Asymptotic model for velocity dip position in open channels

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Abstract An empirical model is proposed to predict the velocity dip position at the central section of open channels. The model is fitted based on asymptotic matching technique and validated by using a wide range of aspect ratios (channel width/flow depth) from 0.155 to 15. The matching approach, which relies on dividing the trend into smaller segments that can be combined into an overall relation, employs regression technique and thus warrants the best-fit accuracy results. The obtained model satisfies the upper and lower bounds of dip positions equal to 0.5 and 1, respectively. A comparison with other formulas widely reported in the literature is provided. The model is also applied to predict Reynolds shear stress and velocity distribution in open channels. This model will help extending our ability for analyzing velocity field in open channels under different flow and boundary conditions.

Keywords Velocity distribution · Secondary current · Asymptotic matching · Aspect ratio · Sidewall effect

Abbreviations

Ar	Aspect ratio (—)
b	Channel width (m)
b_0, b_1	Parameters (—)
h	Flow depth (m)
m	Matching constant (—)
MASE	Mean absolute standard error (—)
N	Total number of data points (—)
r	Average percentage relative error (—)

RMSE	Root mean square error (—)
s_1	Sum of squared relative error (—)
s_2	Sum of logarithmic deviation error (—)
u	Mean longitudinal velocity (m/s)
u'	Longitudinal velocity fluctuation (m/s)
u^*	Shear velocity (m/s)
v'	Vertical velocity fluctuation (m/s)
x	Longitudinal direction (m)
y	Vertical direction (m)
y_d	Location of dip position (m)
z	Lateral direction (m)
Y_i	Asymptotes (—)
λ, β	Parameters (—)
κ	Von Karman coefficient (—)
Π	Coles' wake parameter (—)
ξ_d	Dimensionless dip position (—)
ξ_0	Zero velocity bed level (m)
$\xi_{d,c}$	Computed value of ξ_d (—)
$\xi_{d,o}$	Observed value of ξ_d (—)
φ_i	Matching functions (—)

Introduction

The vertical water velocity distribution in open channels is considered three-dimensional due to the presence of large-scale circular secondary currents. In many cases, the location of the maximum streamwise velocity appears below the free surface (Francis 1878; Stearns 1883; Murphy 1904; Gibson 1909; Vanoni 1946; Nezu and Nakagawa 1993). An accurate estimation of the velocity dip position with respect to channel bottom is required for many practical applications including the assessment of sediment

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transport rates (Einstein 1950) and contamination (Guo and Julien 2008), and the precise definition of hydrodynamic flow conditions in urban drainage sewers (Lee and Julien 2006; Guo et al. 2015).

Nezu and Rodi (1985) found that the velocity dip position in open channels is related to the aspect ratio Ar , which is the ratio of channel width b to water depth h . It is evident that at the central section of open channels the velocity dip phenomenon occurs when sidewall shear stresses dominate for $Ar < 5$. However, when $Ar > 5$, channels exhibit no velocity dip, i.e. the maximum water velocity appears at the top surface (Stearns 1883; Montes and Ippen 1973).

Predicting the velocity dip position at the central section of open channels within a wide range of aspect ratios is a difficult task. Several researchers provided empirical models to predict it. Based on data obtained from nine literatures, Wang et al. (2001) proposed an empirical relation applicable to narrow channels. Yang et al. (2004) proposed a model valid for aspect ratios from 4.1 to 15 showing that even for large aspect ratios the velocity dip occurs very close to the channel sidewall region. Bonakdari et al. (2008) showed that both models overestimate experimental data when the aspect ratio is small and suggested a new relation. Later, Guo (2013) studied vertical water velocity profiles in smooth rectangular channels and proved that the velocity dip position decreases exponentially from the water surface to the middle depth, as the aspect ratio is varied from infinity to zero. Guo (2013) suggested that the centreline vertical shear stress distribution is expressed as

$$\tau = \tau_0(1 - \xi_d) - \tau_1\xi \quad (1)$$

where $\tau_1 = \alpha\tau_0$ is called the ‘apparent’ shear stress at the free surface. Like Reynolds shear stress, τ_1 only functions like a shear stress although it is essentially a momentum transfer by secondary currents near the water surface (Guo 2013). The parameter α has the range $0 \leq \alpha \leq 1$. If the condition $\tau = 0$ at $\xi = \xi_d$, is imposed into Eq. (1), and from the bounds of parameter α , one can obtain that $0.5 \leq \xi_d \leq 1$. He also found that the two models, besides that of Bonakdari et al. (2008), do not satisfy the following boundary conditions: $\xi_d \rightarrow 0.5$ when $Ar \rightarrow 0$ and $\xi_d \rightarrow 1$ when $Ar \rightarrow \infty$, where ξ_d denotes dimensionless distance of dip-position from channel bed. Pu (2013) proposed an empirical model for smooth and rough channel beds. Only those models for Guo (2013) and Pu (2013) satisfied asymptotically the abovementioned boundary conditions. Apart from these empirical model, some researchers suggested numerical methods and entropy based models to predict it. Guo and Julien (2008) envisaged that near the free surface, velocity profile is likely parabolic type and therefore a parabola can be fitted to find the dip-position.

Recently, Kundu (2017) suggested an entropy based approach to predict the dip-phenomena over entire cross section of open channels. Most of these previous models are tested with limited number of data sets and, therefore, may not produce good results for other data sets. Therefore, a more appropriate model is required for a wide range of data sets. In this study, we develop an empirical model based on the technique of asymptotic matching method proposed by Almedeij (2008, 2009, 2010) with a large number of data sets. The method has been applied to model the shields diagram and drag coefficient for large Reynolds number and has proven to provide good results.

This study suggests a new empirical formula to compute the velocity dip position in open channels. The formula will be validated by using field and experimental data covering a wide range of aspect ratios and compared with models found in the literature. A detailed accuracy analysis will be conducted to evaluate the results. The best applicability of the model will be discussed in estimating parameters for prediction of Reynolds shear stress and mean velocity distribution in open channels.

Model development

Asymptotic matching technique by Almedeij (2008, 2009, 2010)

A model for velocity dip position at central section of open channels can be developed by using asymptotic matching technique. The approach is based on dividing the possible entire range of the dip-position data into smaller segments not necessarily of same length. In each of the formed region, straight lines are fitted by linear regression, and then the segments for each region are combined together asymptotically into an overall relation (Almedeij 2008, 2009, 2010).

The matching procedure is described in Fig. 1a. In this matching method two possibilities may arise. These two cases are presented with simple schematic diagrams, each composed of linear segments Y_1 and Y_2 where each of them is expressed as $Y = b_0X + b_1$; where b_0 and b_1 are slope and intercept, respectively. For the case on the left side figure, the b_0 value increases from Y_1 to Y_2 , and these two segments can be considered to have Monotone Increasing Slopes (MIS) with respect to x -axis. Similarly, for the other case on the right side figure the b_0 value decreases from Y_1 to Y_2 , and hence these segments have Monotone decreasing slopes (MDS). For the case of MIS, Almedeij (2008) suggested that the linear asymptotes can be match asymptotically using the function $\varphi = Y_1^m + Y_2^m$, with the entire relation being $Y = (\varphi)^{1/m}$; where m is a matching constant. If m value is large enough, then this expression

Fig. 1 Schematic diagram of asymptotic matching method. *Solid line* represents the final model and the *dashed lines* represent the asymptotes with linear segments: **a** a series of two segments of Y_1 and Y_2 with the *dash-dot lines* being the entire relation of Y_1 and Y_2 fitted by using the corresponding exponent m ; **b** a series of four linear segments of monotone increasing slopes (MIS)

will automatically match the upper parts of the intersected asymptotes of Y_1 and Y_2 within the corresponding ranges of X . In general, the larger the m value, the smoother is the matching at the joints of the segments. However, the value of φ becomes excessively large for a very large m value that cannot be calculated numerically, and the model fails to work. Similarly, for the MDS, the matching attempt is attained rather by the function $\varphi = 1/(Y_1^{-m} + Y_2^{-m})$ with $Y = (\varphi)^{1/m}$. Similarly, for large values of m , this expression will satisfy the lower parts of the intersected asymptotes.

For data sets having a large range, it can be divided into small parts. In each parts, the above procedure can be applied repeatedly. An example for a series of four segments of MIS is shown in Fig. 1b. In general, a series of segments of MIS is collected by the function

$$\varphi = Y_1^m + Y_2^m + \dots + Y_k^m \quad (2)$$

and for MDS by

$$\varphi = \frac{1}{Y_1^{-m} + Y_2^{-m} + \dots + Y_k^{-m}} \quad (3)$$

where k is the total number of segments used in the series. For more details readers can go through Almedeij (2008, 2009, 2010).

Proposed model

Twenty six datasets obtained from the literature are considered here to examine the distribution pattern of velocity dip position in open channels. Details of data are given in Table 1, which shows that the range of aspect ratios is from 0.155 to 15. For channel with oval shape (sewer shape), the aspect ratio is calculated by taking the ratio of width of the free surface to flow depth (for Larrarte (2006) data set). Figure 2 plots the relationship between the aspect ratio and velocity dip position. Lower and upper boundary limits are apparent for dip positions equal to 0.5 and 1, respectively. In between, dip position data increase as aspect ratio becomes larger. It is worth noting that the data near the lower and upper boundaries match the limits asymptotically. The existence of the lower and upper limits has also

Table 1 Details of selected datasets in steady and uniform flow

No.	Source	Type	Channel cross section	Bed roughness	Flow depth (cm)	Aspect ratio	Data no.
1	Murphy (1904)	Experimental	Rectangular	Smooth	178.2–285.2	0.155–1.35	5
2	Gibson (1909)	Experimental	Rectangular	Smooth	14–57.3	0.256–2	5
3	Vanoni (1946)	Experimental	Rectangular	Rough	7.1–16.9	4.997–11.895	28
4	NHRI (1957)	Experimental	Rectangular	Smooth	8–29.27	4.1–15	24
5	Guy et al. (1966)	Experimental	Rectangular	–	61–243.84	7.94–8.54	6
6	Rajaratnam and Muralidhar (1969)	Experimental	Rectangular	Smooth	–	3.283–10.839	4
7	Montes and Ippen (1973)	Experimental	Rectangular	Smooth	–	5.505–7.656	15
8	Knight and Macdonald (1979)	Experimental	Rectangular	Rough	–	1.012–1.012	1
9	Sarma et al. (1983)	Experimental	Rectangular	Smooth	3.8–30.5	2.024–7.984	3
10	Zippe and Graf (1983)	Experimental	Rectangular	Rough	12.3–16.8	6.173–7.634	5
11	Hu (1985)	Experimental	Rectangular	Rough	–	1.664–11.132	13
12	Nezu and Rodi (1985)	Experimental	Rectangular	Smooth	10–60	1.009–6.001	3
13	Coleman (1986)	Experimental	Rectangular	Smooth	16.7–17.2	2.058–2.132	7
14	Nezu and Rodi (1986)	Experimental	Rectangular	Smooth	3.6–10.5	0.189–5.42	68
15	Cardoso et al. (1989)	Experimental	Rectangular	Smooth	27.2–42.8	4.673–7.353	5
16	Wang and Qian (1989)	Experimental	Rectangular	Smooth	8–10	3–3.75	3
17	Tominaga et al. (1989)	Experimental	Rectangular	Smooth	2.5–10	2.005–7.995	3
18	Wang and Fu (1991)	Experimental	Rectangular	–	–	4.166–4.291	4
19	Kironoto and Graf (1994)	Experimental	Rectangular	Rough	8.7–28.9	2.069–6.897	8
20	Song and Graf (1994)	Experimental	Rectangular	Rough	13.08–20	3.004–4.588	12
21	Wang and An (1994)	Experimental	Rectangular	Rough	7.2–7.8	3.797–4.167	5
22	Yang (1996)	Experimental	Rectangular	Smooth	11	4.47–9.84	88
23	Graf and Cellino (2002)	Experimental	Rectangular	Smooth	12	5	17
24	Larrarte (2006)	Field	Oval with a bench	Smooth	56–122	1.756–2.925	25
25	Ghoshal (2004)	Experimental	Rectangular	Rough	35	1.43	1
26	Yan et al. (2011)	Experimental	Rectangular	Smooth	6–18	2.306–6.999	5

been observed by others such as the experimental work of Hu and Hui (1995).

Figure 2 also provides possible linear segments of Y_1 , Y_2 and Y_3 that can be used to match the velocity dip position data. The asymptotes Y_1 and Y_3 were chosen to satisfy the lower and upper boundary conditions. From the selected data in Table 1, the highest value of aspect ratio is obtained as 15. From the pattern of data in Fig. 4, one can observe that for $Ar > 10$, dip-position vanishes which indicates that after this value, dip-position becomes independent of aspect ratio of channel. Therefore, the highest value of aspect ratio is chosen as 15 as from data. This value is chosen as fixed for the present study. Initially, the entire range of aspect ratios from 0 to 15 is divided into two regions of $0 < Ar < 5$ and $5 < Ar < 15$. In the first region, data are fitted by two linear segments as $Y_1 = 0.48$ and $Y_2 = a_0 Ar + b_0$, where $a_0 (> 0)$ and b_0 are constants to be determined from data. The asymptote Y_1 is chosen in such a way that the final developed model satisfies the lower asymptotic boundary condition. In the second region, data are fitted with only one linear segment as $Y_3 = 1$. The

segments Y_1 and Y_2 can be combined by using the expression $\varphi_1 = (Y_1^m + Y_2^m)^{1/m}$ for MIS. Both φ_1 and Y_3 can then be combined by using $\varphi = [1/(\varphi_1^{-m} + Y_3^{-m})]^{1/m}$ for MDS where m is the matching constant. Combining all the asymptotes, the overall model becomes

$$\xi_d = \varphi = \left[\frac{1}{\{0.48^m + (a_0 Ar + b_0)^m\}^{-1} + 1} \right]^{1/m} \quad (4)$$

where $\xi_d (= y_d/h)$ is dimensionless distance of dip position from channel bed, and y_d denotes distance of dip position from channel bed. The values of the parameters $a_0 (> 0)$ and b_0 and m are obtained from the least square method in MATLAB and they are given as $a_0 = 0.0895$, $b_0 = 0.4718$ and $m = 16.09$. The coefficient of regression R^2 for the model is obtained as 0.85. The low value of coefficient of regression is due to the fluctuation of the data points for $Ar \leq 8$. The fluctuation of data may occur due to measurement error, change of roughness of the bottom boundary and due to change of bed elevation at the central section.

Fig. 2 The asymptotes and the final model **Fig. 3** Variation of model with parameters

It can be observed from the model that it contains three parameters a_0 (> 0) and b_0 and m . The variations of the model with these parameters are shown in Fig. 3. In

Fig. 3a the variation with a_0 for four different values of it are shown. Similarly in Fig. 3b and in Fig. 3c the variation for three different values of b_0 and m are shown

Table 2 Velocity dip position formulas and flow characteristics

No.	Source	Equation	Applicability	Boundary conditions	Prediction accuracy			
					MASE	r (%)	s ₁	s ₂ RMSE
1	This study	$\xi_d = \left[\frac{1}{\left\{ \frac{0.48(16.09 + (0.089Ar + 0.472)^{16.09})}{2} + 1 \right\}^{-1} + 1} \right]^{1/16.09}$	Wide and narrow channels	Satisfies upper and lower bounds	1.0727	6.9297	2.2948	2.1534 0.0633
6	Wang et al. (2001)	$\xi_d = 0.44 + 0.212 \left(\frac{Ar}{2} \right) + 0.05 \sin \left(\frac{2\pi Ar}{2.6} \right)$	Narrow channels	Not satisfied	1.1449	14.3165	11.1974	7.7521 0.1893
3	Yang et al. (2004)	$\xi_d = \left[1 + 1.3 \exp \left\{ -\frac{Ar}{2} \right\} \right]^{-1}$	Wide and narrow channels	Not satisfied	1.0748	7.2222	2.6328	2.3069 0.0658
5	Bonakdari et al. (2008)	$\xi_d = \frac{42.4 + Ar^{1.2}}{94.7 + Ar^{1.2}}$	Wide and narrow channels with smooth boundary	Not satisfied	1.1161	9.6799	4.5034	5.6513 0.0949
2	Guo (2013)	$\xi_d = \left[1 + \exp \left\{ -\left(\frac{Ar}{\pi} \right)^{1.5} \right\} \right]^{-1}$	Wide and narrow channels	Satisfies upper and lower bounds	1.0765	7.0479	2.2513	2.3429 0.0671
4	Pu (2013)	$\xi_d = \frac{40.1 + Ar^{1.4}}{80.5 + Ar^{1.4}}$	Wide and narrow channels with rough and smooth boundaries	Satisfies upper and lower bounds	1.1007	9.3256	3.7711	3.7918 0.0869

Bold values denote minimum errors of all models

respectively. From the figure it can be seen that model is very sensitive for a_0 and b_0 and comparatively less sensitive for the parameter m . Optimizing the values of parameters, the above-mentioned values are obtained for the best effective result.

Comparison with other formulas

The model can be compared with other empirical formulas reported in the literature. The formulas are those of Wang et al. (2001), Yang et al. (2004), Bonakdari et al. (2008), Guo (2013) and Pu (2013). Table 2 shows the mathematical expressions for the models. The model of Wang et al. (2001), which is applicable to narrow open channels, has a sinusoidal wave component. The models of Bonakdari et al. (2008) and Pu (2013), applicable to wide and narrow open channels, are of sigmoid type. The model of Pu (2013) is a modification of Bonakdari et al. (2008) for rather rough boundaries. The models of Yang et al. (2004) and Guo (2013), applicable to both wide and narrow open channels, are of exponential decay.

All models are presented graphically in Fig. 4. From the figure, it can be seen that the models provide nearly similar patterns. However, only Eq. (4), Guo (2013) and Pu (2013) satisfy the lower and upper bounds.

To assess the models' goodness of fit in an objective manner, the following statistical parameters are chosen: mean absolute standard error (MASE), average percentage relative error ($r\%$), sum of squared relative error (s_1), sum of logarithmic deviation error (s_2), and root mean square error (RMSE). These statistics are expressed correspondingly as,

$$\text{MASE} = \frac{1}{N} \sum_{i=1}^N M_i \quad (5)$$

$$M_i = \begin{cases} \xi_{d,c}/\xi_{d,o} & \text{if } \xi_{d,c} > \xi_{d,o} \\ \xi_{d,o}/\xi_{d,c} & \text{if } \xi_{d,o} > \xi_{d,c} \end{cases}$$

$$r = \frac{1}{N} \sum_{i=1}^N \frac{|\xi_{d,c} - \xi_{d,o}|}{\xi_{d,o}} \times 100 (\%) \quad (6)$$

$$s_1 = \sum_{i=1}^N \frac{(\xi_{d,c} - \xi_{d,o})^2}{\xi_{d,o}^2} \quad (7)$$

$$s_2 = \sum_{i=1}^N (\log |\xi_{d,c}| - \log |\xi_{d,o}|)^2 \quad (8)$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\xi_{d,c} - \xi_{d,o})^2} \quad (9)$$

where N denotes the total number of data points, and $\xi_{d,c}$ and $\xi_{d,o}$ denote the computed and observed values of dimensionless velocity dip position, respectively. The

Fig. 4 Comparison with velocity dip position formula

computed accuracy results are shown in Table 2, suggesting that the proposed model has the best statistical results.

Applications of the model

The appropriate prediction of the velocity-dip-position is required to predict the Reynolds shear stress distribution and the vertical velocity distribution in open channels. This section explores the application of the proposed model and proposes its rational superiority compared to the other models.

Prediction of total shear stress

In turbulent flow through open channels, the total shear stress, composed of viscous shear stress and the Reynolds shear stress, is maximum at the channel bed and it gradually decreases linearly with vertical distance y which is generally modeled as,

$$\tau_{\text{total}} = \mu \frac{du}{dy} - \rho \overline{u'v'} = \rho u_*^2 \left(1 - \frac{y}{h}\right) \quad (10)$$

where u' and v' are fluctuation parts of the fluid velocities along longitudinal and vertical direction respectively, and u_* is the shear velocity and ρ is the fluid density. For wide open channels, the maximum velocity occurs at the free water surface and Eqs. (10) and (11) show that shear stress vanishes there. For a narrow channel, the maximum velocity at the central section always occurs below the water surface. In such a case the shear stress vanishes below the water surface where the maximum velocity occurs. Therefore, for narrow open channels, Eqs. (10) and (11) cannot be used as these do not satisfy the boundary condition $\frac{\overline{u'v'}}{u_*^2} = 0$ at $\xi = \xi_d$ and hence Eqs. (10) and (11) need to be modified. Yang et al. (2004) and Kundu (2015) proposed a modification to this equation as,

$$\begin{aligned} \tau_{\text{total}} &= \mu \frac{du}{dy} - \rho \overline{u'v'} = \rho u_*^2 \left(1 - \frac{y}{h} - \lambda \frac{y}{h}\right) \\ &= \rho u_*^2 (1 - \xi - \lambda \xi) \end{aligned} \quad (11)$$

Fig. 5 Prediction of total shear stress by Eqs. (10) and (11) using velocity-dip formulas

where $\lambda (> 0)$ is the dip-correction parameter. They found that λ changes with the aspect ratio of the channel as,

$$\lambda = \frac{1}{\xi_d} - 1 \quad (12)$$

Kundu (2016) studied the effect of lateral roughness variation on the suspension profile. He analyzed the experimental data of Wang and Cheng (2006) and found that in wide open channels the presence of cellular secondary current affects the Reynolds shear stress which can be modeled as,

$$\frac{\tau_{\text{total}}}{\rho u_*^2} = 1 - \xi - \beta \xi (1 - \xi) \quad (13)$$

where β is a parameter whose value depends on location of dip-position. At two ends of a circular secondary cell, vertical velocity has opposite direction. It is well known that dip phenomenon occurs with the downward direction of the vertical velocity where $v < 0$. Therefore, at one end of a circular secondary cell dip occurs, whereas on the other end vertical velocity $v > 0$ and maximum velocity appear at the free surface. Accordingly, β value can be taken as ξ_d over the length of a circular secondary cell.

In Fig. 5, experimental data of the total shear stress of Imamoto and Ishigaki (1988) are plotted together with the Eqs. (10) and (11). The experiment was carried out by Imamoto and Ishigaki (1988) using a laser Doppler anemometer, the aspect ratio, b/h was 5. In the figure the solid lines are plotted from Eqs. (10) and (11) where λ is calculated from Eq. (12) using the present model of dip-position. From the figure it can be observed

that the proposed model provides good results for dip-position. It can be mentioned here that in the last figure, the model did not agree well with the data points. It happens due to the fluctuation of dip-position data points for $Ar \leq 5$.

Figure 6 shows the validity of the proposed dip position model with the experimental data of Wang and Cheng (2006). The experiments were carried out in a straight rectangular tilting flume 18 m long, 0.6 m wide and 0.6 m deep. The bed comprised five rough and four smooth longitudinal strips, placed in an alternate manner. Each strip was 0.075 m in width except for two sidewall strips, which had half of the original width. The rough strips were prepared with densely packed fine gravel of uniform medium diameter of 2.55 mm. The flow depth was 0.075 m and the aspect ratio Ar was maintained at 8 (wide open channel). The Reynolds shear stress is computed from Eq. (13) where β is computed for each of the models of dip-position. The results in this figure show that the proposed model is effective for predicting the dip-position.

Prediction of vertical velocity profile

Several authors proposed models for vertical velocity distribution in sediment-laden open channel flows (Guo and Julien 2008; Yang et al. (2004); Bonakdari et al. (2008); Kundu and Ghoshal 2012). Kundu (2015) compared those models and found that the vertical velocity is best described by the model of Kundu and Ghoshal (2012) as:

Fig. 6 Prediction of total shear stress by Eq. (13) using proposed dip formula

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \left(\frac{\xi}{\xi_0} \right) + \frac{\lambda}{\kappa} \ln(1 - \xi) + \frac{2\Pi}{\kappa} (3\xi^2 - 2\xi^3) - \frac{4\lambda\Pi}{\kappa} \xi^3 \quad (14)$$

where ξ_0 is the distance from the bed at which the velocity is hypothetically equal to zero, $\kappa = 0.41$ is the von Karman coefficient, Π is the Cole's wake parameter and λ is the dip correction parameter computed from Eq. (12). Equation (14) predicts the velocity profile if the parameters Π and λ are correctly chosen. The value of Π is taken from experimental data. The value of λ is calculated from Eq. (12) for all the models of dip-position.

Figure 7 shows the comparison of velocity profiles computed from Eq. (14) with value of λ from Eq. (12) for the experimental data of Coleman (1986). Coleman (1986) did experiments in a smooth flume which was 356 mm wide and 15 m long. During the experiments, the energy slope was kept to be 0.002. Among 40 test cases, test cases 1, 21, and 32 were performed in clear water flow. In the experiment, the location of dip-position was also obtained. Figure 7 shows the result for RUN 1 of Coleman (1986) data. The value of Π is computed as follows: initially the value of ξ_d is taken from experimental data set of Coleman (1986), and then the value of λ is calculated from Eq. (12). After that the value of Π is computed by using the least squares method using experimental data. The value of Π thus obtained is kept fixed for all dip-position models. It can be

observed from the figure that when value of λ is computed from model of Wang et al. (2001) and Yang et al. (2004), it overestimate the maximum velocity and model of Bonakdari et al. (2008) underestimates the maximum velocity. Whereas the proposed model, and models of Guo (2013) and Pu (2013) give satisfactory results. To get a quantitative result, the RMSE error is computed for all the models which are shown in Table 3. Table shows that the velocity model gives the best result when parameter λ is computed from the model proposed in this study.

Conclusions

The velocity dip position model proposed in this study has been fitted by asymptotic matching technique with data obtained from a wide range of aspect ratios for open channels of both smooth and rough bed surfaces. The employed fitting asymptotic technique has allowed matching the lower and upper boundaries of dip positions equal to 0.5 and 1, respectively. The comparison performed with other formulas showed that the proposed model provides the best-fit accuracy results. It is thus advocated here that the proposed model is capable of predicting velocity dip position in open channels under different conditions of aspect ratio and bed roughness. Also, the application results show that the proposed model provides better

Fig. 7 Prediction of mean velocity by Eq. (14) using dip formulas

Table 3 Accuracy of the model for predicting velocity profile

Coleman (1986) runs	Wang et al. (2001)	Yang et al. (2004)	Bonakdari et al. (2008)	Guo (2013)	Pu (2013)	This study
1	0.4781	0.4417	1.1527	0.3134	0.4172	0.2926*
21	0.5273	0.5060	1.1681	0.3956	0.4608	0.3765*
32	0.4962	0.4551	1.0898	0.2788	0.3787	0.2640*

* denotes minimum error

parameter estimation than the other models in terms of computing the Reynolds shear stress and mean velocity distribution over the full water depth of open channels.

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