Numerical simulation of an acoustic field generated by a phased arc array in a fluid-filled borehole

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Abstract: The acoustic tools widely used in borehole well logging and being developed in borehole acoustic reflection imaging do not have the function of azimuthal measurement due to a symmetric source, so they can not be used to evaluate the azimuthal character of borehole formation. In this paper, a 3D finite difference method was used to simulate the acoustic fields in a fluid-filled borehole generated by a traditional monopole source and a phased arc array. Acoustic waveforms were presented for both cases. The analysis of the simulated waveforms showed that different from the monopole source, the acoustic energy generated by the phased arc array transmitter mainly radiated to the borehole in a narrow azimuthal range, which was the key technique to implement azimuthal acoustic well logging. Similar to the monopole source, the waveforms generated by the phased arc array in the fluid-filled borehole also contain compressional (P) waves and shear (S) waves refracted along the borehole, which is the theoretical foundation of phased arc array acoustic well logging.

Key words: Phased arc array, acoustic field, acoustic well logging, azimuth, finite difference method

1 Introduction

With the development of complicated hydrocarbon exploration, azimuthal acoustic logging has been urgently needed (Li and Chu, 1997). The acoustic monopole source, widely used in acoustic well logging, can be treated approximately as a point source, so the acoustic field generated by the point source in a fluid-filled borehole is axially symmetric. In a fluid-filled borehole surrounded by a soft formation, shear waves can not be generated by the monopole source. To implement shear wave acoustic logging in a soft formation, excited flexural motion was presented (Brie et al, 1998). The subsequent acoustic well logging technology combined quadrupole and dipole sources with the monopole source, which expanded the functions and application scope of acoustic well logging (Kessler and Varsamis, 2001; Tang and Cheng, 2004; Tang, 2004). Unlike the monopole source, the dipole and quadrupole sources are directional, but they have limited azimuthal measurement ability. For example, the directivity curve of the dipole source is like a “∞” shape. Another problem is multiple solutions in formation inversion. A great deal of research on acoustic well logging with phased linear arrays has been carried out (Che et al, 2005; 2006; Chen and Xu, 2008; Qiao et al, 2002; 2003). It is shown that the signal-to-noise ratio of the measurement signal can be improved by using a phased linear array, but the acoustic field generated by the phased linear array is also axially symmetric. The key to making acoustic well logging able to achieve azimuthal measurements is to

develop a special downhole source (Qiao et al, 2006a), which can radiate acoustic energy azimuthally onto the sidewall. Azimuthal radiation may fundamentally resolve the difficult problems of azimuthal measurement ability, evaluation of heterogeneous and anisotropic formations, and improvement of the signal-to-noise ratio. Qiao et al (2008) designed a phased arc array acoustic transmitter and obtained its radiation directivity curve by both experimental measurement and numerical calculation. The results showed that the acoustic energy generated by the phased arc array transmitter was in a specific azimuthal range. This study is to calculate the acoustic field generated by a phased arc array transmitter in a fluid-filled borehole and to compare it with the acoustic field generated by a traditional monopole source.

2 Working principle of a phased arc array transmitter

The schematic diagram of a 14-element acoustic phased arc array transmitter is shown in Fig. 1. Each element is a piezoelectric vibrator. If the five elements centering on the No. 4 element are excited, the acoustic travel time from the No. 2 element and the No. 3 element to line AB are τ_2 and τ_1 respectively.

When the phased arc array is activated, the No. 2 and the No. 6 elements are excited simultaneously and this moment is marked as zero time. Then the No. 3 and the No. 5 elements are excited simultaneously at $\tau_2 - \tau_1$, and the No. 4 element is excited at τ_2 . In this way, the wave fronts generated by all active elements can reach line AB at τ_2 simultaneously. It seems like a 5-element equivalent linear phased array centering on the No. 4 element, with each element located

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Fig. 1 Schematic diagram of a phased arc array acoustic transmitter

along a line parallel to the y -axis. If at a specific moment the No. 2–No. 6 elements emit, at the next moment the No. 3–No. 7 elements emit, and in the same way different elements are excited in turn at different moments, then azimuthally scanning radiation of acoustic energy can be achieved.

3 Numerical calculation

3.1 Method

The finite difference method is extensively used to simulate acoustic fields in acoustic well logging (Liu et al, 1996; Lin et al, 2006; He et al, 2006). In this paper, the acoustic field generated by a phased arc array in a fluid-filled borehole is simulated by the 3D finite difference method. Because the elastic wave equations of fluid can be regarded as a special case of those of a solid, the following finite difference equations are derived from the elastic wave equations of solids. When referring to fluid, the elastic coefficient $\mu=0$.

In Cartesian coordinates, for a homogeneous elastic medium with mass density of $\rho=\rho(x, y, z)$ and Lamé constants of $\lambda=\lambda(x, y, z)$ and $\mu=\mu(x, y, z)$, v_x, v_y , and v_z are particle velocity components in the x, y , and z directions, T_{xx}, T_{yy} , and T_{zz} are normal stresses of x, y , and z axes, T_{xy}, T_{xz} , and T_{yz} are shear stresses in the planes of xy, xz , and yz , and g_{ij} ($j = x, y, z$) is the acoustic source density of stress rate (Pa/s). The first-order partial differential equations (Liu et al, 1996; Lin et al, 2006) for the velocity and stress are given below:

$$\rho \frac{\partial v_x}{\partial t} = \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \quad (1)$$

$$\rho \frac{\partial v_y}{\partial t} = \frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} \quad (2)$$

$$\rho \frac{\partial v_z}{\partial t} = \frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial y} + \frac{\partial T_{zz}}{\partial z} \quad (3)$$

$$\frac{\partial T_{xx}}{\partial t} = \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) + 2\mu \frac{\partial v_x}{\partial x} + g_{xx} \quad (4)$$

$$\frac{\partial T_{yy}}{\partial t} = \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) + 2\mu \frac{\partial v_y}{\partial y} + g_{yy} \quad (5)$$

$$\frac{\partial T_{zz}}{\partial t} = \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) + 2\mu \frac{\partial v_z}{\partial z} + g_{zz} \quad (6)$$

$$\frac{\partial T_{xy}}{\partial t} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \quad (7)$$

$$\frac{\partial T_{xz}}{\partial t} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \quad (8)$$

$$\frac{\partial T_{yz}}{\partial t} = \mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \quad (9)$$

Assuming the unit size in the staggered grids is $\Delta x \times \Delta y \times \Delta z$ (Fig. 2 shows 1/8 of a unit in the staggered grids), the normal stress T_{mm} ($m=x, y, z$) is in the center of the unit, and the time step is Δt . Then in the staggered grids, the wave field components in the discrete space and time domains are defined as follows:

$$T_{mm}^n(i, j, k) = T_{mm} \left[\left(i + \frac{1}{2} \right) \Delta x, \left(j + \frac{1}{2} \right) \Delta y, \left(k + \frac{1}{2} \right) \Delta z; n \Delta t \right] \quad (10)$$

$m = x, y, z$

$$T_{xy}^n(i, j, k) = T_{xy} \left[i \Delta x, j \Delta y, \left(k + \frac{1}{2} \right) \Delta z; n \Delta t \right] \quad (11)$$

$$T_{xz}^n(i, j, k) = T_{xz} \left[i \Delta x, \left(j + \frac{1}{2} \right) \Delta y, k \Delta z; n \Delta t \right] \quad (12)$$

$$T_{yz}^n(i, j, k) = T_{yz} \left[\left(i + \frac{1}{2} \right) \Delta x, j \Delta y, k \Delta z; n \Delta t \right] \quad (13)$$

$$v_x^{n+1/2}(i, j, k) = v_x \left[i \Delta x, \left(j + \frac{1}{2} \right) \Delta y, \left(k + \frac{1}{2} \right) \Delta z; \left(n + \frac{1}{2} \right) \Delta t \right] \quad (14)$$

$$v_y^{n+1/2}(i, j, k) = v_y \left[\left(i + \frac{1}{2} \right) \Delta x, j \Delta y, \left(k + \frac{1}{2} \right) \Delta z; \left(n + \frac{1}{2} \right) \Delta t \right] \quad (15)$$

$$v_z^{n+1/2}(i, j, k) = v_z \left[\left(i + \frac{1}{2} \right) \Delta x, \left(j + \frac{1}{2} \right) \Delta y, k \Delta z; \left(n + \frac{1}{2} \right) \Delta t \right] \quad (16)$$

Fig. 2 Diagram of 1/8 of a unit in the staggered grids

After discretization (second-order in the time domain and eighth-order in the space domain), Eq. (1) is approximated as below:

$$\begin{aligned}
 v_x^{n+1/2}(i, j, k) = & v_x^{n-1/2}(i, j, k) \\
 & + \frac{\Delta t}{\rho \Delta x} \sum_{l=0}^3 a_l [T_{xx}^n(i+l, j, k) - T_{xx}^n(i-l-1, j, k)] \\
 & + \frac{\Delta t}{\rho \Delta y} \sum_{l=0}^3 a_l [T_{xy}^n(i, j+l+1, k) - T_{xy}^n(i, j-l, k)] \\
 & + \frac{\Delta t}{\rho \Delta z} \sum_{l=0}^3 a_l [T_{xz}^n(i, j, k+l+1) - T_{xz}^n(i, j, k-l)]
 \end{aligned} \quad (17)$$

In Eq. (17), the weight coefficients of a_l ($l=0-3$) are 1225/1024, $-245/3072$, $49/5120$, and $-5/7168$, respectively. Similar approximation can be applied to Eqs. (2)–(9).

3.2 Model well

The dimensions of the model well used for numerical calculation is $1.2 \text{ m} \times 1.2 \text{ m} \times 1.5 \text{ m}$. The distance from the borehole axis to the outer wall of the model well is 0.6 m, and the borehole diameter is 0.2 m. The P-wave velocity and S-wave velocity of the borehole formation are 4500 m/s and 2500 m/s respectively, and the density of the borehole formation is 2200 kg/m^3 . The velocity and density of the borehole fluid are 1500 m/s and 1000 kg/m^3 respectively. The space step is $\Delta x = \Delta y = \Delta z = 5 \text{ mm}$, the center frequency of pulse source $f_0 = 12 \text{ kHz}$, and the time step is $\Delta t = 2.5 \times 10^{-7} \text{ s}$.

Fig. 3(a) illustrates the locations of the acoustic transmitter and receiver array in the xy -plane of the fluid-filled borehole. “T” is the acoustic transmitter with a radius of 35 mm, “R” is the azimuthal receiver array centering on the acoustic transmitter with a radius of 90 mm. The azimuthal angle between the two adjacent receivers is 3.75° . The center of the phased arc array is in the direction of $\theta = 0^\circ$. Fig. 3(b) and Fig. 3(c) show the locations of the receiver array corresponding to the monopole source and the phased arc array transmitter in the fluid-filled borehole respectively, in which the axial distance of two adjacent receivers is 250 mm.

Fig. 3 Locations of acoustic transmitters and receiver array in the fluid-filled borehole

3.3 Numerical results

When the phased arc array acoustic transmitter is excited in the fluid-filled borehole, the waveforms received by the azimuthal receiver array (Fig. 3(a)) are shown in Fig. 4. In Fig. 4, the horizontal coordinates denote time and the vertical coordinates denote azimuthal angle θ . We define the 3dB main lobe width as $\theta_{3dB} = \theta_2 - \theta_1$, where θ_1 and θ_2 are the angles corresponding to 0.707 of the maximum peak to peak amplitude of the signal. The acoustic energy generated by the phased arc array is mainly focused in the main lobe with the 3dB main lobe width of 47.5° (corresponding to the waveforms marked by red arrows in Fig. 4), while the acoustic energy outside the azimuthal range is very weak. It is obviously different from the acoustic field generated by the monopole source, whose radiation energy is uniformly distributed in the circle direction (Qiao et al, 2006b).

Fig. 4 Waveforms received by the azimuthal receiver array when the phased arc array acoustic transmitter is excited in the fluid-filled borehole

Fig. 5(a) and Fig. 5(b) show the waveforms received by the axial receiver array, when the monopole source (Fig. 3(b)) and the phased arc array (Fig. 3(c)) are excited respectively. The horizontal coordinates denote time and the vertical coordinates denote axial offset, which increases gradually from 0.25 m to 1.5 m from top to bottom. It can be seen that the first arrivals with the smallest amplitude are the refracted P waves, followed by the refracted S waves with larger amplitude, for both acoustic sources. The velocities of the refracted P waves obtained from Fig. 5(a) and Fig. 5 (b) are both about 4460 m/s, and the velocities of the refracted S waves obtained from Fig. 5(a) and Fig. 5(b) are about 2460 m/s and 2400 m/s respectively. In detail, the waveforms generated by the two sources with the axial offset of 0.75 m in Fig. 5(a) and Fig. 5(b) are redrawn in Fig. 5(c). It can be seen that, similar to the traditional symmetric source, the waves generated by the phased arc array in the fluid-filled borehole also contain refracted P waves and refracted S waves, which establishes the theoretical foundation of acoustic well logging using a phased arc array. Because the phased arc array is closer to the sidewall than the point source in the numerical

calculation, all kinds of mode waves generated by the phased arc array arrive earlier than those generated by the monopole source in the borehole. However, the velocities of refracted P waves and S waves inverted from the waveforms are basically the same for the two sources, meaning that the inverted formation information will not be affected.

Fig. 5 Waveforms received by the axial receiver array when the sources are excited in the fluid-filled borehole (a) monopole source (b) phased arc array acoustic transmitter (c) comparison of waveforms with an axial offset of 0.75 m

4 Conclusions

1) Different from the traditional monopole source, a phased arc array acoustic transmitter is able to radiate acoustic energy to the borehole in a narrow azimuthal range, so evaluating the azimuthal character of the borehole formation is feasible.

2) Similar to the symmetric source, the waves generated by the phased arc array in the fluid-filled borehole also contain refracted P waves and S waves of the formation, and this is the theoretical foundation of the phased arc array acoustic logging.

3) Phased arc array azimuthal acoustic logging may become a new generation of acoustic logging technology, but the complexity of transducers and the corresponding electronic circuit must be faced.

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