in tight oil reservoirs boundary layer is up to 60 % (Li et al. 2011) of the throat, and pore-to-throat ratio reaches up to 400 (Li et al. 2012), no existing solutions take them into account. Therefore, existing solutions are not suitable for tight oil reservoirs.

Purcell (1949) developed an equation to calculate multiphase relative permeability. In two-phase flow, the relative permeability can be calculated as follows:

$$K_{rwe} = \frac{\int_0^{S_{we}} \frac{1}{p_c} dS_{we}}{\int_0^1 \frac{1}{p_c} dS_{we}} \quad (1)$$

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$$K_{rwe} = \frac{\int_{S_{we}}^1 \frac{1}{p_c} dS_{we}}{\int_0^1 \frac{1}{p_c} dS_{we}}, \quad (2)$$

where K_{rwe} is relative permeability of wetting phase; S_{we} is saturation of wetting phase; p_c is capillary pressure, MPa; K_{rnwe} is relative permeability of nonwetting phase.

Purcell's solution neglects the effect of pore-throat structure on the relative permeability. Therefore, Burdine (1953) modified Purcell's solution by introducing a tortuosity factor as a function of wetting phase saturation. The relative permeability can be computed as follows:

$$K_{rwe} = (\lambda_{rwe})^2 \frac{\int_0^{S_{we}} \frac{1}{p_c^2} dS_{we}}{\int_0^1 \frac{1}{p_c^2} dS_{we}} \quad (3)$$

$$K_{rnwe} = (\lambda_{rnwe})^2 \frac{\int_{S_{we}}^1 \frac{1}{p_c^2} dS_{we}}{\int_0^1 \frac{1}{p_c^2} dS_{we}}, \quad (4)$$

where λ_{rwe} is tortuosity ratio of wetting phase; λ_{rnwe} is tortuosity ratio of nonwetting phase.

According to Burdine, λ_{rw} and λ_{rnw} could be calculated as follows:

$$\lambda_{rwe} = \frac{\tau_{we}(1)}{\tau_{we}(S_{we})} = \frac{S_{we} - S_m}{1 - S_m} \quad (5)$$

$$\lambda_{rnwe} = \frac{\tau_{nwe}(1)}{\tau_{nwe}(S_{we})} = \frac{1 - S_{we} - S_e}{1 - S_m - S_e}, \quad (6)$$

where $\tau_{we}(1)$ is tortuosities of wetting phase when wetting phase saturation is equal to 100 %; $\tau_{we}(S_{we})$ is tortuosities of wetting phase when wetting phase saturation is equal to S_{we} ; S_m is minimum wetting phase saturation from capillary pressure curve; τ_{nwe} is tortuosity of nonwetting phase; S_e is equilibrium saturation of nonwetting phase.

Honarpour et al. (1986) pointed out that the expression for the wetting phase relative permeability fits the experimental data much better than the expression for the nonwetting phase.

According to Burdine's solution, an analytical expression for the wetting and nonwetting phase relative permeability can be obtained if capillary pressure curves can be represented by a simple mathematical function. Corey (1954) found that oil–gas capillary pressure curves can be expressed approximately using the following linear relation:

$$1/p_c^2 = CS_{we}^* \quad (7)$$

where C is a constant.

S_{we}^* could be expressed as follows for the drainage case

$$S_{we}^* = \frac{S_{we} - S_{wer}}{1 - S_{wer}} \quad (8)$$

where S_{we}^* is normalized wetting phase saturation; S_{wer} is residual saturation of wetting phase.

Although originally the Corey's solution was not developed for the imbibition case, it was used to calculate the imbibition relative permeability by defining the normalized wetting phase saturation as follows:

$$S_{we}^* = \frac{S_{we} - S_{wer}}{1 - S_{wer} - S_{nwer}}, \quad (9)$$

where S_{nwer} is residual saturation of nonwetting phase.

Substituting Eq. 7 into Eqs. 3 and 4 with the assumption that $S_e = 0$ and $S_m = S_{wr}$, Corey obtained the following equations to calculate the wetting (oil) and nonwetting (gas) phase relative permeability for drainage cases:

$$K_{rwe} = (S_{we}^*)^4 \quad (10)$$

$$K_{rnwe} = (1 - S_{we}^*)^2 [1 - (S_{we}^*)^2]. \quad (11)$$

A constraint to the use of Corey's solution (Eqs. 10 and 11) is that the capillary pressure curve should be represented by Eq. 7. Therefore, Brooks and Corey (1966) modified the representation of capillary pressure function to a more general form as follows:

$$p_c = p_e (S_{we}^*)^{-1/\lambda} \quad (12)$$

where p_e is entry capillary pressure, MPa; λ is pore size distribution index.

When λ is equal to 2, the Brooks–Corey's solution reduces to the Corey's solution. Substituting Eq. 12 into the Burdine's solution (Eqs. 3 and 4) with the assumption that $S_e = 0$, Brooks and Corey derived equations to calculate the wetting and nonwetting phase relative permeability as follows:

$$K_{rwe} = (S_{we}^*)^{\frac{2+3\lambda}{\lambda}} \quad (13)$$

$$K_{rnwe} = (1 - S_{we}^*)^2 [1 - (S_{we}^*)^{\frac{2+\lambda}{\lambda}}]. \quad (14)$$

Li and Horne (2002) tested different solutions of calculating relative permeability were. It is found that the Purcell's solution is the best fit to the experimental data of the wetting phase relative permeability for the cases studied, as long as the measured capillary pressure curve has the same residual saturation as the relative permeability curve. The differences between the results of experiments and the Purcell's solution are almost negligible.

Li and Horne (2002) found that the Purcell's solution is proposed to calculate the wetting phase relative permeability and the Brooks–Corey's solution is proposed to calculate the nonwetting phase relative permeability. However, the sum of the wetting and nonwetting phase relative permeability is equal to one when using Purcell's solution and the sum of irreducible water saturation and residual oil saturation are equal to zero when using either of the two solutions. These may not be true in tight oil reservoirs (Yang and Wei 2007). In addition,

no existing solutions take wettability into account. In this paper, two correlations to estimate boundary layer thickness and permeability reduction, respectively, are obtained. Then by incorporating the two correlations into the Purcell's solution, an improved solution considering the effect of boundary layer, Jamin and wettability is developed.

Assumptions

The major assumptions are as follows: (1) This improved solution is based on the bundle of capillary tubes. The bundle of capillary tubes consists of tubes whose radius ranges from $r_{\min}$ to $r_{\max}$. The length of all tubes is then same. (2) Fluids are immiscible. (3) Rocks and fluids are slightly incompressible. (4) Fluids have a constant viscosity. (5) When water displaces oil, if water is the wetting phase and the displacement pressure is p , the tubes whose radius is less than r are filled with water and the others are filled with oil. The relation between p and r is shown Eq. 15.

$$p = \frac{2\sigma \cos \theta}{r} \quad (15)$$

where p is displacement pressure, MPa; σ is interface tension, mN/m; θ is contact angle; r is the radius whose capillary pressure is p , μm .

When water displaces oil, if water is the nonwetting phase and the displacement pressure is p , the tubes whose radius is less than r are filled with oil and the others are filled with water. (6) A tube only contains one fluid. (7) The flow behavior in bundle of capillary tubes obeys Darcy's equation and that in a capillary tube obeys Poiseuille's equation.

Estimation of irreducible water saturation and residual oil saturation

Estimation of boundary layer saturation

Figure 1 shows the results of microtube experiments to study the effect of viscosity on boundary layer thickness (Li and He 2005). Figure 2 shows the results of microtube experiments to study the effect of pressure gradient on boundary layer thickness (Li et al. 2011). Figure 3 shows the results of microtube experiments to study the effect of tube radius on boundary layer thickness (Li et al. 2011). It is known that the ratio of boundary layer thickness to tube thickness has an exponential relation with tube radius, a power relation with pressure gradient and a linear relation with viscosity. However, when the

pressure gradient is larger than 1 MPa/m, the ratio of boundary layer thickness to tube radius stays constant. Figure 4 shows the maximal throat radius from 259 mercury injection experiments and shows that the maximal throat radius is less than 10 μm in tight oil reservoirs. However, the tube radius in the experiments to study the effect of viscosity does not belong to tight oil reservoirs. Thus, experimental data whose tube radius is less than 10 was used to obtain a correlation. Notice that the relation of boundary layer thickness and viscosity is the same when tube radius is less than 10 μm . Therefore, boundary layer thickness and viscosity is simplified to linear relation with intercept boundary layer thickness is 0, when viscosity is 0. Based on the experimental results, we obtain a new correlation (Eq. 16) to estimate boundary layer thickness.

$$h = r \cdot 0.25763e^{-0.261r}(\nabla p)^{-0.419} \cdot \mu \quad (16)$$

Fig. 1 Boundary layer thickness vs. viscosity obtained from experiments. h is boundary layer thickness, μm ; μ is viscosity, $\text{mPa} \cdot \text{s}$

Fig. 2 Ratio of boundary layer thickness to tube radius vs. pressure gradient from experiments. r is tube radius, μm ; ∇p is pressure gradient, MPa/m

Fig. 3 Ratio of boundary layer thickness to tube radius vs. tube radius from experiments

Fig. 4 The maximal throat radius from mercury injection experiments. $r_{\max}$ is the maximal throat radius, μm . K is gas permeability, $\times 10^{-3} \mu\text{m}^2$

where h is boundary layer thickness, μm ; r is tube radius, μm ; ∇p is pressure gradient, MPa/m ; μ is viscosity, mPa s .

Equation 16 shows that the boundary thickness is simply correlated with pressure gradient and viscosity. However, the relation between boundary layer thickness and tube radius is complex. The derivation of Eq. 16 with respect to r is

$$\frac{dh}{dr} = 0.25763(\nabla p)^{-0.419} \cdot \mu \cdot e^{-0.261r}(1 - 0.261r), \quad (17)$$

when r is 3.8 μm , the boundary layer thickness achieves a maximum (Fig. 5).

Because pressure gradient in tight oil reservoirs is larger than 1 MPa/m , the boundary layer thickness is the value when the pressure gradient is 1 MPa/m . The boundary layer saturation is Eq. 18.

$$S_b = \sum_{r_{\min}}^{r_{\max}} \left[\left(\frac{h_i}{r_i} \right)^2 \cdot \alpha_i \right], \quad (18)$$

where S_b is boundary layer saturation, dimensionless; $r_{\min}$ is the minimal throat radius, μm ; $r_{\max}$ is the maximal throat

Fig. 5 Boundary layer thickness vs. tube radius from Eq. 16

radius, μm ; h_i is boundary layer thickness in the throat whose radius is r_i , μm ; α_i is proportion of throats whose radius is r_i , dimensionless.

Estimation of stagnant pore saturation

Fluid saturation in stagnant pores is calculated according to Eq. 19,

$$S_s = 1 - S_{H_{g\max}}, \quad (19)$$

where S_s is fluid saturation in stagnant pores, dimensionless; $S_{H_{g\max}}$ is the maximal mercury saturation, dimensionless.

Discrimination of irreducible water saturation and residual oil saturation

Irreducible water and residual oil consist of fluid in stagnant pores and boundary layer fluid. According to the classification standard of wettability in Table 1, assuming that the wettability index is I , then the ratio of irreducible water saturation to residual oil saturation is as follows.

$$S_{or}^* : S_{wc}^* = (1 - I) : (I + 1), \quad (20)$$

where S_{or}^* is residual oil saturation ratio, dimensionless; S_{wc}^* is irreducible water saturation ratio, dimensionless; I is wettability index, dimensionless.

The boundary layer thickness of oil and water is calculated according to Eq. 16. Oil saturation and water saturation in boundary layer is calculated according to Eqs. 21 and 22, respectively. Because the pressure gradient in tight oil reservoirs is larger than 1 MPa/m , the boundary layer thickness is the value when the pressure gradient is 1 MPa/m .

Table 1 Classification standard of wettability

Wettability	Wettability index
Strongly water-wet	0.7–1
Water-wet	0.3–0.7
Slightly water-wet	0.1–0.3
Middle	–0.1 to 0.1
Slightly oil-wet	–0.3 to –0.1
Oil-wet	–0.7 to –0.3
Strongly oil-wet	–1 to –0.7

$$S_{or}^1 = \frac{S_{or}^*}{S_{or}^* + S_{wc}^*} \cdot \sum_{r_{min}}^{r_{max}} \left[\left(\frac{h_{oi}}{r_i} \right)^2 \cdot \alpha_i \right] \quad (21)$$

$$S_{wc}^1 = \frac{S_{wc}^*}{S_{or}^* + S_{wc}^*} \cdot \sum_{r_{min}}^{r_{max}} \left[\left(\frac{h_{wi}}{r_i} \right)^2 \cdot \alpha_i \right], \quad (22)$$

where S_{or}^1 is oil saturation in boundary layer, dimensionless; S_{wc}^1 is water saturation in boundary layer, dimensionless; h_{oi} is boundary layer thickness of oil in the throat whose radius is r_i , μm ; h_{wi} is boundary layer thickness of water in the throat whose radius is r_i , μm .

The residual oil saturation and irreducible water saturation in stagnant pores are calculated according to Eqs. 23 and 24.

$$S_{or}^2 = \frac{S_{or}^*}{S_{or}^* + S_{wc}^*} \cdot S_s \quad (23)$$

$$S_{wc}^2 = \frac{S_{wc}^*}{S_{or}^* + S_{wc}^*} \cdot S_s, \quad (24)$$

where S_{or}^2 is oil saturation in stagnant pores, dimensionless; S_{wc}^2 is water saturation in stagnant pores, dimensionless.

Therefore, the residual oil saturation and irreducible water saturation are obtained in Eqs. 25 and 26.

$$S_{or} = S_{or}^1 + S_{or}^2 \quad (25)$$

$$S_{wc} = S_{wc}^1 + S_{wc}^2, \quad (26)$$

where S_{or} is residual oil saturation, dimensionless; S_{wc} is irreducible water saturation, dimensionless.

Fraction of permeability reduction

Fraction of permeability reduction at co-seepage point

Figure 6 is the experimental results to study the effect of Jamin (Tang et al. 2009) and shows that as gas permeability decreases, the maximal fraction of permeability reduction increases. When gas permeability is less than $0.1 \times 10^{-3} \mu\text{m}^2$, the maximal fraction of permeability reduction changes more rapidly. Then, we obtain a

Fig. 6 The maximal fraction of permeability reduction vs. gas permeability from Jamin experiments. R is fraction of permeability reduction, dimensionless

correlation to estimate the maximal fraction of permeability reduction (Eq. 27).

$$R_{max} = (-4.7544 \ln K + 77.831)/100, \quad (27)$$

where R_{max} is maximal fraction of permeability reduction, dimensionless; K is gas permeability, $\times 10^{-3} \mu\text{m}^2$.

When oil and water flow together, the sum of effective permeability of oil and water is less than the absolute permeability due to Jamin's effect. The maximal fraction of permeability reduction appears at the co-seepage point. Therefore, it can be obtained according to Eq. 27.

Fraction of permeability reduction at residual oil saturation point

Water relative permeability at residual oil saturation point cannot reach 1 due to irreducible Jamin's effect. Because ejection efficiency shows the result that capillary traps mercury, the value of 100 minus ejection efficiency can reflect the fraction of permeability reduction at residual oil saturation point.

$$R_{or} = 1 - W_e \quad (28)$$

where R_{or} is fraction of permeability reduction at the reducible oil saturation point, dimensionless; W_e is ejection efficiency, dimensionless.

Fraction of permeability reduction vs. water saturation

According to preceding part, the fraction of permeability reduction at co-seepage point and residual oil point is obtained. And the fraction of permeability reduction at irreducible water saturation is 1. Therefore, the relation of the permeability reduction fraction and water saturation

Fig. 7 Fraction of permeability reduction vs. water saturation from cores with different permeability. The cores are taken from the tight oil reservoir in Ordos Basin

Table 2 Fitting coefficient of quadratic form: Ten relative permeability curves

No.	Porosity, %	Permeability, $\times 10^{-3} \mu\text{m}^2$	Fitting coefficient of quadratic	
			$S_{wi} < S_w < S_{co}$	$S_{co} < S_w < 1 - S_{or}$
1	10.8	0.53	1	0.9981
2	10.1	0.46	0.9999	0.9996
3	10.9	0.403	1	0.9997
4	11.7	0.384	0.9999	0.9998
5	11.1	0.383	1	0.998
6	10.7	0.283	0.9999	0.9976
7	13.5	0.254	0.9999	0.9978
8	13.3	0.205	1	0.9947
9	9.4	0.131	0.9996	0.9678
10	11.8	0.122	1	0.9991

can be obtained. Figure 7 is the results of relative permeability experiments of 10 cores from the tight oil reservoir in Ordos Basin. Table 2 shows the fitting coefficient of quadratic. When the water saturation ranges from irreducible water saturation point to the co-seepage point and the water saturation ranges from the co-seepage point to reducible oil saturation, the relations of $1-R$ and water saturation are both quadratic. The expression of the fraction of permeability reduction is Eq. 29. The constant can be determined according to Eq. 30.

$$1 - R = \begin{cases} aS_w^2 + bS_w + c, & S_{wc} \leq S_w < S_{co} \\ dS_w^2 + eS_w + f, & S_{co} \leq S_w \leq 1 - S_{or} \end{cases}, \quad (29)$$

where R is fraction of permeability reduction, dimensionless; S_w is the water saturation, dimensionless; S_{co} is the water saturation at the co-seepage point, dimensionless; a, b, c, d, e, f are the constant.

$$\begin{cases} aS_{wi}^b = 1 \\ aS_{co}^b = 1 - R_{\max} \\ cS_{co}^2 + dS_{co} + e = 1 - R_{\max} \\ cS_{or}^2 + dS_{or} + e = 1 - R_{or} \end{cases}. \quad (30)$$

Relative permeability estimation

Due to the effect of boundary layer, the effective flow radius is less than the tube radius (Eq. 31).

$$r' = r - h \quad (31)$$

where r' is effective flow radius, μm .

Mercury–gas capillary transforms into oil–water capillary according to Eq. 32.

$$p'_c = \frac{2\sigma_{ow} \cos \theta_{ow}}{r'}, \quad (32)$$

where p'_c is capillary pressure considering boundary layer, MPa; σ_{ow} is interface tension, mN/m; θ_{ow} is contact angle of oil and water.

Because Purcell's solution coincides with the experimental results best (Li and Horne 2002), we develop the improved solution based on Purcell's solution. For water-wet reservoir ($I > 0.1$), when water displaces oil, the capillary is the power. As the throat radius decreases, the capillary increases. As a result, water entries into the tinier throats first. Therefore, when the pressure is p_i , the throats whose radius are less than r_i are filled with water. Therefore, Eqs. 33 and 34 are the relative permeability of oil and water accounting for the effect of the boundary layer.

$$K_{rw}(S_{wi}) = \frac{\int_{S_{wc}}^{S_{wi}} \frac{1}{(p'_c)^2} dS}{\int_{S_{wc}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS} \quad (33)$$

$$K_{ro}(S_{wi}) = \frac{\int_{S_{wi}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS}{\int_{S_{wc}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS}, \quad (34)$$

where K_{ro} is relative permeability of oil, dimensionless; K_{rw} is relative permeability of water, dimensionless; S_{wi} is water saturation when water entries into the throat whose radius is r_i , dimensionless.

Equations 35 and 36 are the equations to calculate relative permeability considering the effect of boundary layer and Jamin.

$$K_{rw}(S_{wi}) = \frac{\int_{S_{wc}}^{S_{wi}} \frac{1}{(p'_c)^2} dS}{\int_{S_{wc}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS} \cdot \left(1 - \frac{R}{100}\right) \quad (35)$$

$$K_{ro}(S_{wi}) = \frac{\int_{S_{wi}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS}{\int_{S_{wc}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS} \cdot \left(1 - \frac{R}{100}\right). \quad (36)$$

For oil-wet reservoir ($I < 0.1$), when water displaces oil, the capillary is the resistance. As the throat radius decreases, the capillary increases. As a result, water enters into the larger throats first. Therefore when the pressure is p_i , the throats whose radius are larger than r_i are filled with water. Therefore, Eqs. 37 and 38 are the relative permeability of oil and water.

$$K_{rw}(S_{wi}) = \frac{\int_{S_{wc}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS}{\int_{S_{wc}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS} \cdot \left(1 - \frac{R}{100}\right) \quad (37)$$

$$K_{ro}(S_{wi}) = \frac{\int_{S_{wc}}^{S_{wi}} \frac{1}{(p'_c)^2} dS}{\int_{S_{wc}}^{1-S_{or}} \frac{1}{(p'_c)^2} dS} \cdot \left(1 - \frac{R}{100}\right). \quad (38)$$

Results and discussion

Figure 8 shows the relative permeability curves whose maximal throat radius are 3.1, 1.9 and 1.2 μm , respectively, from the improved solution with the 0.25- μm median throat radius. Figure 9 shows the relative permeability curves whose median throat radius are 0.42, 0.25 and 0.14 μm with the 1.9- μm maximal throat radius. The maximal mercury saturation is 76.6 % and the ejection efficiency is 58 %. The wettability is slightly water-wet ($I = 0.2$). When water displaces oil, capillary pressure is the power if the wettability phase is water, so water enters into the tinier throats first. In addition, the contribution of large throats to permeability is larger than that of tiny throats. As a result, relative permeability changes more and more rapidly as water saturation increases. As the maximal throat radius becomes larger, relative permeability changes far slower at first and far faster when water saturation is high. However, as the median throat radius becomes larger, relative permeability changes less slower at first and less faster when water saturation is high. In addition, as the maximal throat radius and the median throat radius increase, the saturation of boundary fluid decreases. As a result, two-phase region extends. Therefore, the crossing point of curves with different maximal throat radii appears when water saturation is low. However, the crossing point of curves with different median throat radii appears when water saturation is high. Because the permeability with larger maximal throat radius and the median throat radius is higher, the maximal fraction of permeability reduction is smaller. Therefore, the relative permeability at co-seepage point is higher. When the maximal throat radius is larger, the throat radius is larger at co-seepage point. Meanwhile, the proportion of tiny throats is the same. Therefore, the water saturation at co-seepage point is larger. When the median throat radius is larger, the throat radius is larger at co-seepage point. However, the proportion of tiny throats is

Fig. 8 Relative permeability curves of different maximal throat radius

Fig. 9 Relative permeability curves of different median throat radius

the lower when median throat radius is larger. Therefore, the water saturation at co-seepage point is smaller. Due to the same ejection efficiency, water relative permeability at residual oil point is the same.

Figure 10 shows the relative permeability curves with different wettabilities from the improved solution (Table 3). The maximal throat radius is 1.9 μm and the median throat radius is 0.25 μm . The maximal mercury saturation is 76.6 % and ejection efficiency is 58 %. Table 3 shows that the two-phase region extends when wettability index increases, because boundary layer thickness of oil is larger than that of water. Figure 7 shows that as wettability index decreases, relative permeability curve moves right entirely. Differing from the case of water-wet, water enters larger throats first because capillary pressure is the resistance in the case of oil-wet. As a result, relative permeability changes more and more slower as water saturation increases. When wettability index is 0, the solution cannot be used. Because it cannot tell whether capillary is power or resistance, it cannot tell whether water enters tinier throats or larger throats first.

Fig. 10 Relative permeability curves of different wettability indexes

Table 3 Two-phase region of different wettabilities

Wettability	Wettability index	Two-phase region (%)
Strongly water-wet	0.8	44.2
Water-wet	0.5	42.4
Slightly water-wet	0.2	40.5
Slightly oil-wet	−0.2	38
Oil-wet	−0.5	36.1
Strongly oil-wet	−0.8	34.2

Verification

Relative permeability experiments and mercury injection experiments were conducted, respectively, using two cores from the tight oil reservoir in Chang 7 Member, Yanchang Formation, Ordos Basin. Porosity and permeability of the first core are 12.7 % and $0.67 \times 10^{-3} \mu\text{m}^2$, respectively. Those of the second core are 14 % and $0.25 \times 10^{-3} \mu\text{m}^2$, respectively. The wettability indexes of No. 1 and No. 2 core are assumed to be -0.2 and 0.04 , respectively. The wetting phase relative permeability of the existing solution is Purcell's solution and the nonwetting phase relative permeability of the existing solution is Brooks–Corey's solution. Compared to the experimental results (Figs. 11, 12), it is found that the improved solution is more accurate and reliable. However, water cut usually does not reach 100 %, 10PV, when experiments end, therefore the relative permeability at residual oil saturation from improved solution is higher than that from experiments. The water enters the largest throats at last when the reservoirs are water-wetting, while it enters the tiniest throats at last when the reservoirs are oil-wetting. In addition, the contribution to permeability of the largest throats is far larger than that of the tiniest throats. Therefore, the difference between the results of experiments and those of improved solution when reservoir is water-wetting is larger than that when reservoir is oil-wetting.

Fig. 11 Experimental and calculated results of the first core

Fig. 12 Experimental and calculated results of the second core

Application

Wang (2010) proposed that throat distribution of tight oil reservoirs in Changqing Field is different from that in Daqing Field although their permeability is the same. Figure 13 is the capillary pressure curve of the tight oil reservoir in Changqing Field and that in Daqing Field and their permeability is $0.6 \times 10^{-3} \mu\text{m}^2$. It is found that the threshold pressure and median pressure of the tight oil reservoir in Daqing Field is less than that in Changqing Field. This indicates that the throat distribution of the tight oil reservoir in Daqing Field is narrower and more homogeneous than that in Changqing Field. In addition, the ejection efficiency of the tight oil reservoir in Daqing Field is 23.4 % and that in Changqing Field is 38.7 %. The wettability indexes of the reservoirs are 0.2. Then the relative permeability is calculated using the improved solution. Figure 14 shows the results. It is found that the relative permeability is the same at co-seepage point, because the permeability of the reservoirs is the same. The water-phase relative permeability of the tight oil reservoir in Daqing Field is smaller than that in Changqing Field

Fig. 13 The capillary pressure curves of the tight oil reservoirs in Daqing Field and that in Changqing Field. The permeability of the reservoirs is $0.6 \times 10^{-3} \mu\text{m}^2$. p_c is capillary pressure, MPa; S_{Hg} is mercury saturation, %

Fig. 14 The results of the improved solution applied to the tight oil reservoirs in Daqing Field and Changqing Field. The permeability of the reservoirs is $0.6 \times 10^{-3} \mu\text{m}^2$

because the ejection efficiency of the tight oil reservoir in Daqing Field is smaller than that in Changqing Field. The reservoirs are slightly water-wet and the capillary pressure is power. Water enters tinier throats first. The contribution to permeability of tinier throats of the tight oil reservoir in Daqing Field is larger than that in Changqing Field, while the contribution to permeability of larger throats of the tight oil reservoir in Daqing Field is smaller than that in Changqing Field. Therefore, the relative permeability of the tight oil reservoir in Daqing Field changes faster than that in Changqing Field when water saturation is low while that in Daqing Field changes slower than that in Changqing Field when water saturation is high.

The effect of the throat distribution on the productivity is studied using the Eclipse software. The numerical model

Table 4 Basic parameters of the numerical model

Parameters	
Size of grids (m)	10×10
Porosity (%)	12
Permeability ($\times 10^{-3} \mu\text{m}^2$)	0.6
Oil saturation (%)	50
Reservoir thickness (m)	15
Original formation pressure (MPa)	15
Reservoir capacity	0.91
Oil viscosity (mPa s)	1.5
Oil density (g/cm^3)	0.83

Fig. 15 The productivity of the tight oil reservoirs in Daqing Field and Changqing Field. The permeability of the reservoirs is $0.6 \times 10^{-3} \mu\text{m}^2$. Q_o is productivity of oil, m^3/days

contains a production well and a injection well. The distance of these two wells is 480 m. This is the same with Changqing Field. The basic parameters of the numerical model are in Table 4.

Figures 15 and 16 are the results of the numerical models. It is found that the development effect is different although the permeability of the tight oil reservoirs in Daqing Field and Changqing Field is the same. The productivity of the tight oil reservoir in Daqing Field is less than that in Changqing Field at first, because the oil-phase relative permeability of the tight oil reservoir in Daqing Field is less than that in Changqing Field. However, the productivity decreases and water cut increases suddenly because the injected water reaches the production well. The inhomogeneity of the tight oil reservoir in Daqing Field is less serious than that in Changqing Field, because the throat distribution of the tight oil reservoir in Daqing Field is narrower than that in Changqing Field. This leads to a longer breakthrough time in the tight oil reservoir of Daqing Field.

Fig. 16 The water cut of the tight oil reservoirs in Daqing Field and Changqing Field. The permeability of the reservoirs is $0.6 \times 10^{-3} \mu\text{m}^2$

Conclusions

- (1) When the maximal throat radius and median throat radius become larger, the two-phase region extends and relative permeability at co-seepage point and residual oil saturation point are higher. The location of co-seepage point moves right as the maximal throat radius increases, but it is on the contrary to median throat radius.
- (2) As the wettability index increases, the relative permeability moves right entirely and two-phase region extends.
- (3) In this paper, an improved solution is presented to calculate the relative permeability and it is only applied to tight oil reservoirs. The improved solution considers the effect of boundary layer, Jamin and wettability. Compared to the experimental results, it is found that the improved solution has a higher accuracy. The boundary layer and Jamin have a significant effect on flow behavior.
- (4) Because the throat distribution of the tight oil reservoir in Daqing Field is narrower than that in Changqing Field, the productivity of the tight oil reservoir in Daqing Field is less than that in Changqing Field at first. However, the breakthrough

time of the tight oil reservoir in Daqing Field is longer than that in Changqing Field.

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