Erratum to: Oscillation of second-order delay dynamic equations on time scales

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In the above-referenced paper [1], we stated and proved four theorems on the sufficient conditions for the oscillation of all solutions of second order delay dynamic equations

$$\left(p(t)(x^\Delta(t))^\gamma\right)^\Delta + q(t)f(x(\tau(t))) = 0 \quad (0.1)$$

on a time scale $\mathbb{T}$, where $\gamma \geq 1$ is a quotient of odd positive integers, p and q are positive, real-valued rd-continuous functions defined on $\mathbb{T}$, $\tau : \mathbb{T} \rightarrow \mathbb{T}$ is a strictly

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increasing function such that $\tau(t) \leq t$ and $\tau(t) \rightarrow \infty$ as $t \rightarrow \infty$, $f \in C(\mathbb{R}, \mathbb{R})$ which satisfies for some positive constant L , $\frac{f(x)}{x^\gamma} \geq L$, for all nonzero x . There were mistakes in the statements of the four theorems, and we wish to correct the mistakes below.

We define the time scale interval of the form $[t_0, \infty)_{\mathbb{T}}$ by $[t_0, \infty)_{\mathbb{T}} = [t_0, \infty) \cap \mathbb{T}$, and consider the two cases

$$\int_{t_0}^{\infty} \left(\frac{1}{p(t)} \right)^{\frac{1}{\gamma}} \Delta t = \infty, \quad (0.2)$$

and

$$\int_{t_0}^{\infty} \left(\frac{1}{p(t)} \right)^{\frac{1}{\gamma}} \Delta t < \infty. \quad (0.3)$$

The four theorems are corrected as follows:

Theorem 1 Assume (0.2) holds and $\tau \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{T})$, $\tilde{\mathbb{T}} := \tau(\mathbb{T}) \subset \mathbb{T}$ is a time scale, $\tau(\sigma(t)) = \sigma(\tau(t))$. Furthermore, assume that there exists a positive function $\delta \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{R})$ such that

$$\limsup_{t \rightarrow \infty} \int_{t_0}^t \left(Lq(s)\delta(s) - \frac{p(\tau(s))((\delta^\Delta(s))_+)^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s)\tau^\Delta(s))^\gamma} \right) \Delta s = \infty, \quad (0.4)$$

where $(\delta^\Delta(s))_+ = \max\{0, \delta^\Delta(s)\}$. Then (0.1) is oscillatory on $[t_0, \infty)_{\mathbb{T}}$.

Theorem 2 Assume (0.2) holds and $\tau \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{T})$, $\tilde{\mathbb{T}} := \tau(\mathbb{T}) \subset \mathbb{T}$ is a time scale, $\tau(\sigma(t)) = \sigma(\tau(t))$. Furthermore, assume that there exists a positive function $\delta \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{R})$ such that for $m \geq 1$

$$\limsup_{t \rightarrow \infty} \frac{1}{t^m} \int_{t_0}^t (t-s)^m \left(Lq(s)\delta(s) - \frac{p(\tau(s))((\delta^\Delta(s))_+)^{\gamma+1}}{(\gamma+1)^{\gamma+1}(\delta(s)\tau^\Delta(s))^\gamma} \right) \Delta s = \infty, \quad (0.5)$$

where $(\delta^\Delta(s))_+ = \max\{0, \delta^\Delta(s)\}$. Then (0.1) is oscillatory on $[t_0, \infty)_{\mathbb{T}}$.

Theorem 3 Assume (0.3) holds and $\tau \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{T})$, $\tilde{\mathbb{T}} := \tau(\mathbb{T}) \subset \mathbb{T}$ is a time scale, $\tau(\sigma(t)) = \sigma(\tau(t))$. Furthermore, assume that there exists a positive function $\delta \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{R})$ such that (0.4) holds. If

$$\int_{t_0}^{\infty} \left[\frac{1}{p(t)} \int_{t_0}^t q(s) \Delta s \right]^{\frac{1}{\gamma}} \Delta t = \infty, \quad (0.6)$$

then every solution of (0.1) is either oscillatory or converges to zero on $[t_0, \infty)_{\mathbb{T}}$.

Theorem 4 Assume (0.3) holds and $\tau \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{T})$, $\tilde{\mathbb{T}} := \tau(\mathbb{T}) \subset \mathbb{T}$ is a time scale, $\tau(\sigma(t)) = \sigma(\tau(t))$. Furthermore, assume that there exists a positive function $\delta \in C_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{R})$ such that (0.5) and (0.6) hold. Then every solution of (0.1) is either oscillatory or converges to zero on $[t_0, \infty)_{\mathbb{T}}$.

Actually, the major results are based on Lemma 2.2 (p. 462, line 26). We think these are not useful in this case because the forward jump operator $\tilde{\sigma}$ in $\tilde{\mathbb{T}}$ is not the same σ in $\mathbb{T}$, so the differentiation in p. 463 line 19 is not correct and also $(x(\tau(t)))^\Delta = x^\Delta(\tau(t))\tau^\Delta(t)$ is not correct in general. Therefore, we intend to correct the mistakes as follows.

Lemma (Chain Rule) Assume that $\tau : \mathbb{T} \rightarrow \mathbb{R}$ is strictly increasing and $\tilde{\mathbb{T}} := \tau(\mathbb{T}) \subset \mathbb{T}$ is a time scale, $\tau(\sigma(t)) = \sigma(\tau(t))$. Let $x : \tilde{\mathbb{T}} \rightarrow \mathbb{R}$. If $\tau^\Delta(t)$ and $x^\Delta(\tau(t))$ exist for $t \in \mathbb{T}^\kappa$, then $(x(\tau(t)))^\Delta$ exists, and

$$(x(\tau(t)))^\Delta = x^\Delta(\tau(t))\tau^\Delta(t). \quad (0.7)$$

Proof Let $0 < \varepsilon < 1$ be given and define $\varepsilon^* = \varepsilon[1 + |\tau^\Delta(t)| + |x^\Delta(\tau(t))|]^{-1}$. Note that $0 < \varepsilon^* < 1$. According to the assumptions, there exist neighborhoods $\mathcal{N}_1$ of t and $\mathcal{N}_2$ of $\tau(t)$ such that

$$|\tau(\sigma(t)) - \tau(s) - (\sigma(t) - s)\tau^\Delta(t)| \leq \varepsilon^*|\sigma(t) - s|, \quad s \in \mathcal{N}_1$$

and

$$|x(\sigma(\tau(t))) - x(r) - (\sigma(\tau(t)) - r)x^\Delta(\tau(t))| \leq \varepsilon^*|\sigma(\tau(t)) - r|, \quad r \in \mathcal{N}_2.$$

Put $\mathcal{N} = \mathcal{N}_1 \cap \tau^{-1}\mathcal{N}_2$ and let $s \in \mathcal{N}$. Then $s \in \mathcal{N}_1$ and $\tau(s) \in \mathcal{N}_2$ and

$$\begin{aligned} & |x(\tau(\sigma(t))) - x(\tau(s)) - (\sigma(t) - s)x^\Delta(\tau(t))\tau^\Delta(t)| \\ &= |x(\tau(\sigma(t))) - x(\tau(s)) - (\sigma(\tau(t)) - \tau(s))x^\Delta(\tau(t)) \\ &\quad + [\sigma(\tau(t)) - \tau(s) - (\sigma(t) - s)\tau^\Delta(t)]x^\Delta(\tau(t))| \\ &= |x(\sigma(\tau(t))) - x(\tau(s)) - (\sigma(\tau(t)) - \tau(s))x^\Delta(\tau(t)) \\ &\quad + [\tau(\sigma(t)) - \tau(s) - (\sigma(t) - s)\tau^\Delta(t)]x^\Delta(\tau(t))| \\ &\leq \varepsilon^*|\sigma(\tau(t)) - \tau(s)| + \varepsilon^*|\sigma(t) - s||x^\Delta(\tau(t))| \\ &\leq \varepsilon^*\{|\sigma(\tau(t)) - \tau(s) - (\sigma(t) - s)\tau^\Delta(t)| \\ &\quad + |\sigma(t) - s||\tau^\Delta(t)| + |\sigma(t) - s||x^\Delta(\tau(t))|\} \\ &= \varepsilon^*\{|\tau(\sigma(t)) - \tau(s) - (\sigma(t) - s)\tau^\Delta(t)| \\ &\quad + |\sigma(t) - s||\tau^\Delta(t)| + |\sigma(t) - s||x^\Delta(\tau(t))|\} \\ &\leq \varepsilon^*\{\varepsilon^*|\sigma(t) - s| + |\sigma(t) - s||\tau^\Delta(t)| + |\sigma(t) - s||x^\Delta(\tau(t))|\} \\ &= \varepsilon^*|\sigma(t) - s|\{\varepsilon^* + |\tau^\Delta(t)| + |x^\Delta(\tau(t))|\} \\ &\leq \varepsilon^*|\sigma(t) - s|\{1 + |\tau^\Delta(t)| + |x^\Delta(\tau(t))|\} \\ &= \varepsilon|\sigma(t) - s|. \end{aligned}$$

The proof is complete. $\square$

References

1. Sun, S., Han, Z., Zhang, C.: Oscillation of second-order delay dynamic equations on time scales. *J. Appl. Math. Comput.* **30**, 459–468 (2009)