2.5-D modeling of cross-hole electromagnetic measurement by finite element method

Shen Jinsong^{1,2 *} and Sun Wenbo¹

¹ Department of Geophysics, School of Resources and Information Technology, China University of Petroleum, Beijing 102249, China

² CNPC Key Lab of Geophysical Exploration, China University of Petroleum, Beijing 102249, China

Abstract: A finite element method is developed for simulating frequency domain electromagnetic responses due to a dipole source in the 2-D conductive structures. Computing costs are considerably minimized by reducing the full three-dimensional problem to a series of two-dimensional problems. This is accomplished by transforming the problem into y -wave number (K_y) domain using Fourier transform and the y -axis is parallel to the structural strike. In the K_y domain, two coupled partial differential equations for magnetic field H_y and electric field E_y are derived. For a specific value of K_y , the coupled equations are solved by the finite element method with isoparametric elements in the x - z plane. Application of the inverse Fourier transform to the K_y domain provides the electric and magnetic fields in real space. The equations derived can be applied to general complex two-dimensional structures containing either electric or magnetic dipole source in any direction. In the modeling of the electromagnetic measurement, we adopted a pseudo-delta function to distribute the dipole source current and circumvent the problem of singularity at the source point. Moreover, the suggested method used isoparametric finite elements to accommodate the complex subsurface formation. For the large scale linear system derived from the discretization of the Maxwell's equations, several iterative solvers were used and compared to select the optimal one. A quantitative test of accuracy was presented which compared the finite element results with analytic solutions for a dipole source in homogeneous space for different ranges and different wave numbers K_y , to validate the code and check its effectiveness. In addition, we addressed the effects of the distribution range τ of the pseudo-delta function on the numerical results in homogeneous medium.

Key words: 2.5-D, electromagnetic modeling, frequency domain, isoparametric finite element, pseudo-delta function

1 Introduction

Electromagnetic methods provide information about the formation resistivity which is one of the most important parameters in saturation evaluation in the oil industry (Farrelly et al, 2004). Forward modeling is one of the most common and effective methods that help us understand the physical significance of the electromagnetic responses. However, there are many restrictions on electromagnetic modeling which are caused by the difficulty of dealing properly with actual 2-D or 3-D subsurface structures and considerable numbers of inhomogeneities (Tompkins, 2004; Avdeeva et al, 2007). In order to simulate actual subsurface formation for surveys in complex geological areas, the configuration irregularity of the target formation must also be considered (Gallardo et al and Meju, 2003), which becomes a difficult problem for traditional modeling methods, such as finite difference method. Moreover, we must be able to put inhomogeneities

near the source to estimate the effect of source overprint (Ueda and Zhdanov, 2005; Zaslavsky, 2006; Johansen et al, 2005).

In this study, we implemented the modeling of the so-called 2.5-D problem which considers a 3-D source over a 2-D earth (MacGregor et al, 2006). With respect to 2.5-D electromagnetic modeling, only a limited number of papers are published because of the difficulty of dealing with the source. Kong et al (2008) calculated the frequency domain responses excited by a magnetic dipole source with the finite element method (FEM). Leppin (1992) and Liu and Sato (2004) estimated the time domain responses generated by a rectangular current loop with the implicit Finite Difference Method (FDM) in a simple medium. The finite element method (FEM) was adopted by David and Morrison (1995) to compute the electromagnetic fields induced by a magnetic dipole source to simulate inhomogeneities of the formation. Sugeng et al (1996) also applied the isoparametric finite element method to determining the secondary electromagnetic fields excited by a rectangle loop.

In the published results, most of the schemes are carried

*Corresponding author. email: shenjinsong@cup.edu.cn

Received June 28, 2007

out by solving the secondary field due to anomalous bodies (Davydycheva et al, 2003). The reason for isolating the primary field is that the dipole source term is singular at the source point and is difficult to be expressed accurately by discrete equations (Goldman et al, 1986). For most cases, the primary field is calculated for the homogeneous or 1-D structure for which analytical equations exist (Hoversten et al, 2003; Hoversten and Newman, 2004; 2006). This classical separation of the primary and the secondary field is not effective for complex subsurface formation because of the difficulty of determining the background structure (Hou and Torres-Verdín, 2003).

In this study, we derived the basic equations of the 2.5-D electromagnetic problem and used a pseudo-delta function to express a dipole source (Constable and Weiss, 2004; Constable, 2006; Constable and Srnka, 2007; Abubakar et al, 2006). We focused on the numerical considerations of finite element electromagnetic modeling. Several experiments were carried out to demonstrate the effect of the approximate pseudo-delta on the numerical accuracy and efficiency (Buell, 1991). By using the approximate pseudo-delta, the problems of separating the total electromagnetic fields into primary and secondary field and determination of the background structure were avoided. In addition, we could easily simulate the complex subsurface formation by using the isoparametric FEM. In order to check the validity and efficiency of the code, the numerical result was first compared with the analytic solution in the homogeneous structure with different linear system solvers. Then the layer model was used to probe the optimal wave number range of K_y . Furthermore, we addressed the effects of the distribution range τ of the pseudo-delta function on the numerical results in homogeneous medium. Finally, we focused attention on the cross-hole electromagnetic responses of the complex formation structures often encountered in oil industry.

2 Basic equations of the 2.5-D electromagnetic problem

The earth model considered in this paper is shown in Fig. 1, where the y-axis is set to be the strike direction. Conductivity σ , dielectric permittivity ϵ and magnetic permeability μ are invariants along the y-axis and they vary only in the x-z plane. The electric or magnetic dipole source can be located anywhere and in any direction. Assuming a time dependence function $e^{i\omega t}$, the electric field E and magnetic field H are described by Maxwell's equations

$$\nabla \times E = -\hat{z}H - M_s \quad (1a)$$

$$\nabla \times H = \hat{y}E + J_s \quad (1b)$$

where M_s and J_s are the impressed magnetic and electric sources respectively, and the impedivity $\hat{z} = i\mu\omega$ and the admittivity $\hat{y} = \sigma + i\epsilon\omega$ (Ward and Hohmann, 1988) are used.

After adoption of the curl operator, we have

$$\begin{aligned} & \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \bar{e}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \bar{e}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \bar{e}_z \\ & = -\hat{z}[(H_x + M_{sx})\bar{e}_x + (H_y + M_{sy})\bar{e}_y + (H_z + M_{sz})\bar{e}_z] \end{aligned} \quad (2a)$$

$$\begin{aligned} & \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \bar{e}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \bar{e}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \bar{e}_z \\ & = (\hat{y}E_x + J_{sx})\bar{e}_x + (\hat{y}E_y + J_{sy})\bar{e}_y + (\hat{y}E_z + J_{sz})\bar{e}_z \end{aligned} \quad (2b)$$

Fig. 1 Two-dimensional medium with y-axis strike

From equations (2a) and (2b), we can obtain six formulas of twelve variables on the electromagnetic field and its derivative.

Applying the Fourier transform to each component of E and H and source terms in equations (2a) and (2b) with respect to y , that is

$$\hat{F}(x, k_y, z, \omega) = \int_{-\infty}^{\infty} F(x, y, z, \omega) e^{-ik_y y} dy, \quad \frac{\partial \hat{F}}{\partial y} = -ik_y \hat{F} \quad (3)$$

where $\hat{F}$ can be H_x, H_y, H_z, E_x, E_y or E_z .

We obtain two coupled governing differential equations for E_y and H_y (Stoyer and Greenfield, 1976).

$$\begin{aligned} & \frac{\partial}{\partial x} \left(\frac{\hat{y}}{k_e^2} \frac{\partial \hat{E}_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\hat{y}}{k_e^2} \frac{\partial \hat{E}_y}{\partial z} \right) - \hat{y} \hat{E}_y + ik_y \left[\frac{\partial}{\partial x} \left(\frac{1}{k_e^2} \right) \frac{\partial \hat{H}_y}{\partial z} \right. \\ & \left. - \frac{\partial}{\partial z} \left(\frac{1}{k_e^2} \right) \frac{\partial \hat{H}_y}{\partial x} \right] = \hat{J}_{sy} - ik_y \left[\frac{\partial}{\partial x} \left(\frac{1}{k_e^2} \hat{J}_{sx} \right) + \frac{\partial}{\partial z} \left(\frac{1}{k_e^2} \hat{J}_{sz} \right) \right] \\ & + \frac{\partial}{\partial x} \left(\frac{k^2}{k_e^2} \hat{M}_{sz} \right) - \frac{\partial}{\partial z} \left(\frac{k^2}{k_e^2} \hat{M}_{sx} \right) \end{aligned} \quad (4a)$$

$$\begin{aligned} & \frac{\partial}{\partial x} \left(\frac{\hat{z}}{k_e^2} \frac{\partial \widehat{H}_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\hat{z}}{k_e^2} \frac{\partial \widehat{H}_y}{\partial z} \right) - \widehat{z} \widehat{H}_y + ik_y \left[-\frac{\partial}{\partial x} \left(\frac{1}{k_e^2} \right) \frac{\partial \widehat{E}_y}{\partial z} \right. \\ & \left. + \frac{\partial}{\partial z} \left(\frac{1}{k_e^2} \right) \frac{\partial \widehat{E}_y}{\partial x} \right] = \widehat{z} \widehat{M}_{sy} - ik_y \left[\frac{\partial}{\partial x} \left(\frac{\hat{z}}{k_e^2} \widehat{M}_{sx} \right) + \frac{\partial}{\partial z} \left(\frac{\hat{z}}{k_e^2} \widehat{M}_{sz} \right) \right] \\ & + \frac{\partial}{\partial x} \left(\frac{\hat{z}}{k_e^2} \widehat{J}_{sz} \right) - \frac{\partial}{\partial z} \left(\frac{\hat{z}}{k_e^2} \widehat{J}_{sx} \right) \end{aligned} \quad (4b)$$

where $k^2 = -\widehat{z}\widehat{y}$ and $k_e^2 = k_y^2 - k^2$.

The other components are obtained from the space derivatives of $\widehat{E}_y$ and $\widehat{H}_y$ as follows

$$\widehat{E}_x = \frac{1}{k_e^2} \left(-ik_y \frac{\partial \widehat{E}_y}{\partial x} - \widehat{z} \frac{\partial \widehat{H}_y}{\partial z} - \widehat{z} \widehat{J}_{sx} - ik_y \widehat{z} \widehat{M}_{sz} \right) \quad (5a)$$

$$\widehat{E}_z = \frac{1}{k_e^2} \left(-ik_y \frac{\partial \widehat{E}_y}{\partial z} + \widehat{z} \frac{\partial \widehat{H}_y}{\partial x} - \widehat{z} \widehat{J}_{sz} - ik_y \widehat{z} \widehat{M}_{sx} \right) \quad (5b)$$

$$\widehat{H}_x = \frac{1}{k_e^2} \left(-ik_y \frac{\partial \widehat{H}_y}{\partial x} + \widehat{y} \frac{\partial \widehat{E}_y}{\partial z} + k^2 \widehat{M}_{sx} + ik_y \widehat{z} \widehat{J}_{sz} \right) \quad (5c)$$

$$\widehat{H}_z = \frac{1}{k_e^2} \left(-ik_y \frac{\partial \widehat{H}_y}{\partial z} - \widehat{y} \frac{\partial \widehat{E}_y}{\partial x} + k^2 \widehat{M}_{sz} - ik_y \widehat{z} \widehat{J}_{sx} \right) \quad (5d)$$

3 Finite element equations with isoparametric elements

3.1 Discretization of the Maxwell's equations

In order to solve the above equations by the numerical method, we adopted the Galerkin method (Zienkiewicz, 1977). First, the calculation area was divided into isoparametric elements. Then, the higher order interpolation function of the finite element method was used to calculate the space derivative of $\widehat{E}_y$ and $\widehat{H}_y$ in equations. Here, a quadratic element with eight nodes which is shown in Fig.

Fig. 2 Isoparametric finite element

2 was adopted. In each element, $\widehat{E}_y$ and $\widehat{H}_y$ are expressed by the quadratic interpolation function and the space derivative is able to be described by a linear interpolation function. Furthermore, in each element σ , μ , ε are constant and the interpolation function is represented by the local coordinates ξ , η (Zienkiewicz, 1977).

The global coordinates and each component F of electromagnetic field can be written as

$$x = \sum_{j=1}^8 N_j^e x_j, z = \sum_{j=1}^8 N_j^e z_j, F = \sum_{j=1}^8 N_j^e F_j \quad (6a)$$

$$N_i^e = \begin{cases} \frac{1}{4}(1+\xi\xi)(1+\eta\eta)(\xi\xi + \eta\eta - 1) & (i = 1, 3, 5, 7) \\ \frac{1}{2}(1-\xi^2)(1+\eta\eta) & (i = 2, 6) \\ \frac{1}{2}(1-\eta^2)(1+\xi\xi) & (i = 4, 8) \end{cases} \quad (6b)$$

where N_i^e is the quadratic interpolation function of the i -th node in the e -th element.

Applying the Galerkin method to equation (4a) and (4b), the linear equations system is obtained

$$\sum_{e=1}^{N_e} \iint_{D_e} \left\{ \frac{\partial N_i^e}{\partial x} \left(\frac{\hat{y}}{k_e^2} \frac{\partial \widehat{E}_y}{\partial x} \right) + \frac{\partial N_i^e}{\partial z} \left(\frac{\hat{y}}{k_e^2} \frac{\partial \widehat{E}_y}{\partial z} \right) + N_i^e \widehat{y} \widehat{E}_y + \frac{ik_y}{k_e^2} \left(\frac{\partial N_i^e}{\partial x} \frac{\partial \widehat{H}_y}{\partial z} - \frac{\partial N_i^e}{\partial z} \frac{\partial \widehat{H}_y}{\partial x} \right) \right\} dx dz = \quad (7a)$$

$$\begin{aligned} & - \sum_{e=1}^{N_e} \iint_{D_e} \left\{ N_i^e \widehat{J}_{sy} + \frac{ik_y}{k_e^2} \left(\frac{\partial N_i^e}{\partial x} \widehat{J}_{sx} + \frac{\partial N_i^e}{\partial z} \widehat{J}_{sz} \right) + \frac{k^2}{k_e^2} \left(\frac{\partial N_i^e}{\partial z} \widehat{M}_{sx} - \frac{\partial N_i^e}{\partial x} \widehat{M}_{sz} \right) \right\} dx dz + \sum_{e=1}^{N_e} \oint_{\partial D_e} N_i^e \widehat{H}_l dl \\ & \sum_{e=1}^{N_e} \iint_{D_e} \left\{ \frac{\partial N_i^e}{\partial x} \left(\frac{\hat{z}}{k_e^2} \frac{\partial \widehat{H}_y}{\partial x} \right) + \frac{\partial N_i^e}{\partial z} \left(\frac{\hat{z}}{k_e^2} \frac{\partial \widehat{H}_y}{\partial z} \right) + N_i^e \widehat{z} \widehat{H}_y + \frac{ik_y}{k_e^2} \left(-\frac{\partial N_i^e}{\partial x} \frac{\partial \widehat{E}_y}{\partial z} + \frac{\partial N_i^e}{\partial z} \frac{\partial \widehat{E}_y}{\partial x} \right) \right\} dx dz = \quad (7b) \\ & - \sum_{e=1}^{N_e} \iint_{D_e} \left\{ N_i^e \widehat{z} \widehat{M}_{sy} + \frac{ik_y}{k_e^2} \left(\frac{\partial N_i^e}{\partial x} \widehat{M}_{sx} + \frac{\partial N_i^e}{\partial z} \widehat{M}_{sz} \right) + \frac{\hat{z}}{k_e^2} \left(\frac{\partial N_i^e}{\partial z} \widehat{J}_{sx} - \frac{\partial N_i^e}{\partial x} \widehat{J}_{sz} \right) \right\} dx dz - \sum_{e=1}^{N_e} \oint_{\partial D_e} N_i^e \widehat{E}_l dl \end{aligned}$$

where the second integrals on the right hand are line integrals around the boundary ∂D_e of element region D_e , $\widehat{E}_l$ and $\widehat{H}_l$ are the tangential components for ∂D_e , and N_e is the total number of the FEM elements.

By setting the above equations numerically according to the number of the FEM elements, we formed the global system matrix equation. Because of the continuity of the tangential components $\widehat{E}_l$ and $\widehat{H}_l$ at the boundary between elements, the last terms on the right hand of equations (7a) and (7b) are canceled and remain only at the boundaries of the discrete area. At these boundaries, according to the simple Dirichlet condition, the field values are assigned to be zero (Druskin and Knizhnerman, 1994).

3.2 Solution to the linear equations system

Here, we solved the global system matrix equation with the iterative method. Since the stiffness matrix is symmetric and banded, the incomplete Cholesky preconditioned conjugate gradient (ICCG) method and the modified Cholesky decomposition method were used (Weiss and Newman, 2003). Strictly speaking, the stiffness matrix is simple symmetric, not Hermitian symmetric. However, the conventional ICCG method with simple transpose is effective for such a matrix equation. In this case, ICCG method becomes the incomplete Cholesky preconditioned biconjugate gradient (ICBCG) method (Zhdanov et al, 2007). In the following experiments the ICBCG method (Smith, 1996) was used.

Once $\widehat{E}_y$ and $\widehat{H}_y$ are obtained, the other components can be obtained from equations (7a) to (7d). These complementary components are calculated from the space derivatives of $\widehat{E}_y$ and $\widehat{H}_y$ which are evaluated from the space derivatives of the interpolation functions $\partial N_e^i / \partial x$ and $\partial N_e^i / \partial z$ in each element. Both of these two space derivatives are most accurate at the corners of the elements in Fig. 2. Whereas the other components obtained from the derivatives of $\widehat{E}_y$ and $\widehat{H}_y$ show serious errors at points of abrupt variations of $\widehat{E}_y$ and $\widehat{H}_y$ (Everett and Edwards, 1993).

3.3 Electromagnetic fields in real space

The electromagnetic fields in the domain are transformed to the real space K_y domain by the inverse Fourier transform. The electromagnetic fields in real space y domain are evaluated by

$$F(x, y = y_0, z, \omega) = \frac{1}{2\pi} \int_{-k_y}^{k_y} \widehat{F}(x, k_y, z, \omega) e^{ik_y y} dk_y, (k_y \rightarrow \infty) \quad (10)$$

The integral in equation (10) is carried out by cubic spline interpolation of the solutions in the logarithmic wave number domain K_y (Constable and Srnka, 2007).

4 Considerations of the singularity at the source point

Electric and magnetic dipole sources present singularity at the source location. For instance, in the near zone ($|kr| < 1$),

along strike asymptotic electric field E_y generated by a vertical magnetic dipole (VMD) source at the surface of the homogeneous half space is described as follows (Spies and Frischknecht, 1991)

$$E_y = -\frac{M_{sz}}{16\pi} \frac{x-x_0}{r} \omega^2 \mu^2 \sigma - i \frac{M_{sz}}{4\pi} \frac{x-x_0}{r^3} \omega \mu \quad (11)$$

where the dipole source is located at $(x_0, y_0, 0)$, and the $r = [(x-x_0)^2 + (y-y_0)^2]^{1/2}$ displacement current is neglected. The imaginary part of E_y has the singularity of order $O(1/r^2)$ along the x-axis.

For a horizontal electric dipole (HED) source directed along the strike y -axis, the asymptotic electric field E_y in the near zone is (Edwards, 2005; Ellingsrud et al, 2002)

$$E_y = -\frac{Idy}{2\pi\sigma} \frac{1}{r^3} - i \frac{Idy}{4\pi} \frac{\omega\mu}{r} \quad (12)$$

The real part of E_y has the singularity of order $O(1/r^2)$, and the error of numerical simulation still exists even by using the higher order interpolation function and finer node spacing.

A simple and effective solution to the source singularity is to employ a source which has a finite area. In this study, we adopted the Herrmann's pseudo-delta function (Herrmann, 1979; Furumura and Takenaka, 1996) instead of the delta function.

$$\delta_s(x-x_0) = \frac{1}{2\tau} \begin{cases} 0 & (x-x_0) \leq -2\tau \\ ((x-x_0+2\tau)/\tau)^2/2 & -2\tau < (x-x_0) \leq -\tau \\ -((x-x_0+2\tau)/\tau)^2/2 + 2(x-x_0+2\tau)/\tau - 1 & -\tau < (x-x_0) \leq \tau \\ ((x-x_0+2\tau)/\tau)^2/2 - 4(x-x_0+2\tau)/\tau + 8 & \tau < (x-x_0) \leq 2\tau \\ 0 & 2\tau < (x-x_0) \end{cases} \quad (13)$$

Fig. 3 The spectrum of the pseudo-delta function

The magnitude and width of the pseudo-delta function depend on parameter τ . By employing Herrmann's pseudo-delta function, a three-dimensional pseudo-delta function in our 2.5-D electromagnetic modeling problem was defined as (Unsworth et al, 1993)

$$\delta_s(r-r_0) = \delta_s(x-x_0) \delta(y) \delta_s(z-z_0) \quad (14)$$

Here, a true delta function was used for the strike direction since it can be processed by the Fourier transform without difficulty. In the x-z plane, the solution for the dipole source was represented by the smoothed result of the convolution of the true delta function and the pseudo-delta function. The spectrum of the pseudo-delta function is shown in Fig. 3.

5 Validation of the FEM code

From equation (10), we know that simulation of the electromagnetic fields needs infinite wave number. However, in our numerical evaluation, we have to select finite number K_y of K_y to approximate the field variations in K_y domain, and the integration in equation (10) is carried out by cubic spline interpolation of the solutions in the wave number K_y domain.

In order to validate the code, we checked the numerical result of the electromagnetic field of a specific K_y with its analytic solution in homogeneous medium. With respect to the selection of the K_y , we pursued an optimal range of K_y by simulating the homogeneous responses with different range of K_y and comparing their analytic solutions.

5.1 Comparisons with analytic solution in homogeneous medium

Fig. 4 shows the Hz and Hx components of the magnetic field evaluated by the 2.5-D FEM code and their counterparts using the analytic formula (Spies and Habashy, 1995) when x is 67m and y is 0m.

Fig. 5 presents the Ey component of the electric field evaluated by the 2.5-D FEM code and that of the analytic solution when x is 67m and 111m respectively. Both results show good agreement with analytic solutions, and our code has sound accuracy to simulate the electromagnetic responses. From Fig. 5, it can be seen that as the receivers move far from the source, the field attenuates gradually and our numerical solutions depart from their true values because of inadequate accuracy of the iterative solver.

Fig. 4 Comparisons of Hz (a) and Hx (b) of the magnetic field evaluated by the 2.5-D FEM code with their counterparts using the analytic equation ($x=67\text{m}$, $y=0\text{m}$)

Fig. 5 Comparisons of Ey component of the electric field evaluated by the 2.5-D FEM code with that of the analytic solution when $x=67\text{m}$ (a) and $x=111\text{m}$ (b)

5.2 Effects of the pseudo-delta function distribution on numerical results

It is shown that the spectrum is flat within the limit of $K_x \times \tau < 1$, and as K_x increases beyond the limit, it decreases sharply with oscillations. Therefore, the function works as a high-cut filter. Moreover, the Herrmann's function of equation (13) is described by a quadratic curve with respect to the x-coordinate and is thus suitable for the quadratic interpolation function of the FEM. For the dipole source, we have to select an appropriate τ to accurately simulate the source current distribution.

Fig. 6 shows comparisons of the real and imaginary parts of the magnetic field with that of the analytic solutions for different τ values. We can know from the numerical results that for a specific distance between source and receiver, τ has an optimal range which provides accurate numerical results. In our case, the optimal range of τ is 0.5 to 2.5.

5.3 Optimal range of K_y values

In the case of a homogeneous space model, the effective K_y range directly depends on the node spacing. To simplify our problem, we consider only J_{sy} in equation (6a) and (6b) and assume that $|K_y^2| \ll K^2$. The governing differential equation can be reduced to

$$\frac{\partial}{\partial x} \left(\frac{\partial \widehat{E}_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\partial \widehat{E}_y}{\partial z} \right) - k_y^2 \widehat{E}_y = \widehat{J}_{sy} \quad (15)$$

Taking the Fourier transform with respect to x and z, we obtain

$$(k_x^2 + k_y^2 + k_z^2) \widehat{E}_y = -\widehat{J}_{sy} \quad (16)$$

When $K_y^2 K_x^2$ and $K_y^2 K_z^2$, equation (15) no longer depends on the x and z coordinates. Therefore, there is a proper upper limit of K_y in numerical computations and it should be comparable to the upper limits of K_x and K_z which depend on space discretization. In numerical evaluation, the effective ranges of K_x and K_z are definitely smaller than the Nyquist wave number $K_N = \pi/\Delta$, where Δ is the node spacing. Since the derivatives of the field are included in equation (6a) and (6b), we only need to investigate the accuracy of the derivatives. When we consider the x-derivative of the field, we can assume $f(x) = \exp(ik_x x)$ generally, so the effective wave number K_{ex} is defined as follows (Buell, 1991; Srnka et al, 2006)

$$f'(x) = ik_{ex} \exp(ik_x x) \quad (17)$$

Fig. 6 Comparisons of the simulated magnetic field with that of the analytic solutions (solid line) for different τ values

Fig. 7 Five-layer model when $L=45.2m$

By using the central difference, the effective wave number is given by

$$k_{ex} = \frac{e^{ik_x(x+\Delta)} - e^{ik_x(x-\Delta)}}{2\Delta i e^{ik_x x}} = \frac{\sin(k_x \Delta)}{\Delta} \quad (18)$$

Therefore, the maximum of the effective wave number K_{ex} is $K_{ex} = 1/\Delta$, which is a reasonable value that can determine the effective range. It is advisable to use K_{ex} as a rough threshold of K_y . Fig. 7 indicates a five-layer model which can be used to address the effects of K_y on the numerical results. Fig. 8(a) shows the effects of K_y range on the numerical results. In our case, the appropriate range is 0.001 to 1 and the results become deteriorated when we narrow the K_y range. Fig. 8(b) shows the effects of different K_y on the numerical results. It

Fig. 8 Comparison of the numerical results with those of the analytic solutions (a) ($f=370\text{Hz}$) when $K_y \in (0.001, 1.0)$ and $K_y \in (0.01, 1.0)$, and that with different number of K_y values when $K_y \in (0.001, 1.0)$ (b)

is suggested that when we decrease the number of K_y to 12, we cannot obtain an accurate solution of the electromagnetic field. In our numerical modeling, we have to select K_y appropriately to ensure accurate results and minimum computing costs.

6 Simulation of cross-hole electromagnetic responses

From equations 5(a) to 5(d), we know that other components of the electric and magnetic fields should be calculated from the derivatives of the E_y component of the electric field and the H_y component of the magnetic field. Therefore, the other components will show serious errors at points of abrupt variations of $\widehat{E}_y$ and $\widehat{H}_y$. Fig. 9(a) provides a model of a deviated bed in a layer background.

Fig. 10 indicates several simulated results with different source locations. From the results of the source one (S1), we

can see that at a depth the same as the source, the response presents spikes of error because of the step variations of the conductivity which simulate the slope of the formation. When the source is located within the target zone, the response shows similar features to the horizontal layer model, whereas the responses of the upper and lower sources indicate considerable differences from those of the horizontal layer model. Therefore we can derive the conductivity distribution by comparing with the response of the layer model.

Fig. 11 indicates comparisons of the simulated results of the fault model with different source locations. From the results, we can see that the responses at different locations present pronounced differences. When the source is located below the target zone, the response shows abrupt variations within the fault shift interval. From this anomaly, we can derive the conductivity change by comparison with the response of the layer model.

Fig. 9 Deviated layer (a) and fault (b) models to check the electromagnetic responses

Fig. 10 Electromagnetic responses of the deviated layer when the source in three different locations

Fig. 11 Electromagnetic responses of the fault model when the source in three different locations

7 Conclusions

A method was developed to calculate the electromagnetic responses of a dipole source in 2-D conductive structures. The scheme adopted a pseudo-delta function to distribute the dipole source current and circumvent the singularity problem at the source point. Moreover, isoparametric finite elements were used to accommodate the complex subsurface formation. Numerical results of the homogeneous medium and layer model showed that the pseudo-delta function could approximate the source current satisfactorily when an appropriate τ value was selected. By properly selecting the y-wave number (K_y), we could considerably minimize the computing costs by reducing the full three-dimensional problem to a series of two-dimensional problems. Also, within a specific K_y range, the value of K_y had considerable effects on the simulation accuracy. So we have to distribute

the K_y value corresponding to the field variations. Simulation results of the slope model and fault model showed that it was possible to derive cross-hole conductivity by comparisons with that of the layer model. Numerical tests showed that our equations could be applied to general complex two-dimensional structures containing either an electric or magnetic dipole source in any direction.

References

- Abubakar A., Habashy T., Druskin V., et al. Two-and-half dimensional Forward and Inverse Modeling for Marine CSEM Problems. 76th Ann. Internat. Mtg. Soc. Expl. Geophys. Expanded Abstracts. 2006: 750–754
- Avdeeva A., Commer M., and Newman, G. A. Hydrocarbon reservoir detectability study for marine CSEM methods: time domain versus frequency domain, 77th Ann. Internat. Mtg. Soc. Expl. Geophys., Expanded Abstracts. 2007: 628–632

- Buell J C. A hybrid numerical method for three-dimensional spatially developing free shear flows. *Journal of Computational Physics*. 1991. 95(2): 313–338
- Constable K S. and Weiss C. Mapping 3D salt using 2D marine MT: Case study from Gemini Prospect, Gulf of Mexico: 74th Annual International Mtg. Soc. Expl. Geophys., Expanded Abstracts, 2004: 596–599
- Constable, S C. Marine electromagnetic methods—A new tool for offshore exploration: *The Leading Edge*. 2006. 25: 438–444
- Constable S C and Srnka L J. An introduction to marine controlled-source electromagnetic methods for hydrocarbon exploration. *Geophysics*. 2007. 72 (2): WA3–WA12
- David L A and Morrison H F. Theoretical and practical considerations for cross well electromagnetic tomography assuming a cylindrical geometry. *Geophysics*. 1995. 60(3): 846–870
- Davdycheva S., Druskin V. and Habashy T. An efficient finite-difference scheme for electromagnetic logging in 3D anisotropic inhomogeneous media. *Geophysics*. 2003. 68(5): 1525–1536
- Druskin V and Knizhnerman L. Spectral approach to solving three-dimensional Maxwell's diffusion equations in the time and frequency domains. *Radio Science*. 1994. 29(4): 937–953
- Edwards N. Marine controlled source electromagnetics: Principles and methodologies—future commercial applications: *Surveys in Geophysics*. 2005. 26(2): 675–700
- Ellingsrud S., Eidesmo T., Johansen S., et al. Remote sensing of hydrocarbon layers by seabed logging (SBL): Results from a cruise offshore Angola. *The Leading Edge*. 2002. 7: 972 – 982
- Everett M E and Edwards R N. Transient marine electromagnetics: the 2.5-D forward problem. *Geophysical Journal International*. 1993. 113(3): 545–561
- Farrelly B., Ringstad C., Johnstad S E., et al. Remote characterization of hydrocarbon filled reservoirs at the Troll Field by Sea Bed Logging. EAGE Fall Research Workshop. Rhodes. Greece 19th–23rd September. 2004: 662–668
- Furumura T and Takenaka H. 2.5-D modeling of elastic waves using the pseudo-spectral method. *Geophysical Journal International*. 1996. 124: 820–832
- Gallardo L A. and Meju M A. Characterization of heterogeneous near-surface materials by joint 2D inversion of DC resistivity and seismic data. *Geophysical Research Letters*. 2003. 30: 1658–1670
- Goldman Y, Hubans C, Nicoletis S, et al. A finite element solution for the transient electromagnetic response of an arbitrary two-dimensional resistivity distribution. *Geophysics*. 1986. 51: 1450–1461
- Herrmann R B. SH-wave generation by dislocation source- A numerical study. *Bulletin of the Seismological Society of America*. 1979. 69(1): 1–15
- Hou J. and Torres-Verdín C. Finite-difference modeling of EM fields using coupled potentials in 3D anisotropic media: Application to borehole logging: 73rd Annual International Mtg., Soc. Expl. Geophys., Expanded Abstracts. 2003: 522–525
- Hoversten G M., Gritto R., Washbourne J. et al. Pressure and fluid saturation prediction in a multi-component reservoir, using combined seismic and electromagnetic imaging. *Geophysics*. 2003. 68(5): 1580–1591
- Hoversten G M. and Newman G A. 3D numerical simulation of a deepwater EM exploration survey. 74th Annual International Mtg., Soc. Expl. Geophys., Expanded Abstracts. 2004: 672–675
- Hoversten G M., Newman G. A., Geier N., et al. 3D modeling of a deepwater EM exploration survey. *Geophysics*. 2006. 71(5): G239–G248
- Johansen S E., Amundsen H E., Rosten T., et al. Subsurface hydrocarbons detected by electromagnetic sounding. *First Break*. 2005. 23: 31–36
- Kong F N., Johnstad S E., Røsten T. et al. A 2.5D finite-element modeling difference method for marine CSEM modeling in stratified anisotropic media. 2008. *Geophysics*. 73(1): F9–F19
- Leppin M. Electromagnetic modeling of 3-D sources over 2-D inhomogeneities in the time domain. *Geophysics*. 1992. 57(8): 994–1003
- Liu S X and Sato M. Application of borehole radar for subsurface physical measurement. *Journal of Geophysics and Engineering*. 2004. 1(3): 221–227
- MacGregor L M., Andreis D., Tomlinson J., et al. Controlled-source electromagnetic imaging on the Nuggets-1 reservoir. *The Leading Edge*. 2006. 25: 984–992
- Smith J T. Conservative modeling of 3-D electromagnetic fields, Part2: Biconjugate gradient solution and an accelerator. *Geophysics*. 1996. 61(5): 1319–1324
- Spies B R and Frischknecht F C. Electromagnetic sounding. *Electromagnetic methods in applied geophysics: Application. in Nabighian, M. N., Ed., Electromagnetic methods in applied geophysics-theory*. 1991. Vol.2: Soc. Explor. Geophys: 431–443
- Spies B R and Habashy T M. Sensitivity analysis of cross-well electromagnetics. *Geophysics*. 1995. 60(3): 834–845
- Srnka L J., Carrazzone J J., Eriksen E A. et al. Remote reservoir resistivity mapping — An overview. *The Leading Edge*. 2006. 25: 972–975
- Stoyer C H. and Greenfield R J. Numerical solutions of the response of a two-dimensional earth to an oscillating magnetic dipole source, *Geophysics*. 1976. 41(2): 519–530
- Sugeng F, Raiche A, and Xiong Z. Modeling topographic effects using an isoparametric finite element method. *The 13th Workshop on Electromagnetic Induction in the Earth. Abstracts*. 1996: 109–110
- Tompkins M J. Marine controlled source electromagnetic imaging for hydrocarbon exploration: Interpreting subsurface electrical properties. *First Break*. 2004. 22: 45–51
- Ueda T and Zhdanov M S. Fast numerical modeling of marine controlled source electromagnetic data using quasi-liner method. 75th Annual International Mtg., Soc. Expl. Geophys., Expanded Abstracts. 2005: 231–235
- Unsworth M J, Travis B J, and Chave A D. Electromagnetic induction by a finite electric dipole source over a 2-D earth. *Geophysics*. 1993. 58(2): 198–214
- Ward S H. and Hohmann G W. Electromagnetic theory for geophysical applications. in Nabighian M. N., Ed., *Electromagnetic methods in applied geophysics-theory*. Soc. Explor. Geophys. 1988. 1: 131–311
- Weiss C J. and Newman G A. Electromagnetic induction in a generalized 3D anisotropic earth, part 2: The LIN preconditioner. *Geophysics*. 2003. 68(3): 922–930
- Zaslavsky M, Davdycheva S., Druskin V. et al. Finite difference solution of the three-dimensional electromagnetic problem using divergence free preconditioners. 76th Ann. Internet. Mtg. Soc. Expl. Geophys., Expanded Abstracts. 2006: 775–778
- Zhdanov M.S., Gribenco A. and Cuma M. Regularized focusing inversion of marine CSEM data using minimum vertical support stabilizer, 77th Ann. Internet. Mtg. Soc. Expl. Geophys., Expanded Abstracts. 2007: 579–583
- Zienkiewicz O C. *The finite element method: third edition*. McGraw-Hill. 1977

(Edited by Yang Lei)