Editorial

B Sury, Associate Editor

We invite every one interested in research – especially in mathematics – to ponder on two of its aspects described below. One concerns the ‘crowd-sourcing of ideas’ and the other deals with the difficulty of proof verification.

The ‘Polymath Collaborative Project’ may not be well known among non-mathematicians. Here is what it is about. The British mathematician Timothy Gowers conducted a social experiment in January 2009 through his blog. He selected an important unsolved problem in mathematics and invited others to contribute to the solution. Gowers also raised the pertinent question: “Is massively collaborative mathematics possible?” Thus began the Polymath Project. The first problem proposed to find a combinatorial proof of the density Hales–Jewett theorem. This theorem can be informally described as saying that the n -in-a-row generalization of the tic-tac-toe game featuring several players cannot end in a draw, irrespective of which player plays each turn, provided the board is of sufficiently large dimension. This first Polymath Project and a few others that got completed were published under the interesting pseudonym of D H J Polymath. Many Polymath Projects started with a level of participation comparable to that of the first one, but some of them eventually got stalled, seemingly due to unsurmountable obstacles, and the level of participation fell below a critical number. Till date, 16 Polymath Projects have been proposed, 5 have been completed successfully, and one other – though in limbo – has contributed to the resolution of a different problem.

We point out that Polymath Project 14 has an Indian connection. The project arose accidentally from a question raised by Apoorva Khare at the end of 2017. Following that, there was furious activity and the project has just been completed with inputs from 14 people. The final resolution is expected to appear as a

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publication by D H J Polymath, and 6 people including Apoorva Khare and Siddharth Gadgil of the Indian Institute of Science, Bangalore would be the ‘authors’ if this were a traditional publication. It seems unlikely that a difficult question of the kind treated here would have been answered so quickly without the sustained and combined interest of several experts. I should like to believe that this method of research is here to stay, and in the future, majority of big questions in mathematics will be dealt with in this fashion.

We mention now a completely different aspect of mathematical research that originated with the mathematician Vladimir Voevodsky in a sense. Voevodsky, who passed away in September 2017 at the age of 51 was a very gifted mathematician at the Institute of Advanced Study, Princeton. He had won the Fields Medal in 2002 for the resolution of the Milnor and Bloch–Kato conjectures. His work completely revolutionized the subject and created a new area called the ‘motivic homotopy theory’. Much earlier, in 1990, Voevodsky and Mikhail Kapranov had written a famous paper, and later a mistake was found in the proof of a key lemma of this paper. In fact, the main result of that paper on infinite groupoids was itself found not to be true. More and more examples of mathematical errors in the works of mathematicians became a matter of consuming concern for Voevodsky. He asked in a public lecture: “Who would ensure that I did not forget something and did not make a mistake, if even the mistakes in much more simple arguments take years to uncover?” As mathematical proofs had become so complicated, the details were seldom checked completely. On top of this, a very high reputation of the author only compounds this problem because every one assumes that the proofs must be right. Voevodsky’s perception was that the foundations of mathematics based on set theory were far removed from the way mathematicians actually worked. Thereafter, he started essentially rewriting the foundations of mathematics. He started thinking of computer representation of mathematical proofs and created a theory named the ‘univalent foundations’. It provides the foundation for a mathematics repository

and is (potentially) an alternative to set theory as the foundation for all of mathematics. This would help mathematicians to verify and share each other's work even if their approaches to the underlying concepts were different.

The July issue of *Resonance* features Alan Baker, a British mathematician who won the Fields Medal at the age of 31 for his path-breaking work on transcendental number theory. Even though this is a highly technical subject, the article by Yann Bugeaud, one of the present-day experts in this area, describes in reasonably elementary terms, some of Baker's seminal work. Another article on transcendence by Senthil Kumar, R Thangadurai and Veekesh Kumar launches itself from a fascinating elementary problem that appeared in the Putnam Undergraduate Mathematics Competition. The problem asks for a proof that a real number c for which the c -th power of every positive integer is a positive integer, must itself be a non-negative integer.

In a gripping article, Anantha Ramakrishna and Arun Jayannavar provide an approach to answering the fundamental question as to how much time a quantum particle or wave spends in a given region. The series of illuminating classroom articles on fluid dynamics by Chirag Kalelkar has reached its 15th episode and talks about fluid instabilities. Sneha Titus reviews Sheshagiri Rao's book *A Gentle Man Who Taught Infinity* which recounts in fascinating detail the influence his school mathematics teacher Chanakeshava had on him, while D Balasubramanian reviews *Indian Science: Transforming India – A look back on its 70-year journey; impact of science in independent India*, recently published by the Indian National Science Academy. Finally, a three-part masterly description of breakthroughs in information and communication technologies has been sketched by V Rajaraman, and the first part appears in this issue.