The influence of pore structure on P- & S-wave velocities in complex carbonate reservoirs with secondary storage space

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Abstract: Secondary storage spaces with very complex geometries are well developed in Ordovician carbonate reservoirs in the Tarim Basin, which is taken as a study case in this paper. It is still not clear how the secondary storage space shape influences the P- & S-wave velocities (or elastic properties) in complex carbonate reservoirs. In this paper, three classical rock physics models (Wyllie time-average equation, Gassmann equation and the Kuster-Toksöz model) are comparably analyzed for their construction principles and actual velocity prediction results, aiming at determining the most favourable rock physics model to consider the influence of secondary storage space shape. Then relationships between the P- & S-wave velocities in carbonate reservoirs and geometric shapes of secondary storage spaces are discussed from different aspects based on actual well data by employing the favourable rock physics model. To explain the influence of secondary storage space shape on V_p - V_s relationship, it is analyzed for the differences of S-wave velocities between derived from common empirical relationships (including Castagna's mud rock line and Greenberg-Castagna V_p - V_s relationship) and predicted by the rock physics model. We advocate that V_p - V_s relationship for complex carbonate reservoirs should be built for different storage space types. For the carbonate reservoirs in the Tarim Basin, the V_p - V_s relationships for fractured, fractured-cavernous, and fractured-hole-vuggy reservoirs are respectively built on the basis of velocity prediction and secondary storage space type determination. Through the discussion above, it is expected that the velocity prediction and the V_p - V_s relationships for complex carbonate reservoirs should fully consider the influence of secondary storage space shape, thus providing more reasonable constraints for prestack inversion, further building a foundation for realizing carbonate reservoir prediction and fluid prediction.

Key words: Complex carbonate reservoir, secondary storage space, velocity prediction, V_p - V_s relationships

1 Introduction

In the rock physics study, P- & S-wave velocity prediction is the primary issue because it directly reflects the “bridge” role of rock physics linking reservoir physical properties with elastic properties. And the predicted velocities also provide key constraint data for the following pre-stack inversion. Currently, many rock physics models regarding velocity prediction have been established. The most basic and classic models are the Wyllie time-average equation (proposed based on effective average theory by Wyllie et al in 1956), the Gassmann equation (considering the effect of pore fluids

proposed by Gassmann in 1951 and re-expounded by Biot in 1956), and the Kuster-Toksöz model (considering the effect of pore shape through adjusting pore aspect ratio proposed by Kuster and Toksöz in 1974). Other rock physics models for velocity prediction are mostly the extended models on the basis of the three basic models, such as the Xu-White model (Xu and White, 1995) for clay-sand mixtures (integrating the Gassmann equation and the Kuster-Toksöz model), differential effective medium model (incrementally adding inclusions to the matrix on the basis of the Kuster-Toksöz model). Therefore, this paper focuses on the analysis of these basic classical rock physics models, and discusses how the secondary storage space shape influences the P- & S-wave velocities in carbonate reservoirs.

The reason why we focus on discussing the influence of secondary storage space shape is derived from the specific

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research and practices on Ordovician carbonate reservoirs in the Tarim Basin in recent years. As the oilfields located in eastern China gradually go into the later stage of high water cut, the Tarim Basin that contains abundant oil and gas resources will take over as the important oil and gas frontier in China. However, these oil and gas resource are deeply buried (averaging below 6,000 m). Furthermore, these reservoirs

experienced multiphase tectonic movements, improvements, and compactions, resulting in the disappearance of primary pores and the fine development of secondary storage spaces (Lü et al, 2009; Pang et al, 2010). Fig. 1 shows some typical secondary storage space types including dissolution caverns (including large-sized cave and core-sized vug), dissolution holes, and fractures.

Fig. 1 The photos of dissolution caverns, dissolution holes, and fractures developed in Ordovician carbonate reservoirs in the Tarim Basin (these pictures are provide by the Tarim Oilfield Co., China)

All of the characteristics of carbonate reservoirs in the Tarim Basin make reservoir prediction in this area a world-class scientific tough problem. The solution to the problem requires prestack analysis, which is also the core idea expressed in this special issue. For prestack analysis, S-wave information is needed while it is generally not available due to the high measurement cost and the negative effect of secondary pores on sonic logging for this type of deep-buried complex carbonate reservoirs. Therefore, rock physics technology is required to reconstruct or predict the available P- & S-wave velocities. Velocity prediction for this type of carbonate reservoirs is concentrated on how to depict and reflect the influence of various secondary space shapes on velocities. Many experiments and theoretical research have proven the objective existence of the influence of pore geometry on rock velocity (Toksöz et al, 1976; Cheng and Toksöz, 1979; Xu and White, 1995; Yan et al, 2002; Kumar and Han, 2005; Xu and Payne, 2009; Carlson, 2010). For carbonate rocks dominated by secondary storage spaces, based on laboratory measurement results, Eberli et al (2003) pointed out that the change laws between velocity and porosity are very different for rocks with different storage

spaces such as micro-pores, moldic pores, and intercrystal pores; Han et al (1986) also pointed out that caves almost had no contribution to the velocity change due to their high rigidity while micro-pores or cracks had significant influence on the velocity decrease due to their low rigidity through laboratory measurements and investigations on 52 different carbonate samples. On the basis of the previous experimental and theoretical results, through comparatively selecting appropriate rock physics model, this paper will discuss the influence of secondary storage space shape on P- & S- wave velocities from multiple aspects using actual well data.

2 Rock physics models

The Wyllie time-average equation (Wyllie et al, 1956), the Gassmann equation (1951) and the Kuster-Toksöz model (Kuster-Toksöz, 1974) are three basic classical rock physics models. However, not all of these models are suitable for considering the influence of pore geometry on elastic wave velocities because their assumptions on the pore structures of rock have obvious differences. The Wyllie time-average equation is the simplest volume-average model. In this model, the whole rock is equivalent to a mixture of several

parts including gas, oil, water, and matrix in a layered structure. This equivalence is only appropriate for pure rock and there is no concept of pore geometry. The Gassmann equation is a classical fluid substitution model and considers well the connectivity among pores. This model also can be used for velocity prediction when the elastic moduli of dry rocks are known (Sun et al, 2004). However, because it is free of assumption of the pore geometry change (generally spherical pores are assumed), it may be very difficult to use this model to predict the wave velocities in carbonate

reservoirs with various secondary storage spaces (e.g., cracks, holes, and caves). The Kuster-Toksöz model is derived on the basis of long-wavelength first-order scattering theory and describes various pore geometries using different pore aspect ratios, thus well characterizing the actual storage spaces in reservoirs. Therefore, the Kuster-Toksöz model may be very suitable for considering the influence of secondary storage spaces on elastic wave velocities in carbonate reservoirs. The equivalent schemes for the above three rock physic models are shown in Fig. 2.

Fig. 2 Equivalent schemes for the Wyllie time-average equation, the Gassmann equation, and the Kuster-Toksöz model

The formulas of the Wyllie time-average equation and the Gassmann equation can be easily understood (see Wyllie et al, 1956; Gassmann, 1951; Biot, 1956; Sun et al, 2004; Wang et al, 2009). The key formulas of the Kuster-Toksöz model are expressed as follows:

$$(K^* - K_m) \frac{(3K_m + 4\mu_m)}{(3K^* + 4\mu_m)} = \sum_{i=1}^N (K_i - K_m) P^{mi} \quad (1)$$

$$(\mu^* - \mu_m) \frac{(\mu_m + \zeta_m)}{(\mu^* + \zeta_m)} = \sum_{i=1}^N x_i (\mu_i - \mu_m) Q^{mi} \quad (2)$$

where K^* and μ^* are the unknown bulk and shear moduli of the saturated rock; K_m and μ_m are the bulk and shear moduli of the rock matrix; K_i and μ_i are the bulk and shear moduli of the i th inclusion; P and Q are the coefficients describing elastic properties of the pore phase. There are two kinds of calculation algorithms for the coefficients P and Q . One is Wu's arbitrary pore aspect ratio (Wu, 1966), and the other is Berryman's 3D special pores (sphere, needle, penny-shaped crack, and disk^①) (Berryman, 1995). In this paper, the Kuster-Toksöz model with arbitrary pore aspect ratio is called the 2D KT model, and the one with 3D special pores is called the 3D KT model.

Then the availability of the three rock physics models above will be comparatively analyzed for two actual wells (Well A in the Tazhong area and Well B in the Lungu area, the Tarim Basin).

A comparison of the results predicted by the three rock

physics models and measured by sonic logging is shown in Fig. 3. The moduli and densities of minerals and fluids used for the velocity prediction are listed in Table 1, and they are also the basic parameters used for other studies in this paper. From Fig. 3 we can see that all of the three rock physics models above can predict the velocities (the inverse of slowness) of carbonate reservoirs in different accuracies. However, there are apparent deviations between the data measured and that predicted by the Wyllie time-average equation and the Gassmann equation, especially for the former. Significantly, the results predicted by the Kuster-Toksöz model agree well with those measured. The reason why the Kuster-Toksöz model has the best predicted results is that through adjusting pore aspect ratio or the proportion of various 3D pores it can effectively model the geometries of various storage spaces, which is the dominant factor influencing the velocities of the elastic-wave propagating in this type of complex carbonate reservoirs. Next, we will employ the best model (the Kuster-Toksöz model) to discuss the influence of secondary storage space shape on P- & S-wave velocities in complex carbonate reservoirs in the Tarim Basin.

Table 1 The moduli and densities of minerals and fluids used in this paper

Minerals and fluids	Density g/cm ³	Bulk modulus GPa	Shear modulus GPa	References
Calcite	2.71	76.80	32.000	Simmons (1965)
Dolomite	2.87	94.90	45.000	Nur and Simmons (1969)
Clays	2.55	25.00	9	Han et al (1986)
Brine (3%)	1.03	2.50	—	Adams (1931)
Oil	0.87	1.58	—	Standing (1952)
Gas	0.14	0.029	—	Thomas et al (1970)

^① When discussing with James G. Berryman in 2009, he definitely indicated that the disk-shaped pores are actually artificial. Hence, the disk-shaped pores are not used in this study.

Fig. 3 Comparisons of P-wave slowness between predicted by the Wyllie time-average equation (WL), the Gassmann equation (GM), and the Kuster-Toksöz model (KT) respectively and measured by sonic logging (AC). For each part of the figure, the first three columns denote the results of logging interpretation; in the fourth column (Pre.-AC), symbols ‘TKT’ (green curve), TGM (pink curve), and TWL (blue curve) denote the slowness predicted by the Kuster-Toksöz model, the Gassmann equation, and the Wyllie time-average equation respectively; in the fifth column (Pre.-AC error), symbols ‘WTWT’, ‘WTGM’, and ‘WTWL’ denote the corresponding relative errors of the predicted results compared with the measured results

3 The analysis of secondary storage space shape influencing P- & S-wave velocities

This part will employ the Kuster-Toksöz model with arbitrary pore aspect ratio (i.e., the 2D KT model) and that with 3D special pores (i.e., the 3D KT model) respectively to analyze the influence of secondary storage space shape on velocity prediction.

3.1 Arbitrary pore aspect ratio

For the 2D KT model, the storage space shape is described by the pore aspect ratio, the value of which is ranging from 0 to 1. In this part, Well C with very complex secondary storage spaces is selected as an example. When using the 2D KT model to predict velocities in the reservoir, the pore aspect ratio alters from 0.05 to 0.9. Fig. 4 shows the corresponding calculated P- & S-wave slowness of the well for each pore aspect ratio. The figure shows that both P- & S-wave slowness decrease with an increase in the pore aspect ratio, illustrating that the spherical pores with high aspect ratio tends to be stiffer than the oblate pores with low aspect ratio. That is to say, P- & S-wave velocities in the reservoir with spherical pores tend to be higher than that with oblate pores. It can be easily understood because stiffer pores contribute to the stiffness of the whole rock while oblate pores contribute to the compliance of the whole rock. The figure also shows that the increase in P- & S-wave velocities

is sharper when the pore aspect ratio increases from 0.05 to 0.3. However the increase trend is reduced when the pore aspect ratio increases from 0.3, and it tends to stop when the pore aspect ratio increases approximately to 0.5. Now, the predicted P- & S-wave slowness is more consistent with the measured slowness although there still is some deviation. All above illustrate that the dominant secondary storage spaces are dissolution holes or caves with high pore aspect ratio. Additionally, this paper defines 0.3 as the critical value of the pore aspect ratio, which is the turning point of the increase trend of predicted velocities for this type of carbonate rocks. The value is determined by the inherent elastic properties of rock and should be varied for different lithologies. However, for carbonate rocks, the critical aspect ratio (0.3) may be constant because Sun et al (2004) also obtained a similar value for western Canada carbonate rocks.

To account for the relationship between the pore aspect ratio and the P- & S-wave velocities more clearly, three relatively ideal depth points in Fig. 4 are chosen. They are respectively the depth of 6,186 m (Depth I) with a high porosity and a low clay content, the depth of 6,190 m (Depth II) with a low porosity and a high clay content, and the depth of 6,200 m (Depth III) with a low porosity and a low clay content. The predicted P- & S-wave slowness for the three depth points with different pore aspect ratios are extracted and converted into the corresponding P- & S-wave velocities (Velocity (m/s)=304,800/Slowness (μs/ft)). The

final velocities for the three depth points under different pore aspect ratios are plotted in Fig. 5.

The figure supports the following results:

- 1) As the whole, both P- & S-wave velocities increase with the pore aspect ratio, but obviously the influence of pore geometry on the P-wave velocity is greater than that on the S-wave velocity. The varying range of the P-wave velocity is 983-2,285 m/s while that of the S-wave velocity is 261-680 m/s. The result gives us an implication: due to the influence of pore geometry, not only the respective values of P- and S-wave velocities (V_p and V_s) but also the relative value between them will be altered. That is to say, the V_p - V_s relationships of the complex carbonate reservoirs may be not the same for different secondary storage space types. The detailed relationships will be discussed in Section 4.
- 2) Similarly to the results from Fig. 4, for the reservoirs with low pore aspect ratio (smaller than 0.3) the P- & S-wave velocities will significantly increase with the pore aspect

ratio while for the reservoirs with high pore aspect ratio the increasing trend is lower. This phenomenon illustrates that the major factor influencing the large-scale velocity change is oblate pores (or cracks). For deep-buried carbonate reservoirs, the brittleness of rock is very strong, so cracks always occur more or less. It further illustrates the significance of the study of how secondary storage spaces influence the velocities of the elastic wave propagating in complex carbonate reservoirs.

3) It can be also found that for different reservoir conditions (e.g., porosity and clay content) the ranges of P- & S-wave velocities are different. Under the same pore aspect ratio, the P- & S-wave velocities (V_{PI} , V_{SI}) for reservoir condition I (high porosity and low clay content) are the lowest and those (V_{PIII} , V_{SIII}) for reservoir condition III (low porosity and low clay content) are the highest. And the reservoir condition II (low porosity and high clay content) has intermediate P- & S-wave velocities (V_{PII} , V_{SII}). Those above mainly reflect the influence of lithology and reservoir

Fig. 4 The change characteristics of P- & S-wave slowness predicted by the 2D KT model when the pore aspect ratio increases from 0.05 to 0.9. Symbols ‘TKT’ and ‘DSKT’ denote the predicted P- & S-wave slowness using the KT model respectively; Symbols ‘AC’ and ‘DS’ denote the measured P- & S-wave slowness respectively; ‘a’ in each bracket denotes the pore aspect ratio

Fig. 5 The characteristics of P- & S-wave velocities varying with the pore aspect ratio. Reservoir condition I-high porosity, low clay content; reservoir condition II-low porosity, high clay content; reservoir condition III-low porosity, low clay content

physical properties on P- & S-wave velocities. However, the wave velocities under the same reservoir conditions may be also significantly different if the pore aspect ratios are different. Meanwhile, even there are obvious differences in different reservoir conditions (dominating the range of reservoir velocities), the velocities may also be similar to each other if the difference of pore aspect ratios is big enough. For example, the P-wave velocity in a low-porosity and high-clay-content reservoir (corresponding to reservoir condition II) with a pore aspect ratio of 0.9 is approximately to that in a low-porosity and low-clay-content reservoir (corresponding to reservoir condition III) with a pore aspect ratio of 0.1. Both the P- & S-wave velocities in a high-porosity and low-clay-content reservoir (corresponding to reservoir condition I) with pore aspect ratios more than 0.9 are approximately the same as those in a low-porosity and high-clay-content reservoir (corresponding to reservoir condition II) with a pore aspect ratio less than 0.2, and even the P-wave velocity of the former is higher than the latter when the difference of their pore aspect ratios is big enough (e.g., 0.9 for the former and 0.1 for the latter). All of the phenomena above illustrate that the influence of secondary storage space shape on P- & S-wave velocities may outweigh that of lithology and reservoir physical properties on P- & S-wave velocities, similarly to the conclusions presented by Wang (2001).

3.2 Combination of 3D special pores in certain proportion

In the 3D KT model, storage space shape is described by Berryman's 3D special pores (Berryman, 1995). To model the secondary storage space shape in the complex carbonate reservoir discussed in this paper, the storage spaces in each set of the reservoirs are assumed to be the combination of cracks, needles (representing needle-shaped dissolution holes), and spheres (representing dissolution caverns) in certain proportion. To model various secondary storage spaces, three storage space combining models are established in this

part (shown in Fig. 6(a)). In the first model, the proportion of crack, needle, and sphere is 0.8: 0.1: 0.1 (namely, crack as the dominate storage space). In the second model, the proportion is 0.1: 0.8: 0.1 (namely, needle-shaped void as the dominate storage space). In the third model, the proportion is 0.1: 0.1: 0.8 (namely, sphere as the dominate storage space). The P- & S-wave slowness (TKT and DSKT) predicted by the 3D KT model for the combination of different storage spaces are shown in Fig. 6(b) (Well C is still selected as an example). Fig. 6 shows that if the dominant storage space type is crack-shaped pore, the predicted P- & S-wave slowness (see the curves named "TKT" and "DSKT") are far from the corresponding measured P- & S-wave slowness (see the curves named "AC" and "DS"). If sphere-shaped pore or needle-shaped pore is assumed as the dominant storage space type, the difference between the predicted and the measured is reduced. And especially at the assumption of needle-shaped pore, the difference is minimum. Incorporating with the results obtained when employing the 2D KT model (the storage space in the interest interval has high pore aspect ratio), it can be concluded that the dominant storage space type for the interest interval should be dissolution holes. Therefore, the integrated application of the 2D and 3D KT models can help us properly determine the storage space types of the complex carbonate reservoir, thus providing much more information to enable us to have a deeper insight into the reservoir.

Additionally, it also should be noticed that the dominant secondary pores in different intervals in a well are generally different. The difference can significantly influence the predicted results of wave velocities. To illustrate this issue, Well D (a well with relative complex storage spaces) in the study area is taken as an example. Under the assumptions of crack-shaped pore and sphere-shaped pore as the dominant pore respectively, the 3D KT model is employed to predict the P- & S-wave slowness of the well (shown in Fig. 7).

According to the figure, it can be seen that for the upper and lower intervals marked by the light blue pane, the predicted P- & S-wave slowness (green lines: TKT(C), DSKT(C)) under the assumption of crack-shaped pores as the dominant storage space are closer to the corresponding measured slowness (dark lines: AC, DS) than those (purple lines: TKT(N-S), DSKT(N-S)) under the assumption of needle- and sphere-shaped pores as the dominant storage space. However, for the middle interval marked by light red pane, the predicted slowness under the latter assumption is closer to the measured than those under the former assumption. Hereby we can judge that the dominant storage space in the upper and lower intervals of the well are fractures (or cracks), so the geometric coefficient of Berryman's penny-shaped crack can be used for the modeling. Similarly, it can be judged that the dominant storage space types in the middle interval of the well are dissolution holes and vugs, so the geometric coefficient of Berryman's sphere- and needle-shaped pores can be used here. The differences of the dominant storage space types of different intervals in the same well indicate that the velocity prediction process needs to be done interval by interval.

(a)

(b)

Fig. 6 Three sets of secondary storage space combination models (a) and the comparison between the predicted P- & S-wave slowness ('TKT' and 'DSKT') by employing 3D KT model and the corresponding measured P- & S-wave slowness ('AC' and 'DS') (b). Symbol 'N' denotes needle-shaped pore; Symbol 'S' denotes sphere-shaped pore; Symbol 'C' denotes crack-shaped pore; other symbols are the same to those above

Fig. 7 The influence of the difference of dominant storage space types in different intervals in a well on predicted results of wave velocities

4 The V_p - V_s relationships relating with secondary storage space shape

4.1 Analysis of S-wave velocity estimation using common V_p - V_s relationships

In the following simultaneous inversion based on prestack P-wave data (discussed in the next paper of the special edition (Zhang et al, 2011), the P- & S-wave velocities (V_p and V_s) of each well are not only taken as the constraint for well-seismic-calibration and wavelet extraction but also the startup term to build the relationship between P-impedance (PI) and S-impedance (SI). For accurate P- & S-wave velocity prediction, we resort to employ or establish a proper rock physics model that can consider various factors including porosity, pore shape, fluid types, etc. After the standardization of the predicted velocities, the V_p - V_s relationship fitting for the study area can be built through mathematical regression.

These results not only consider more factors but also represent the special elastic characters of the study area well, thus providing more reasonable constraint information for the subsequent prestack inversion.

However, at present, an improper method is commonly used to obtain the two constraint conditions, especially on-site in oilfields. Due to lack of the measured S-wave data and the limitations of carrying on complex rock physics analysis for velocity prediction, some known empirical relationships, such as Castagna’s mud-rock line (Castagna et al, 1985) and Greenberg-Castagna’s V_p - V_s relationship (Greenberg and Castagna, 1992) (see Eqs. (3) and (4)), are directly treated as the startup term to build the relationship between PI and SI, and are also employed to derive the S-wave velocity based on the measured P-wave velocity.

$$V_s = 0.862V_p - 1.172 \tag{3}$$

$$V_s = -1.03 + 1.017V_p - 0.055V_p^2$$

(4)

where both P- & S-wave velocities are in km/s.

Though Castagna’s mud-rock line and Greenberg-Castagna’s V_p - V_s relationship are well-known, they are built for only one or several areas, so they cannot represent the elastic properties of reservoirs in other areas due to the differences of burial depth, compaction, lithology, pore shapes, etc. Especially for the carbonate reservoirs with very complex storage spaces in the Tarim Basin, the V_p - V_s relationship is seriously dependent on the secondary storage space shape (corresponding to the explanation for Fig. 5). For complex carbonate reservoirs, it would be absurd if those empirical relationships having nothing to do with pore geometry and the derived S-wave data from them were taken as the two important constraint conditions for pre-stack

inversion.

To illustrate the above problem more intuitively, based on actual logging data of a well (Well E), the results of S-wave slowness (reverse of velocities) derived from the above two conventional V_p - V_s relationships that have nothing to do with secondary storage space shape are compared with the measured data. The P- & S-wave slowness predicted by the Kuster-Toksöz model (taking into consideration the effect of secondary storage space geometry) are also plotted in Fig. 8. The figure shows that the DSC (S-wave slowness derived from Castagna’s mud-rock line) is consistently furthest from the DS (the measured S-slowness) because Castagna’s mud-rock line was developed for clastic rocks; the DSGC (S-wave slowness derived from the Greenberg-Castagna V_p - V_s relationships) is relatively closer to the DS because the Greenberg-Castagna V_p - V_s relationship was developed on

Fig. 8 A comparison of S-wave slowness derived from conventional V_p - V_s relationships and that measured. The P- & S-wave slowness ‘TKT’ and ‘DTKT’ predicted by the KT model are also plotted together. Symbols ‘DSC’ and ‘DSGC’ denote the S-wave slowness derived from Castagna’s mud-rock line and the Greenberg-Castagna V_p - V_s relationship respectively; Symbols ‘WDSC’ and ‘WDSGC’ denote the relative errors of the estimated S-wave slowness compared with that measured; other symbols are the same to those above

water-saturated limestones (the main lithology of carbonate rocks); the DSKT (S-wave slowness predicted by the KT model) is consistent best with the DS because lithology and reservoir physical properties are not only considered in the KT model but also more importantly secondary storage space geometries are taken into account.

It should be pointed out that we do not advocate using empirical relationships to derive S-wave information. However, it should not absolutely negate the practical value of empirical relationships because they indeed can provide a ready way to estimate S-wave information when high accuracy is not required in oilfield practice. We just emphasize that empirical relationships should not directly copy the results from other areas but should be built based on the data of the corresponding study area. Meanwhile, the influence of pore geometry should be also taken into account for reservoirs that have complex pore spaces. Thus, with respect to complex carbonate reservoirs in the Tarim Basin, which way is reasonable to build the V_p - V_s relationship? Our answer is that respective V_p - V_s relationship must be built for different secondary storage spaces.

4.2 The building of V_p - V_s relationships related with secondary storage space shape

The Ordovician carbonate reservoirs in the Tarim Basin are representative of complex carbonate reservoirs dominated by secondary storage spaces, and are also very important for China's future oil and gas supplies. Hence, it is important to build proper V_p - V_s relationships for this basin.

On the basis of accurate P- & S-wave velocity prediction and secondary storage space modeling for more than 50 wells in the Tazhong area and the Lungu area (the major production areas in the Tarim Basin), incorporating with some available measured P- & S-wave velocities and referring to reservoir type classification data provided by the Tarim Oilfield Co., we built three sets of quadratic V_p - V_s relationships[®] respectively for fractured, fractured-cavernous, and fractured-hole-vuggy carbonate reservoirs in the Tarim Basin (shown in Fig. 9).

To simplify these relationships, we can write them in a standard form, namely,

$$V_s = \alpha V_p^2 + \beta V_p + \gamma \quad (5)$$

where both V_p and V_s are in km/s; coefficients α , β and γ determine the V_p - V_s relationship that corresponds to the type of carbonate reservoir, and their values are listed in Table 2.

[®] The naming rule is that dominant pore type as the headword is placed in the latter part and subsidiary pore type as the modifier is placed in the former part. In our opinion, due to the strong brittleness of rocks in deep-buried carbonate reservoir, fractures always exist to a greater or lesser extent in the reservoir, so each secondary storage space combining type includes fracture. Furthermore, because fractured-holed reservoir with fracture and pure-needle-shaped pores is few in this study and it is often classified into fractured-hole-vuggy reservoir in oilfield site, so there is no special V_p - V_s relationship built for fractured-holed reservoir here.

Fig. 9 The built V_p - V_s relationships respectively for fractured, fractured-cavernous, and fractured-hole-vuggy carbonate reservoirs in the Tarim Basin

Table 2 The coefficients of quadratic empirical relationships (Eq. (5)) for different carbonate reservoir types in the Tarim Basin

Carbonate reservoir types	α	β	γ
Fractured reservoir	-0.1397	2.2638	-5.4223
Fractured-cavernous reservoir	-0.1048	1.669	-3.1092
Fractured-hole-vuggy reservoir	-0.0375	0.7966	-0.2907

5 Conclusions

The topics discussed in this paper include: 1) Illustration of the conceptual differences between the three classical basic rock physics models (the Wyllie time-average equation, Gassmann equation, and the Kuster-Toksöz model) and the comparison of their velocity prediction results. 2) The influence of secondary storage space shape on velocity prediction results; 3) The establishment of V_p - V_s relationships related to secondary storage space shape.

As for the above topics, a number of assumptions or approximations are performed. We assume the medium of carbonate rocks in this case study is isotropic, which is also the postulation permitting us to employ these rock physics models for our discussion. The fluids are assumed to be zero shear modulus (i.e., to be infinitely compressible) so that pore compressions do not induce changes in pore pressure.

With the above discussed results and limitations in mind, the following conclusions can be derived from the discussion regarding the influence of secondary storage space shape on P- & S-wave velocities for complex carbonate reservoirs in this paper:

1) Wyllie time-average equation, Gassmann equation, and Kuster-Toksöz model are three basic classical rock physics models. Through the comparison for the model-constructing principles and velocity prediction results, the Kuster-Toksöz model is proven to be the best one to discuss the influence of secondary storage space shape on P- & S-wave velocities. And the dominant storage space type can be derived by the integration of the 2D KT model and the 3D KT model.

2) Secondary storage space shape significantly influences the P- & S-wave velocities in carbonate reservoirs. Both P- & S-wave velocities increase with pore aspect ratio, but

obviously the influence of pore geometry on the P-wave velocity is greater than that on the S-wave velocity. Additionally, the increase in amplitude for lower pore aspect ratios is greater than that for higher pore aspect ratios. That is to say, oblate pores or cracks influence the P- & S-wave velocities in reservoirs more significantly.

3) Secondary storage space shape also influences the relationship between the P-wave velocity (V_p) and S-wave velocity (V_s) in carbonate reservoirs. On the basis of velocity prediction and secondary storage space type modeling, the corresponding V_p - V_s relationships are built respectively for fractured, fracture-cavernous, and fracture-hole-vuggy carbonate reservoirs, thus making common simple V_p - V_s relationship to relate with secondary storage space shape. This work has important practical significance for simultaneous inversion using PP data, to help improve complex carbonate reservoir prediction results.

But we have to point out that there are some limitations in the Kuster-Toksöz model. Firstly, this model is appropriate to ultrasonic laboratory condition, so the velocities predicted by this model generally exhibit differences with seismic wave velocities in low-frequency bands (this is the velocity dispersion issue that will be discussed in the future). Secondly, this model is limited to dilute concentrations of pores. That is to say, the porosity could not be too high. To solve the problem, differential effective medium theory can be used to modify this model for better velocity prediction results. But the two problems will not obviously influence the results discussed in this paper because the secondary storage space shape plays a dominant role in influencing the P- & S-wave velocities of the reservoir for carbonate reservoirs with complex storage space types in the Tarim Basin.

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