Erratum

Title: Guidelines for Tilt Measurements Realized by MEMS Accelerometers

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Typographical errors in the symbols used in the above article have been corrected. The corrected version of the article is included in this issue starting from the following page.

Guidelines for Tilt Measurements Realized by MEMS Accelerometers

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The paper discusses thoroughly a problem of determining tilt over various measurement ranges by measuring Cartesian components of the gravity acceleration with application of accelerometers that are Microelectromechanical Systems (MEMS). The presented standard and original methods are based on various mathematical relationships between the components and the pitch and the roll. Their application results in the fact that the realized measurements are characterized by different sensitivity and accuracy (experimentally evaluated to be of 0.18°-2° for a tilt sensor built of standard MEMS accelerometers). Each method is discussed in detail with regard to the respective sensitivity and accuracy of the measurement, and its application scope is determined. Essential part of the paper are guidelines for an optimal selection of the number and type of MEMS accelerometers, as well as appropriate mathematical formula, to be used in a specific case of tilt measurement.

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NOMENCLATURE

g = gravitational acceleration
 g_x, g_y, g_z = Cartesian components of acceleration g
 α = pitch
 β = roll
 γ = alternative roll (according to aeronautical nomenclature)
 φ = arbitrary tilt angle
 ψ = either pitch α or roll β
 $u(q)$ = standard uncertainty of quantity “ q ”
 $u_c(q)$ = combined uncertainty of quantity “ q ”
 k = sensitivity
 e = indication error of the accelerometer
 q_1 = quantity “ q ” related to arc sin function
 q_2 = quantity “ q ” related to arc cos function
 q_3 = quantity “ q ” related to arc tan function
 q_4 = quantity “ q ” related to combination of arc sin and arc cos function
 q_5 = quantity “ q ” related to weighted average function

1. Introduction

Tilt measurements are employed in many devices, usually for the purposes of positioning, aligning, leveling or navigating. They can be realized by means of various types of sensors and measurement systems, which may be divided into: magnetic, gyroscopic (gimballed or strapdown systems) or gravitational. The first two can operate under static and dynamic conditions. The last employ various phenomena related either to a motion of a sensitive element or a change of a particular physical property (e.g., natural frequency) of an active element affected by a component of the gravitational force; their operation is correct only under static or quasi-static conditions. The sensitive element can be a fluid, a free element (e.g., a rolling ball) or a seismic mass supported in elastic suspension. In order to detect motion of the sensitive element various techniques are applied: resistive, capacitive, optical, etc.

As far as mechatronic and electronic devices are concerned, the most interesting solution for tilt measurements is application of MEMS sensors, as these are characterized by miniature dimensions, satisfactory metrological parameters, low power consumption and significantly lower cost compare to their conventional counterparts. As classified by Djuric-Jovicic et al.,¹ tilt measurements can be realized using the following inertial MEMS sensors:

- accelerometers,
- paired accelerometers,
- accelerometers combined with gyroscopes,
- Inertial Measurement Unit (IMU) – comprising accelerometers, rate gyroscopes and magnetometers (sometimes pressure sensors too).

On the one hand, devices like e.g., drone-type aerial vehicles driven by quadrotors sometimes require application of a state-of-art IMU (e.g., while keeping a stabilized position over a long term). Such unit is described e.g., by Lee et al.,² focusing on improvement of its metrological parameters owing to a novel calibration procedure proposed. On the other hand, there exist many devices that employ very simple tilt measurements (e.g., landscape or portrait orientation of a picture in a cellular telephone), where application of MEMS accelerometers is possible without any sophisticated signal processing.

However, accelerometers can be applied in still another group of devices employing tilt measurement, which may be realized under quasi-static conditions. These are e.g.: mobile microrobots³ or some scuba diving computers (e.g., Vyper Air or Cobra 3 by Suunto company) equipped with MEMS accelerometers enabling tilt compensation for the embedded electronic compass.⁴ Still another example of devices of this kind, which have emerged quite recently, are exoskeleton robots presented e.g.,^{5,6} In the case of such devices, two crucial aspects of using a single accelerometer are its smaller overall dimensions and lower power consumption, compare to the other abovementioned MEMS systems. Nevertheless, to ensure a satisfactory accuracy remains a challenge.

It is worth mentioning that increasing the number of accelerometers up to 6 or even 12,⁷ makes it possible to build an orientation/navigation system capable of operating under dynamic conditions, without using MEMS gyroscopes. Even though such approach is complicated, sometimes it is worthwhile undertaking.

Another important advantage of using accelerometers in tilt measurements is a possibility of employing various mathematical relations while determining the tilt. In numerous related papers various formulas are used, yet their choice is usually not substantiated, and sometimes it seems even not justified, e.g.,⁸ where nonlinear formula had been used and then additional computation was employed to overcome the resultant inaccuracy, instead of having used a linear formula at first.

The paper answers the question what mathematical formula should be used in order to meet requirements related to certain tilt measurement problem. It provides explanation based on the sensitivity and uncertainty of the measurements. It also helps to find an appropriate kind of MEMS accelerometer (i.e., its number of sensitive axes and its measurement range).

Owing to the guidelines presented in the paper, it is possible to develop the simplest tilt measurement unit based on MEMS accelerometers, being at the same time the most cost-efficient, yet ensuring satisfactory accuracy.

2. Tilt vs. Components of the Gravity Acceleration

The most common way of expressing tilt is to determine its two component angles pitch α and roll β , defined in Fig. 1 against the

Fig. 1 Components of the tilt and the gravitational acceleration

components of the gravitational acceleration.

The designations used in Fig. 1 mean:

g – gravitational acceleration;

g_x, g_y, g_z – Cartesian components of acceleration g ;

α – pitch;

β – roll;

γ – alternative roll (according to aeronautical nomenclature);

ϕ – arbitrary tilt angle.

Fig. 1 shows an arbitrary tilt angle ϕ , included between acceleration g and axis z , resolved into two component angles α (pitch) and β (roll), which are contained within vertical planes x_0xz_0 and lyz_0 (the planes are generally not orthogonal with respect to one another) – i.e. these are angles between horizontal plane x_0y_0 and axis x or y , respectively. The coordinate system $x_0y_0z_0$ is immobile (its axis z_0 is vertical), whereas the coordinate system xyz is fixed to the mobile tilt sensor (which can be represented by the cuboid constructed by vectors g_x, g_y and g_z). Angles α and β are interdependent and their values result from the value of angle ϕ . In order to illustrate angle β , line l has been created as a result of the intersection of vertical plane yz_0 and horizontal plane x_0y_0 , as proposed by the author et al.⁹

According to the standards accepted in aeronautics, roll has been defined in a different way - as angle γ between axis y and y_0 .¹⁰ In fact, this is the angle by which the tilt sensor physically rotates around axis x , while tilted in two phases (the first phase is rotation by pitch α around axis y when it still overlaps axis y_0). However, such definition of the roll is inconvenient while applying accelerometers for tilt measurements, for it is only α and β angle that can be determined on the basis of a single output signal of the accelerometer [see Eq. (2) and (3)]. Additionally, the related formulas presented later in the text are analogical for angle α and β . Besides, defining β as roll has been commonly accepted, e.g.,¹¹⁻¹⁵ Relation between the considered angles is as follows:^{10,16}

$$\gamma_1 = \arcsin\left(\frac{\sin\beta}{\cos\alpha}\right) \quad (1)$$

From this point on, angles α, β and γ will be designated with additional subscripts, depending on the formula employed for their determination, introduced in order to allow us to unequivocally distinguish between the formulas later in the text.

It is possible to determine angle γ knowing pitch and roll angles, yet

such operation usually results in a decrease of the related accuracy;¹⁶ the same applies to Eq. (8) introduced later in the text (the exception is a case when the pitch is approximately of -180° , 0° or 180°), what is sometimes overlooked by some researchers, e.g.,¹⁷ A converse problem, when the roll is determined on the basis of pitch and angle γ , is relevant in the case of using a dual-axis test rig like the one described e.g.,^{17,18}

As results from Fig. 1, the tilt angles can be calculated as follows:^{9,11}

$$\alpha_1 = \arcsin \frac{g_x}{g} \quad (2)$$

$$\beta_1 = \arcsin \frac{g_y}{g} \quad (3)$$

The vertical component acceleration g_z allows us to create the following additional formulas, reported in:^{9,19}

$$\alpha_2 = \arccos \frac{\sqrt{g_y^2 + g_z^2}}{g} \quad (4)$$

$$\beta_2 = \arccos \frac{\sqrt{g_x^2 + g_z^2}}{g} \quad (5)$$

and formulas introduced in:^{12-14,20}

$$\alpha_3 = \arctan \frac{g_x}{\sqrt{g_y^2 + g_z^2}} \quad (6)$$

$$\beta_3 = \arctan \frac{g_y}{\sqrt{g_x^2 + g_z^2}} \quad (7)$$

and additionally:²¹

$$\gamma_3 = \arctan \frac{g_y}{g_z} \quad (8)$$

Since the formulas that have been quoted later in the text apply to the pitch in the same measure as to the roll, from now on, the symbols α and β have been replaced with ψ (meaning any of these two angles).

It is also possible to determine the tilt angles in more complicated, yet effective ways. The first way is to choose one of the values determined according to arc sine or arc cosine function, depending on the magnitude of the angle, as proposed:⁹

$$\begin{cases} \psi_1, \psi_2 < 45^\circ \Rightarrow \psi_4 = \psi_1 \\ \psi_1, \psi_2 > 45^\circ \Rightarrow \psi_4 = \psi_2 \end{cases} \quad (9)$$

The second way is to determine the tilt angles as a weighted average (whose coefficients a and b have a variable value, depending on the magnitude of the respective tilt angle). Such approach has been proposed e.g., by Grepl,²² and can be expressed in the considered case as follows:

$$\psi_5 = a\psi_1 + b\psi_2 \quad (10)$$

Assuming that each sensitive axis of the accelerometers is characterized by the same accuracy, it can be proven that in order to obtain the highest accuracy of tilt measurements, the variable coefficients are to be determined as in the following formula:¹⁹

$$\psi_5 = \psi_1 \cdot \cos^2 \psi_4 + \psi_2 \cdot \sin^2 \psi_4 \quad (11)$$

If the tilt is to be measured over one axis only, Eq. (4)-(7) get simplified, as one of the component accelerations (either g_x or g_y) will be then of zero value (e.g., Eq. (7) transforms then to Eq. (8)).

As results from the preceding formulas, indications of each of the three component accelerometers (which constitute the whole tilt sensor) reach all the possible values within the angular range of 180° (for trigonometric functions are periodic). An unequivocal determination of each component tilt angle is possible by observing the sign of the vertical component acceleration g_z , which is positive over the domain of $<-90^\circ; 90^\circ>$, while negative over the remaining angular range. Moreover, indications of the accelerometers are symmetric within the basic range, so further considerations can be limited to a sub-range of $<0^\circ; 90^\circ>$.

3. Sensitivity of the Measurements

Even though the nominal values of the tilt angles resulting from the presented equations are the same, yet their accuracy is different.

3.1 Uncertainty of determining the tilt

In order to evaluate accuracy of determining tilt angles, a notion of uncertainty can be used, which estimates the random errors dominating in the related measurements. Such approach has been accepted and discussed by the author et al.^{9,19}

The basic relations between tilt angles and the respective components of the gravitational acceleration [Eq. (2) and (3)] are strongly nonlinear, especially within the range of $<70^\circ; 90^\circ>$. Nowadays, it is easy to deal with nonlinearity of indications due to application of e.g., microprocessor technology. However, it should be noted that if a characteristic of a sensor is nonlinear, the sensitivity of its indications would vary over its measurement range.

According to the guidelines of ISO, the square of the standard uncertainty associated with the output estimate of a given quantity is to be calculated as the sum of the squares of its partial derivatives with respect to all the input estimates (sensitivity coefficients), each multiplied by the standard uncertainty of the respective input estimate.²³ Thus, while determining the standard uncertainty of the component tilt angles we can use the following general equation:

$$u_c(\psi) = \sqrt{\left(\frac{\partial \psi}{\partial g_x} u(g_x)\right)^2 + \left(\frac{\partial \psi}{\partial g_y} u(g_y)\right)^2 + \left(\frac{\partial \psi}{\partial g_z} u(g_z)\right)^2 + \left(\frac{\partial \psi}{\partial g} u(g)\right)^2} \quad (12)$$

where $u(g_x)$, $u(g_y)$, $u(g_z)$ are the standard uncertainties related to the measurement of the Cartesian components of gravitational acceleration g_x , g_y or g_z , and $u(g)$ is the standard uncertainty related to determining the value of the output signal of the accelerometer, which corresponds to gravitational acceleration g (that can be obtained during calibration of the tilt sensor). The values of the standard uncertainties $u(g_{x...z})$ characterize performance of a particular accelerometer and are related to its quality; they can be easily determined in an experimental way. For further analyses, it can be assumed that,

$$u(g_x) = u(g_y) = u(g_z) = u(g_{x...z}) \gg u(g) \quad (13)$$

The above assumption helps to simplify further considerations, and is based on accepting the fact that the employed accelerometers feature the same accuracy. It can be expected that the standard uncertainty $u(g)$ will be rather smaller than the standard uncertainties $u(g_{x...z})$, what results from the fact that the indication of the accelerometer that corresponds to the gravitational acceleration is determined in the process of calibration of the tilt sensor (usually a necessary procedure to be realized in the case of commercial MEMS accelerometers), which requires to perform much more measurements than in the case of determining the component accelerations during a standard operation of the sensor.

Assuming that the relation between the standard uncertainties can be expressed by Eq. (13), it has been accepted that the value of the gravitational acceleration is constant. Considerations pertaining to variability of the gravitational acceleration have been discussed by the author et al.^{9,19}

By using appropriate formulas, we obtain reduced forms of Eq. (12) for particular cases:

$$u_c(\psi_1) = \frac{u(g_{x...z})}{|g|} \frac{1}{\cos \psi_1} \quad (14)$$

$$u_c(\psi_2) = \frac{u(g_{x...z})}{|g|} \frac{1}{\sin \psi_2} \quad (15)$$

$$u_c(\psi_3) = \frac{u(g_{x...z})}{|g|} = \text{const} \approx u_c(\psi_5) \quad (16)$$

Uncertainties of angles α_4 and β_4 can be of course expressed by a respective combination of Eq. (14) and (15). As far as α_5 and β_5 are concerned, the author et al proved¹⁹ that the related uncertainties are approximately equal to Eq. (16).

3.2 Sensitivity of determining the tilt

Since variations of the uncertainty of determining the roll and the pitch, which are described by Eq. (14)-(15), grow to infinity, it is more convenient to use a notion of sensitivity of the sensor, as it often appears in the related catalog data, e.g.,^{13,14} The appropriate formulas can be derived from Eq. (14)-(16), or from a transformed form of Eq. (2)-(7), (11). So, a relation between the sensitivity and the tilt angle ψ_1 can be defined as:⁹

$$k_1 = g \cos \psi_1 \quad (17)$$

Likewise, for tilt angles ψ_2 , ψ_3 and ψ_5 we can obtain:

$$k_2 = g \sin \psi_2 \quad (18)$$

$$k_3 = g = \text{const} \approx k_5 \quad (19)$$

Graphs of Eq. (17) and (18) are presented in Fig. 2(a) and in Fig. 2(b) for Eq. (19), accepting that $g = 9.81 \text{ m/s}^2$. As in the case of uncertainty $u_c(\psi_4)$, sensitivity k_4 related to angles α_4 and β_4 is a

Fig. 2 Accelerometer sensitivity in tilt measurements

combination of Eq. (17) and (18), represented by the k_4 curve in Fig. 2(b). Again, because of the symmetry of the sensor indications, a smaller, yet fully representative, range of variation of the tilt angles $<0^\circ; 90^\circ>$ has been considered.

As can be clearly observed, sensitivity k_1 and k_2 , related to the arc sine and the arc cosine formulas expressed by Eq. (2)-(5), decrease down to zero for a tilt angle of 90° or 0° respectively, what is very disadvantageous and sometimes even unacceptable. However, while using these formulas as a combined pair [application of Eq. (9)], sensitivity k_4 does not decrease so dramatically. Moreover, application of the arc tangent formula [Eq. (6)-(7)], as well as the weighted average expressed by Eq. (11), ensure a constant, and at the same time the highest value of the sensitivity – the sensitivity curve k_3, k_5 .

4. Experimental Verification

The considerations that have been presented so far are based on theoretical analyses. So, in order to verify their correctness, appropriate experimental studies have been carried out on a physical model of a tilt sensor. It consisted of two dual-axis MEMS accelerometers ADXL 202E manufactured by Analog Devices, Inc.¹⁵ The experiments have been carried out using the test rig and methodology minutely described by the author.^{18,9}

Let us consider the pitch angle – exemplary results are illustrated in Fig. 3 (results obtained for the roll angle were very similar). The indication error e_n ($n = 1, \dots, 5$) expressed over axis y has been defined as a difference between the value of the pitch φ applied by means of the test station and the value determined with respect to the average of relevant indications of the tested sensor, employing Eq. (2)-(7), (9), (11)

Fig. 3 Errors of determining the pitch angle

respectively. So:

$$e_n = |\varphi - \alpha_n| \quad (20)$$

Other ways of evaluating accuracy of tilt sensors, based e.g., on creation of a relevant calibration model and employing its prediction or confidence interval, are discussed by the author.²⁴

Styles of the graphs of the considered indication errors e_n presented in Fig. 3 are consistent with their counterparts illustrated in Fig. 2 [as each pair is related to the same formula, expressed by one of the Eqs. (2)-(7), (9), (11)].

First, let us consider the error graphs e_1 and e_2 related to arc sine and arc cosine formula. As can be observed in Fig. 3(a), in the case of determining small magnitudes of the tilt angles according to the arc sine type of formula [Eq. (2) or (3)] results in a big indication error - here, of ca. 2° , whereas for large magnitudes of the angles, the same applies to using the arc cosine type of formula [Eq. (4) or (5)]; that corresponds to a drastic decrease of sensitivity k_1 and k_2 , respectively (see Fig. 2(a)). Yet, as illustrated in Fig. 3(b), employing combination of these two types of formulas, expressed by Eq. (9), radically reduces the maximal value of the indication error e_4 , which in the presented studies did not exceed 0.27° over the whole measurement range, i.e., $<-180^\circ; 180^\circ>$. This improvement is consistent with the graph of sensitivity k_4 illustrated in Fig. 2(b).

Now, let us consider the error graph e_3 , related to the arc tangent formula [Eq. (6) and (7)], as compared to the error graph e_4 related to the combination of arc sine and arc cosine formula [Eq. (9)]. As could be expected, at the beginning and at the end of the considered angular range, there is almost no difference in the course of these two graphs. However, in the middle of the range, the difference is the most evident.

So, while using the arc tangent formula [Eq. (6) or (7)] instead of the combination of arc sine with arc cosine formula [Eq. (9)], we decrease the related error of the sensor indications by ca. 30%, whose maximal value did not exceed 0.18° in the experiment. A difference between using arc tangent formula and the weighted average [Eq. (11)] is insignificant and cannot be distinguished on the graphs, what justifies the approximation assumed in Eq. (16) and (19).

To summarize, we can say that the experimental results are fully consistent with the presented theoretical approach. Owing to application of the most complicated formulas [i.e., Eq. (6), (7), (9) or (11)] instead of the basic ones [i.e., Eq. (2)-(5)], the error of the sensor indications decreased from 2° down to ca. 0.2° or 0.3° , respectively. So, a significant improvement of the sensor performance can be achieved in this way.

5. Scope of Application

While choosing appropriate number of sensitive axes of the accelerometers, their type and their spatial arrangement, as well as the most convenient mathematical relationship to be used, one should take into account, first of all, the required accuracy and then the simplicity of the related computations (thus, the resultant speed and cost of the measurement unit). Relations between these parameters are discussed in the following sub-sections.

5.1 Number and type of the accelerometers

The most demanding case for any tilt sensor is to feature a measurement range that covers all the possible orientations with respect to the gravity vector (i.e., full angle of pitch and roll). Then, the considered sensor must consist of one or more accelerometers with a total number of sensitive axes equal to at least 3. While employing MEMS accelerometers, four options are reasonable: to use 3 single-axis, 1 single-axis and 1 dual-axis, 2 dual-axis or 1 triaxial accelerometer. Still another option is to employ 1 multi-axial accelerometer (such as e.g., the one presented²⁵) - the resulting advantages of that solution related to the accuracy are discussed by the author et al in.¹⁹ Requirements related to other measurement cases are presented in Table 1, where the numbers in row1, column 3 (r1c3), r1c4 and r2c4 refer to the cases, when it is possible to obtain sensitivity higher than 1 g/rad (ca. $10 \text{ m/s}^2\text{rad}$), due to redundancy of the indications, whereas the numbers in r1c1 and r2c2 denote cases, when the sensitivity is lower than 1 g/rad .

While 3 sensitive axes are required, application of single-axis or dual-axis accelerometers instead of a triaxial one is more laborious and expensive, however it provides an opportunity of building a more accurate tilt sensor. This approach is accepted by such companies as MicroStrain Inc. or Sentra Technology Corp., whose sensors have the sensitive axes of the constituent accelerometers precisely aligned and calibrated²⁶ (especially with respect to the ambient temperature), with the obtained data stored in special look-up tables used during a standard operation of the tilt sensor.

It should be realized that in the case of dual-axis MEMS accelerometers the alignment error of their sensitive axes is very small, e.g., of 0.01° .¹⁵ Usually, this is not the case with triaxial accelerometers, whose z -axis is poorly aligned and additionally features worse accuracy, what results from their mechanical structure determined by the fabrication

Table 1 Measurement ranges of tilt vs. number of the accelerometer sensitive axes

Number of the sensitive axes	1	2	3	4
Single-axis tilt measurement	$\pm 90^\circ$	$\pm 180^\circ$	$\pm 180^\circ$	$\pm 180^\circ$
Dual-axis tilt measurement (pitch & roll)	- $\langle 0^\circ; 180^\circ \rangle^*$	$\pm 90^\circ$	$\pm 180^\circ$	$\pm 180^\circ$

*pseudo-dual axis measurement (see sec.5.2 for details)

process (this shortcoming is evident especially in the case of surface micromachining, which in fact is semi-three-dimensional,²⁷ yet is dominant among commercial triaxial MEMS accelerometers, due to its low production costs).²⁸⁻³⁰

Application of 2 dual-axis accelerometers provides also a higher accuracy due to redundancy of the information (two sensitive axes are associated with one of the three Cartesian axes, so a respective average of the doubled indications can be used).

5.2 Spatial arrangement

In the case of using 3 sensitive axes of the accelerometers, their spatial configuration is the simplest and most preferable, when their axes are orthogonal with respect to one another, and thus form a Cartesian coordinate system xyz , as depicted in Fig. 1.

Any other spatial configuration of the sensitive axes is also acceptable, and sometimes even more advantageous - e.g., in the case of using 3 or more accelerometer sensitive axes for a single-axis measurement, none of the sensitive axes should overlap the tilt axis, in order to increase the related accuracy.

If 1 or 2 sensitive axes of the accelerometers are sufficient, their spatial arrangement must be carefully observed. In the first case, the axis should be oriented horizontally, if small magnitudes of tilt angles are to be detected with higher sensitivity. On the contrary, when higher sensitivity is to be obtained for larger magnitudes of tilt angles ($>45^\circ$), it should be oriented vertically. In the second case, two basic options are possible. While performing a single-axis tilt measurement, one sensitive axis should be vertical and the other horizontal, whereas both sensitive axes should be horizontal when a dual-axis tilt measurement is to be performed.

There is yet another case presented in Tab.1 to be considered. Having only one sensitive axis, it is possible to perform a pseudo-dual-axis tilt measurement. Such case may be interesting e.g., while monitoring an object against losing its balance. Then, the accelerometer should be oriented vertically, as to sense any arbitrary tilt angle φ (see Fig. 1). The related measuring range is then of $\langle 0^\circ, 180^\circ \rangle$, however with no distinction between pitch and roll. Another disadvantage of such application is insensitivity around the vertical position of the monitored object (only tilt angles larger than few degrees arc can be detected this way).

5.3 Mathematical formulas

In majority of the measurements a possibly high accuracy is striven for. So, while using standard MEMS accelerometers the best solution is to use arc tangent formulas, i.e., Eq. (6) and (7), which feature the highest sensitivity, as it results from Table 2.

Table 2 Characteristics of the mathematical formulas

Formula	A	B	C	D	E
Arc sine Eq. (2), (3)	10-0	1	2	$\pm 1g$	Low
Arc cosine Eq. (4), (5)	10-0	1	3	$\pm 1g$	Low
Arc tangent Eq. (6), (7)	10	2	3	$\pm 1g$	Low
Combination Eq. (8)	10-7	2	3	$\pm 0.71g$	Medium
Weighted average Eq. (10)	10	2	3	$\pm 1g$	High

A - Sensitivity k (approximately) [$m/s^2 rad$]; B - Number of the sensitive axes necessary for a single-axis tilt measurement; C - Number of the sensitive axes necessary for a dual-axis tilt measurement; D - Minimal measuring range of the applied accelerometers; E - Complication of the related computation algorithm.

However, there are some cases when other formulas are more convenient. Application of Eq. (9) may be interesting in the case of using accelerometers with a measuring range smaller than $\pm 1g$ or in the case when the accelerometer is subjected to vibration that do not allow to use its full measurement range (these issues are addressed by the author).³⁰ The related sensitivity is then variable and smaller (by ca. 30% in the worst case), and the related algorithms more complicated.

On the other hand, application of the arc sine formulas expressed by Eq. (2)-(3) may be necessary if the tilt sensor measures only one component (single-axis measurements) or two components (dual-axis measurements) of the gravitational acceleration. Moreover, it is the best solution while calibrating the accelerometers, when in fact Eq. (2)-(3) are used in transformed forms (tilt angle is then not determined but applied).

As presented in Table 2, application of Eq. (6) or (7) yields the same accuracy as the weighted average expressed by Eq. (11), yet the later is more complicated. However, it has some advantages over the arc tangent formulas.

It allows us to avoid using arc tangent formula, what may be interesting for the sake of the simplicity of the electronic circuits processing the output signals of the sensor or in the case when some patented algorithms must be replaced with an original approach.

However, the most important advantage of Eq. (11) is a flexibility, which allows us to regard different values of the accuracies associated with particular sensitive axes. Then, the weight coefficients in Eq. (10) must have other form than in Eq. (11), and be determined for a particular case. As already mentioned, it applies, first of all, to triaxial MEMS accelerometers whose output signal related to their vertical sensitive axis is usually less accurate compare to the other two signals.

An interesting option is also to employ few different formulas in turns. For instance, to use Eq. (6) and (7) under purely static conditions, and replacing them with (8) when the tilt sensor is subjected to vibration limiting the useful measurement range of its constituent accelerometers.

6. Summary

A theoretical approach to determining tilt over various measuring ranges has been presented. The accepted idea of using the components of the gravity vector for this purpose allows us to apply a tilt sensor with miniature overall dimensions made of MEMS accelerometers. Some of the proposed methods of determining tilt ensure detection of

tilt angles with accuracy of ca. few tenths of degree arc. The experimental results proved the presented theoretical considerations to be correct and allowed the magnitudes of the accuracy of determining the tilt angles to be estimated by means of a tilt sensor built of MEMS accelerometers.

As it results from the presented considerations, in majority of the applications, the most advantageous way of calculating the tilt angles, as far as the related accuracy is concerned, is to use arc tangent formulas expressed by Eq. (6) and (7). So, this is why the manufacturers of integrated tilt measuring units make use of this relation, even though they usually do not expose this fact (as e.g., in the case of measuring units referred to).³¹ Other measurement cases are minutely considered in sec. 5. The presented considerations allow us to choose the best mathematical relationship and the best configuration of the accelerometers, as far as accuracy of tilt measurement and the related costs are concerned.

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