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ORIGINAL ARTICLE

Professional competences of teachers for fostering creativity and supporting high-achieving students

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Abstract This paper addresses an important task teachers face in class: the identification and support of creative and high-achieving students. In particular, we examine whether primary teachers (1) have acquired professional knowledge during teacher education that is necessary to foster creativity and to teach high-achieving students, and whether they (2) possess the situation-specific skills necessary to do so. For this purpose, (1) the knowledge of German primary school teachers who participated in the TEDS-M study at the end of teacher education is analyzed. (2), a subset of these teachers interpreted classroom video scenes that require identifying and supporting creative and high-achieving students in the longitudinal follow-up study to TEDS-M (TEDS-FU) after 3 years of work experience. Contingency analyses between teachers' professional knowledge and their skills in identifying and supporting mathematically creative and high-achieving students were carried out. The analyses revealed that those teachers who have difficulties in logical reasoning and understanding structural aspects of mathematics also have difficulties in identifying and supporting creative and high-achieving students. It was difficult for them to identify students' thinking processes based on structural reflections and pattern recognition; moreover, they had difficulty in further developing mathematically rich answers by students. In line with these results, teachers with strong professional knowledge were able to identify and support mathematically creative and high-achieving students. Thus, the study reveals that a connection between teachers' professional knowledge and their skills in identifying and supporting mathematically creative and high-achieving students exists but that many future and early career teachers seem to have deficiencies in these respects.

Keywords Teacher competence · Video-based test · Creativity · Giftedness

1 Introduction

In mathematics classrooms, teachers are faced with a multitude of challenges. One of these challenges is to meet the heterogeneous learning requirements of their students. Mathematics teachers, therefore, do not only need to see and react to learning difficulties and misconceptions by students, but they also need to be aware of students' strengths, their creativity and abilities. Teachers need to understand students' diverse learning approaches; they must be able to identify quality and creativity in students' multiple solutions and to draw conclusions about students' mathematical ability. Shayshon et al. (2014) claim that "the teachers' role in nurturing mathematically talented students should be one of the main focal points in teacher preparation and professional development programs" (p. 410).

The present paper focuses on this aspect and examines whether future teachers have developed the mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) during teacher education

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that is necessary to foster creativity and to support mathematically able students. Furthermore, the paper reports the results of research that examined whether these teachers possess situation-specific skills in identifying and fostering creativity and in supporting high-achieving students within mathematics teaching, after about 3 years of teaching practice. The international Teacher Education and Development Study in Mathematics (TEDS-M) assessed mathematics teachers' knowledge at the end of teacher education. Its longitudinal German follow-up study (TEDS-FU) focused on the competence development of these teachers during their first years of work experience. The TEDS-FU study included situation-specific skills of teachers using a video-based test that requires that teachers react to classroom situations. Based on a secondary analysis of these data under the perspective described above, the aim of the present paper is to analyze whether German primary (mathematics) teachers possess the necessary professional competences to foster creativity and act adequately when teaching high-achieving students.

2 Theoretical background

Children bring different prior knowledge and learning strategies to the classroom. However, as Shayshon et al. (2014) point out, teachers often pay more attention to low-achieving students than to high-achieving ones. In order to analyze whether teachers possess the professional competences necessary to foster creativity and act adequately when teaching high-achieving students, the following section characterizes the concepts of mathematical giftedness and creativity. Subsequently, teachers' professional competences are conceptualized.

2.1 Mathematical giftedness and creativity

2.1.1 Conceptualizing mathematical giftedness and creativity

The two terms mathematical giftedness and creativity are sometimes used synonymously (see, e.g., Krutetskii 1976) whereas Renzulli (2004) differentiates between "schoolhouse giftedness" and creativity. Mann (2006) points out that mathematically gifted students are often identified by their classroom performance, by their test scores, or by recommendations. But these characteristics constitute only one part of high achievement in mathematics (Hong and Aqui 2004; Mann 2006). Renzulli's model of giftedness includes three different but interdependent attributes of gifted learners, namely, above-average ability, task commitment, and creativity. According to this model, creativity is a subset of mathematical giftedness. In this regard, Wagner and Zimmermann (1986, p. 276) define mathematical giftedness as "a set of testable abilities of an individual. If he or she scores high in nearly all of these abilities, there is a high probability of successful creative work later on in the mathematical field and related areas. These abilities are defined […], stressing the following complex mathematical activities:

1. organizing material;
2. recognizing patterns or rules;
3. changing the representation of the problem and recognizing patterns and rules in this new area;
4. comprehending very complex structures and working within these structures;
5. reversing processes;
6. finding (constructing) related problems".

Hong and Aqui (2004) describe the state-of-research about gifted children and list various features that distinguish gifted children from their non-gifted peers. Gifted children are stronger cognitively, intrinsically motivated, they are thinking more strategically and are more likely to have conscious control over solution processes, they use more strategies for organizing and transforming information and use them more effectively, they can transfer these strategies to novel tasks, and they use more re-reading, inferring, analyzing structure, predicting, and evaluating strategies.

In the domain of mathematics, the term "creativity" is often used with reference to the work of mathematicians and their novel discoveries. Therefore, creativity in the context of school mathematics is generally related to "problem solving and or problem posing" (Nadjafikhah et al. 2012, p. 290). Haylock (1987) points out that there is no consensus about defining creativity: "Creativity in general is a notion that embraces a wide range of cognitive styles, categories of performance, and kinds of outcomes" (p. 68). However, Nadjafikhah et al. (2012) identify several criteria that many definitions of mathematical creativity have in common: Mathematically creative people develop new prolific mathematical concepts; they discover unknown relations and reorganize the structure of a mathematical theory. Therefore, creativity in mathematics is more than just profound knowledge and the reliable mastery of algorithms: "It entails incorporating experiences and conceptual understanding to solving authentic mathematical problems" (Mann 2006, p. 243).

In the context of assessing mathematical creativity of students, Mann (2009, p. 340) refers to the Creative Ability in Mathematics test developed by Balka (1974) and lists the following sub-abilities to be assessed, namely "the ability to

- formulate mathematical hypotheses concerning cause and effect in mathematical situations;
- determine and identify patterns in mathematical situations;
- break from established mentalities to develop solutions;
- consider and evaluate unusual mathematical ideas and think through their possible consequences for a mathematical situation;
- sense what is missing from a given mathematical situation and to ask questions that will enable one to retrieve the missing mathematical information;
- divide general mathematical problems into specific sub problems."

2.1.2 Supporting mathematical giftedness and creativity

The main element in fostering students' creativity and supporting gifted students in mathematics classrooms is the teacher and the opportunities that he or she offers for the students to learn (Nadjafikhah et al. 2012). However, mathematics instruction often lacks adequate cognitive challenges for gifted learners but provides similar challenges to all students (Rotigel and Fello 2004), i.e., no model exists for supporting gifted students (Shayshon et al. 2014).

Meeting the needs of *each* individual learner should be the highest goal of education (Shayshon et al. 2014; Bolden et al. 2010). This requires that the teacher differentiates the learning opportunities provided. Learning environments that meet the needs of gifted students should include the appreciation of alternative ideas and the acknowledgement of multiple solutions (Nadjafikhah et al. 2012). The teacher should guide the students to ask suitable questions and give them the opportunity to reflect on new ideas and concepts. Furthermore, students should meet opportunities to learn how to make and explore their own conjectures, to hypothesize, refute and adapt heuristic strategies, to devise plans, to conclude, reason and justify the conclusions and reflect on them at a metacognitive level, as mathematicians do (Nadjafikhah et al. 2012).

In this regard, Diezmann et al. (2002; Diezmann and Watters 2000) emphasize the importance of challenging tasks for effective learning processes and the necessity for teachers to select these tasks and support their students in the solution process (see also Shayshon et al. 2014). In order to enable mathematical discoveries and creativity, the teachers themselves need deep insight into the mathematical structures that they want their students to explore, and they need openness to a creative notion, allowing the students to explore mathematical ideas and relations. Finally, the teachers need to identify, encourage and improve the capabilities of mathematically gifted students (Nadjafikhah et al. 2012).

"Thus, it is necessary to pay deeper attention to train teachers especially improving teachers' ability to design and implement educational environments that promote creativity in mathematics" (Nadjafikhah et al. 2012, p. 289). In the following, teachers' professional competences are conceptualized and knowledge is identified that teachers need, in order to identify and support mathematical creativity and giftedness.

2.2 Teachers' professional competence

As Hattie (2009) points out, the quality of instruction depends to a large extent on the teacher and his or her professional competences. Therefore, much research was conducted in this area to conceptualize teachers' professional competences and their development during teacher education. The studies MT21 (Mathematics Teaching in the 21st Century; Blömeke, Kaiser and Lehmann 2008), TEDS-M (Teacher Education and Development Study in Mathematics; Blömeke et al. 2014) and COACTIV (Cognitive Activation in the classroom; Kunter et al. 2011) made important contributions in this area. Referring to the concept of competence defined by Weinert (2001, p. 48) as "cognitive abilities and skills possessed by or able to be learned by individuals that enable them to solve particular problems, as well as the motivational, volitional and social readiness and capacity to utilize the solutions successfully and responsibly in variable situations", these studies include a cognitive and an affect-motivational facet as elements of teacher competences.

With regard to mathematics teachers, the cognitive facet is often distinguished, according to the seminal work by Shulman (1986, 1987), in Mathematics Content Knowledge (MCK), Mathematics Pedagogical Content Knowledge (MPCK) and General Pedagogical Knowledge (GPK). The affect-motivational facet often includes epistemological beliefs about mathematics and about mathematical knowledge acquisition as well as motivational aspects and aspects about the teaching profession (cf. Blömeke et al. 2008; Baumert and Kunter 2011; Peterson et al. 1989; Blömeke and Kaiser 2014).

However, Depaepe et al. (2013) point out that no general consensus exists about MPCK. They identify two different views of this facet. One view identifies MPCK as a dispositional facet which is located "in the head" of the teachers, while the other view emphasizes MPCK as a "social asset" that becomes relevant in the process of teaching. In this regard, Buchholtz et al. (2013) characterize two sub-dimensions of MPCK, namely a more subject-related and a more teaching-related sub-dimension. Other studies followed a similar understanding. For example Ball et al. (2008) developed the Mathematical Knowledge for Teaching (MKT) framework that categorizes the domains of knowledge needed to teach mathematics with various sub-facets.

The German longitudinal TEDS-M follow-up study (TEDS-FU) therefore conceptualizes and assesses teachers' professional competences in addition to the dispositional approach of TEDS-M in a situated way. The analyses reported in the present paper stem from secondary analyses of data from these two studies, and seek answers to the following research questions:

1. To what extent have future teachers acquired during their education the MCK and MPCK necessary to foster creativity and support mathematically able students?
2. To what extent do mathematics teachers possess the situation-specific skills to identify and foster creativity and support high-achieving students after three years of work experience?

In the following section, TEDS-M and TEDS-FU as well as their conceptualization of teachers' professional competences are briefly described, before Sect. 3 presents the methodological approach of the present paper.

2.3 The studies TEDS-M and TEDS-FU

TEDS-M (Blömeke et al. 2014) was an international study aiming at a comparison of teachers' professional competences across countries and the efficiency of teacher education systems. TEDS-M was carried out under the auspices of the International Association for the Evaluation of Educational Achievement (IEA) and assessed future mathematics teachers in 17 participating countries, including about 1200 future primary teachers in Germany.

TEDS-M distinguished mathematics teachers' professional knowledge, based on Shulman's work (1986, 1987) into MCK, MPCK and GPK. MCK and MPCK were internationally assessed by a standardized paper-and-pencil test, while GPK could be assessed as a national option. Regarding future teachers' MCK and MPCK, key points on the scales, so-called anchor points, were identified. For MCK three competence levels could be distinguished, and for MPCK two (Tatto et al. 2012, pp. 136–142). Using the description of teachers' competences at these levels, it is possible to identify indicators for whether the tested teachers possess the competences necessary for promoting creativity and supporting mathematically talented students. It has to be noted that these competence levels provide only indicators on the group level, and it is possible that single (future) teachers are able to act differently than described by their level. However, these competence levels give a first insight into the capability of the tested future teachers to meet the requirements for promotion of creative and talented students.

Concerning the MCK of future primary (mathematics) teachers the following three competence levels were identified: Competence level I—a level with weak mathematical achievement. Future teachers whose scores fit this level missed structural insight, and their example-bound argumentation created difficulties. Competence level II—a level with average mathematical achievement. Future teachers whose score was at this level had sound knowledge and basic ideas at the fundamental level, but experienced problems with argumentative usage in more advanced problems. Competence level III—a level with the highest mathematical achievement. Future teachers at this level were characterized by strong structural mathematical knowledge. They were able to use this knowledge for standard problems in various mathematical areas, and they had skills in argumentation and logical reasoning.

With regard to the MPCK of future primary (mathematics) teachers, two competence levels were defined. Competence level I comprised all future primary mathematics teachers with lower achievement. These teachers had difficulties recognizing the correctness of students' answers and judging the adequacy of specific teaching strategies. Competence level II subsumed all future teachers with higher achievements in the MPCK items. These future teachers were able to interpret students' answers and possible cognitive barriers. In addition, they were able to describe their thinking, and they could identify the more appropriate teaching strategy for specific teaching sequences.

Regarding the requirements necessary to foster creativity and promote talented students, we can state that future (mathematics) teachers, who have difficulties working at an abstract mathematical level, who cannot develop sound mathematical argumentations and proofs or who have difficulties in understanding the adequacy of more complex argumentations by students, are not able to meet these requirements. Under this perspective, the international results of TEDS-M were in many countries not encouraging.

Figure 1 shows that only in six countries were the majority of future primary teachers at competence level III—the level that secures the mathematical knowledge necessary for supporting creativity and high-achieving students—in the field of MCK. The international mean was below a proportion of 50 % at competence level III (Fig. 1). The international MPCK results displayed a similar trend. Only from Taiwan and Singapore, the majority of future primary teachers reached the higher competence level. Internationally, only 27 % reached this competence level (Fig. 2).

In order to get closer to actual teaching and classroom reality, the German follow-up study of TEDS-M extended the theoretical framework of TEDS-M by enriching the knowledge facets with more situated competence

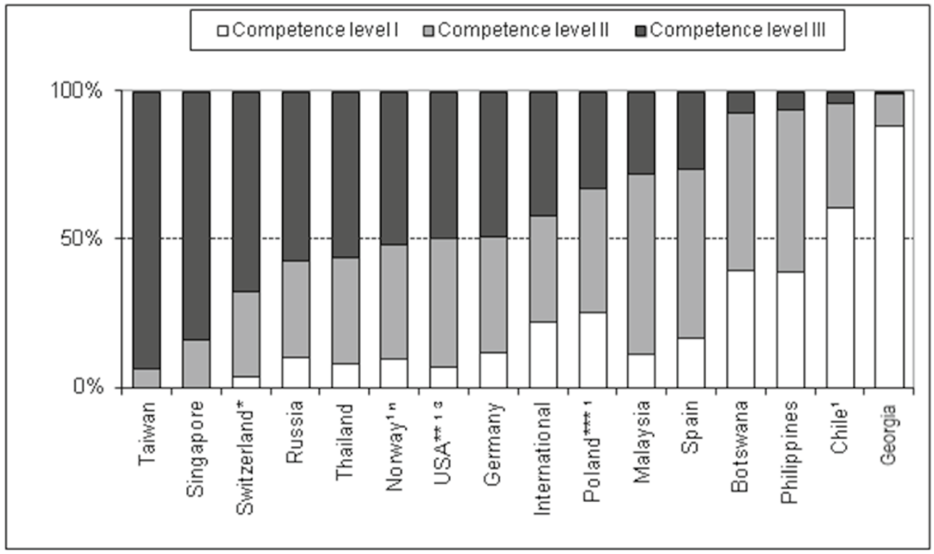

Fig. 1 Competence levels of MCK of future primary (mathematics) teachers (Blömeke et al. 2010, p. 211)

* Pedagogical universities in German-speaking cantons only	1 combined participation rate < 75%
** Public universities only	3 substantial proportion of missing values
*** Institutions with concurrent teacher-educations programs only	
n Sample meets the TEDS-M definition only partly, deviation from the IEA report	

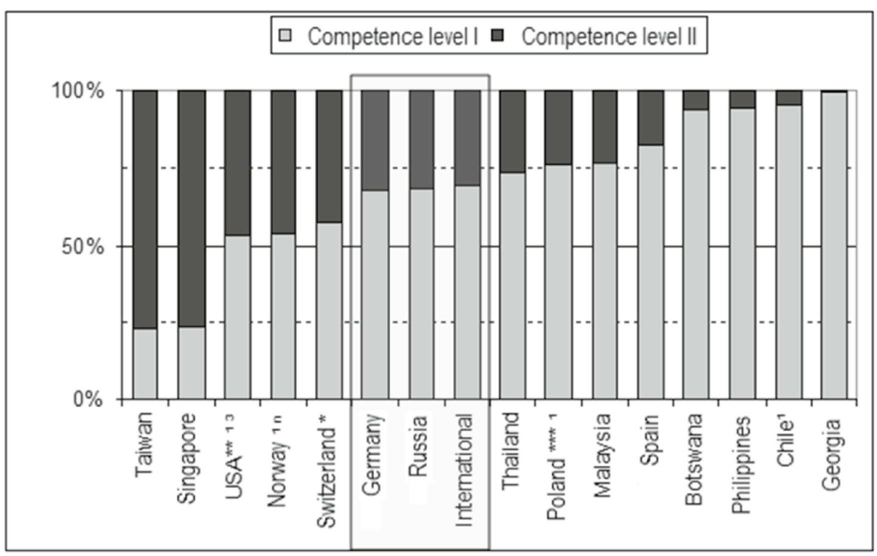

Fig. 2 Competence levels of MPCK of future primary (mathematics) teachers (Blömeke et al. 2010, p. 233)

* Pedagogical universities in German-speaking cantons only	1 combined participation rate < 75%
** Public universities only	3 substantial proportion of missing values
*** Institutions with concurrent teacher-educations programs only	
n Sample meets the TEDS-M definition only partly, deviation from the IEA report	

facets referring to the concept of noticing. Furthermore, the instruments of TEDS-M were complemented by video-based instruments evaluating the situated competence facets.

Referring to the concept of teacher noticing (Sherin et al. 2011; Carter et al. 1988), the TEDS-FU framework distinguishes three situation-specific skills, the so-called PID model (Blömeke et al. 2015; Kaiser et al. 2015, p. 373):

(a) *perceiving* particular events in an instructional setting,
(b) *interpreting* the perceived activities in class and

(c) *decision making*, which either includes to anticipate a response or proposing alternative instructional strategies.

Thus, the model of teachers' professional competence underlying the TEDS-FU study includes these situation-specific skills in addition to knowledge and affect-motivational facets.

The new test instruments developed for the TEDS-FU study assesses these situation-specific skills with a video-based test instrument, which consists of three short video vignettes with corresponding questions. These three video vignettes show mathematics classroom situations and are followed by open and closed questions that assess whether the teachers perceive and interpret relevant aspects of the teaching sequence, and whether they decide on adequate possibilities concerning how to continue the situation or propose adequate alternatives. In addition to these newly developed instruments, the TEDS-FU study tested teachers' knowledge and affect-motivational aspects using a reduced version of the TEDS-M test.

3 Methodological approach

In order to answer the two research questions formulated in Sect. 2.2, we refer to aspects of different test parts from the two previously mentioned studies. Teachers' skills in identifying quality features in students' solutions and in supporting high-achieving students can be analyzed with data from the video-based instrument of TEDS-FU. The data from TEDS-M, which was implemented in the last year of teachers' professional education, can give insight into the extent to which future teachers at the end of their studies have the knowledge to foster creativity and promote the capabilities of high-achieving students.

3.1 Assessment of primary mathematics teachers' knowledge as a basis for identifying mathematical creativity and high-achieving students

We refer to test parts from the TEDS-M study that tested future teachers' MCK and their MPCK. These test parts were implemented as a 60-min paper-and-pencil test. The items assessing the teachers' MCK covered the domains number, algebra, geometry, and data, as well as the three cognitive dimensions of knowing, applying, and reasoning. The items assessing the teachers' MPCK also covered the four content domains and referred in addition to MPCK of curricula and planning or to knowledge about how to enact mathematics in the context of teaching and learning. Thus, two facets were distinguished, one that becomes relevant for planning instruction, and the other that becomes relevant during class. The test included multiple choice items as well as open constructed response items (for details see Blömeke and Kaiser 2014). Scores were created for MCK and MPCK separately in one-dimensional models using item response theory (Blömeke and Kaiser 2014). These results are used in the following analyses as an indication of the teachers' preparation to identify and support creative and high-achieving mathematics students.

3.2 Assessment of primary school mathematics teachers' ability to identify and support creativity and high-achieving students

Teachers' answers to the TEDS-FU video-based instrument that assessed teachers' skills in a more situated way built the basis for evaluating whether they are able to identify quality characteristics in students' solutions and support these students' learning. Three short video vignettes show mathematics education in a German third grade classroom. Before watching the video sequence, the teachers receive context information about the mathematical content and the learning conditions of the students. After observing the classroom scene, the teachers are asked several questions concerning mathematics educational and general pedagogical aspects. The questions are presented in two format types: an open response format, and rating scale items (example items are presented in Figs. 4, 5). Based on the PID model, the questions focus on two aspects: perceiving particular events in the video, interpreting the perceived activities and deciding how to respond; as well as proposing alternative instructional strategies.

For the following analyses, we refer to two of the three video vignettes. In the first video vignette, which covers "Pascal's triangle", the students are working in an open learning environment. All students were asked to calculate and insert numbers into an empty Pascal's triangle. Subsequently, they were asked to color all even numbers and to find structures in the numbers and/or the coloring. The video vignette shows the students working on these tasks each at their individual pace. The teacher finally asks some students to present their findings and selects one for continuation of the work. The video vignette stops at this point. This video sequence, and its corresponding questions that require teachers to identify mathematical structures and patterns in the diverse students' findings, is particularly suitable for analyzing the teachers' ability to identify and support creative and high-achieving students.

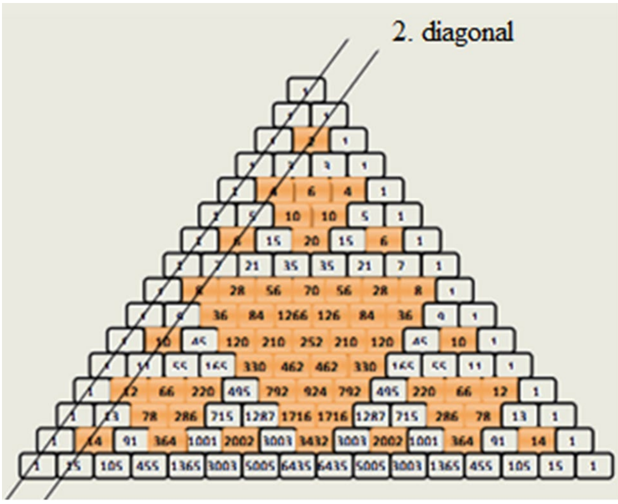

Fig. 3 Patterns and structures in Pascal's triangle

The second video vignette that refers to these requirements covers "Geometry". Here, the students are confronted with Pentominos and are asked to find all possible figures and to determine the number of existing Pentominos. After the introductory scene, a student is shown presenting her solution to the teacher.

All teachers' responses to the video instrument were coded. While extensive coding manuals were developed to evaluate the open response questions, several expert ratings generated the coding references for the rating scale items (see for example, Hoth et al. 2016).

We identified four questions that required the teachers to identify mathematically rich students' solutions and two questions that asked the teachers to support high achievement of students and their creativity. The difference between these two challenges can be explained by the different situation-specific skills that are required to solve the tasks. Identifying rich students' solution necessitates perceiving and interpreting while supporting creative and high-achieving students requires the teachers to decide how to continue their teaching (*decision making*) (see Blömeke et al. 2015; Kaiser et al. 2015, Sect. 2.3). Examples of these items are given below.

3.2.1 Pascal's triangle: Karola's discovery

This rating scale item referring to the video vignette "Pascal's triangle" asks the teachers to rate whether one of the students discovered a structure within the numbers. One girl presented that the second diagonal of Pascal's triangle contains the natural numbers (Fig. 3). The corresponding

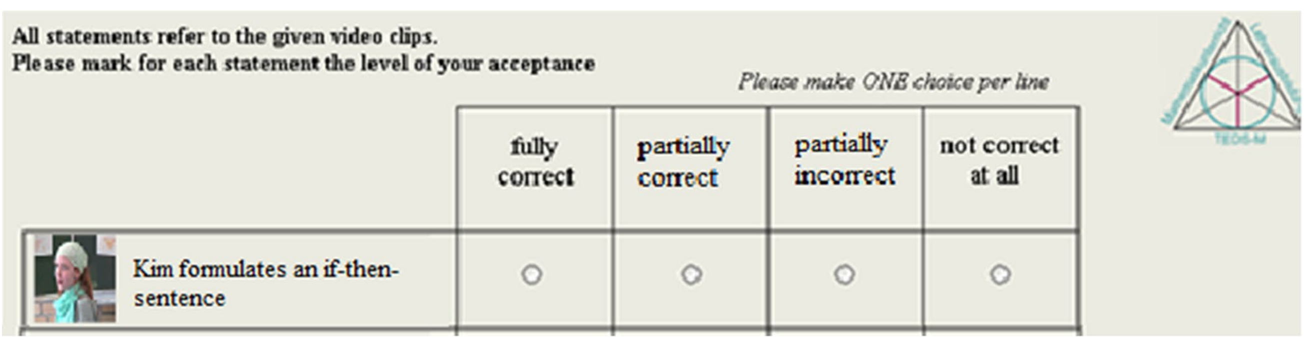

Fig. 4 Rating scale item "Kim's discovery" referring to the video 'Pascal's triangle'

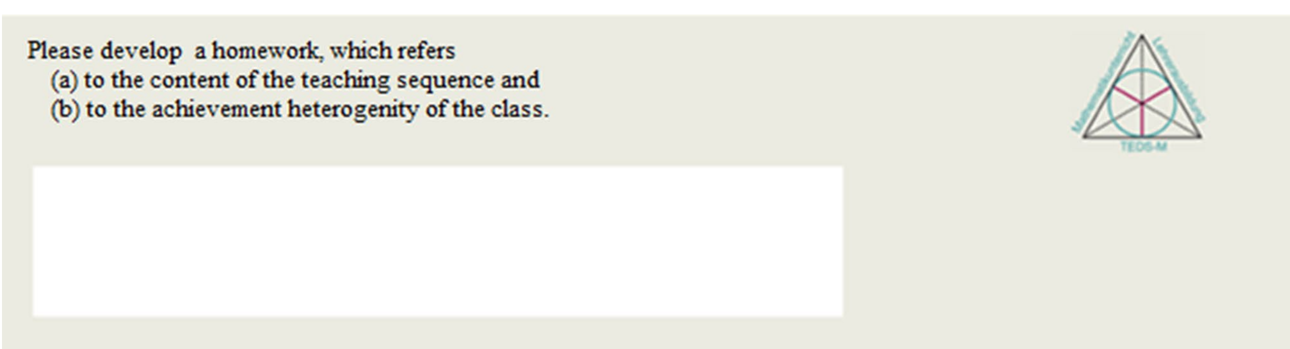

Fig. 5 Open response item "homework for a heterogeneous class" referring to the video 'Pascal's triangle'

item refers to pattern recognition or identification of mathematical structures, which are highly important for fostering creativity and supporting mathematically talented students, because teachers need to identify the mathematical quality and the creative potential of students' answers prior to using them effectively during class.

3.2.2 Pascal's triangle: Kim's discovery

This rating scale item (see Fig. 4) asked the teachers, whether one of the presenting students formulates an if–then-sentence. One girl presents that adding two even numbers results in another even number and adding two odd numbers also results in an even number. Thus, she formulates the if–then-sentence "If I add two even numbers, then I get another even number" and "If I add two odd numbers, then I get an even number". Thus, she reasons about mathematical patterns and reasoning is an essential part of creative mathematical work. It can, therefore, be expected that knowledgeable test persons focus especially on this part of the video. Therefore, this item does not only evaluate remembering, but expertise-guided noticing. Again, teachers need to identify the mathematical potential of the student's finding and abstract from her description the affected mathematical concepts.

3.2.3 Geometry: Identifying problems in the teaching strategy

The teacher's introduction of Pentominos and their mathematical structure to the class is characterized by some problems. This problematic introduction causes mistakes in a student's solution. In her solution, the student disregards the congruency of figures and she generates the different Pentominos based on only one specific Tetromino. This may be caused by several aspects of the teaching such as the material that the teacher presented to the class. This material did not enable the students to turn their Pentominos and, thus, the students were not able to test congruency. There are about eight teaching elements that can be associated with the student's mistakes. The teachers in the study were asked to list three of these teaching elements.

Mathematically correct concept introduction and the usage of rich examples are an indispensable condition for quality-oriented mathematics teaching in general. However, these aspects are especially important for creative and mathematically high-achieving students, who are immediately distracted by restricted or even wrong elaborations of mathematical concepts. In addition, the student in the video presents a creative and profound solution to the proof task. Thus, the teachers are required to identify the quality of the student's solution and ascribe the mistakes to the teacher's introduction.

3.2.4 Geometry: identifying quality features in a student's solution

The student, who presents her solution to the teacher, not only makes mistakes but also develops an abstract mathematical model in answering the proof task. In addition, she gives reasons for her mathematical model on an abstract level. The teachers are subsequently asked to evaluate the quality of the student's solution and name three different aspects indicating its quality. The recognition of mathematical patterns and structures in students' solutions—as evaluated in this item—is especially essential to encourage and further students' creativity.

3.2.5 Pascal's triangle: homework for a heterogeneous class

This question (see Fig. 5) asked the teachers to formulate homework. This homework ought to follow the video sequence and refer on the one hand to the content of the teaching sequence and on the other hand to the performance heterogeneity of the class. This item is based on a sound understanding of the students' solutions that contain various underlying mathematical patterns and structures and are generalized to varying degrees. To notice the variety of the proposed patterns and to identify their value is an important condition for formulating homework that fosters the displayed creativity and supports the talented students.

3.2.6 Pascal's triangle: continuing a student's answer

One of the students in the video presented as result of the work that the coloring of even numbers results in top-down triangle shapes while the shapes of the odd numbers look like bottom-up triangles. The teachers were asked to develop a challenging question that continues this student's discovery. This item addresses a profound mathematical finding of a student and requires that teachers develop a reasonable question that optimally continues the student's approach.

3.3 Data analysis and sample

1032 primary mathematics teachers participated in TEDS-M and 131 were reassessed in TEDS-FU. The following analyses refer to these 131 primary mathematics teachers who participated in both studies. These analyses build the basis to evaluate whether teachers are able to identify and to promote the capabilities of high-achieving and creative students during class. With regard to the first research question, the teachers' scores in the MCK and MPCK tests of TEDS-M and their corresponding competence level provide insight into the teachers' professional knowledge. In order to evaluate the second research question, solution frequencies of the selected items are used to gain insight into teachers' abilities.

To examine the connection between teachers' preparation and their ability to identify and support high-achieving students, the teachers' MCK and MPCK scores are linked with their scores on the selected tasks from the video instrument. Here, we use contingency analyses (see for example, Mayring 2015) to analyze these relations. We chose contingency analyses over correlation analyses because of the small number of competence levels. Especially with regard to teachers' MPCK, only two competence levels were distinguished. The use of the competence levels provides insight into the knowledge facets that the teachers possess or miss.

4 Results

The first section presents general results on teachers' preparation to support the achievements of high-achieving and creative students before we continue to analyze more situation-specific skills.

4.1 Results regarding teachers' preparation at the end of teacher education

Only 50 % of the German future teachers reached the highest MCK competence level in TEDS-M, a competence level strongly needed to promote giftedness, 40 % were at competence level 2 and 10 % at competence level 1. About 70 % of the German future primary teachers were at the lower MPCK competence level in TEDS-M, while only about 30 % belonged to the higher level.

Based on the description of the competence levels in Sect. 3, it can be assumed that teachers with MCK at the lowest and average competence levels will not be able to recognize or understand creative students' solutions and will not be able to support these students' mathematical learning processes. In addition, teachers who reach only the lower competence level in MPCK will not be able to offer learning opportunities for high-achieving and creative students, to develop different representations for a mathematical problem or choose different teaching strategies for their heterogeneous student body. Altogether, the German teachers—but not only they—show deficits regarding structural aspects of mathematics, logical reasoning and the analysis of students' answers. However, these aspects are of special importance, when teaching mathematically high-achieving and creative students. In the next section, we analyze the teachers' ability to identify high-achieving and creative students' solutions.

4.2 Results concerning primary mathematics teachers' ability to identify and support high-achieving and creative students' solutions

The solution frequencies of the six selected items from the TEDS-FU video analysis instrument are shown in Table 1.[1]

The results indicate that the early career teachers had difficulties identifying creative and high-achieving students' responses as well as supporting these learners. Only about one-third of those teachers were able to understand and interpret these students' findings during classroom activities.

In order to analyze the hypothesized relation between teachers' content specific knowledge and their ability to identify and support creative and high-achieving students during class, the following section connects both sets of data.

4.3 Connection between mathematics teachers' content knowledge and their ability to identify and support creative and high-achieving students

The results of the contingency analyses between the teachers' competence levels regarding their MCK at the end of their professional education (TEDS-M data), the competence levels of their MPCK at the end of their education (TEDS-M data), and their scores regarding items that require the teachers to identify and support creative and high-achieving students in a classroom-like situation (TEDS-FU data), are shown in Figs. 6, 7, 8 and 9. For reasons of clarity, the teachers' ability to identify creative and high-achieving students' responses is given as a total score, including the four items "Karola's discovery", "Kim's discovery", "Identifying problems in the teaching strategy" and "Identifying quality features in a student's solution". This total score ranges from min = 0 to max = 6. Similarly, the total score including the two items "Homework for a heterogeneous class" and "Continuing a student's answer" (min = 0, max = 2), indicates the teachers' ability to support high-achieving and creative students in the mathematics classroom. The Figs. 6, 7, 8 and 9 show the contingency analyses between the teachers' professional knowledge facets (represented by the competence levels) and their ability to identify and support high-achieving and creative students (given as the total scores). Each figure shows the percentage of teachers in each of the competence levels and the respective total scores.

[1] Missing responses within a test unit were considered as false answers unless teachers did not respond to or work on an entire test unit (such as one of the three video vignette tests). In that case, missing responses were coded as *missing*.

Table 1 Solution frequencies of the selected items from the TEDS-FU video analysis test

	Frequency of correct responses (%)	Frequency of incorrect responses (%)	Missing responses (%)
Questions assessing teachers' ability to identify high-achieving and creative student's responses			
Pascal's triangle: Karola's discovery	26	57	17
Pascal's triangle: Kim's discovery	34	53	13
Geometry: identifying deficits in the teaching strategy of the teacher	Fully correct (three correct aspects): 8	20	14
	Partially correct (two correct aspects): 31		
	Partially correct (one correct aspects): 27		
Geometry: identifying quality aspects in a student's solution	Fully correct (three correct aspects): 2	43	14
	Partially correct (two correct aspects): 8		
	Partially correct (one correct aspects): 33		
Questions assessing teachers' ability to support high-achieving and creative students			
Pascal's triangle: homework for a heterogeneous class	37	50	13
Pascal's triangle: continuing a student's answer concerning its mathematical potential	12	75	13

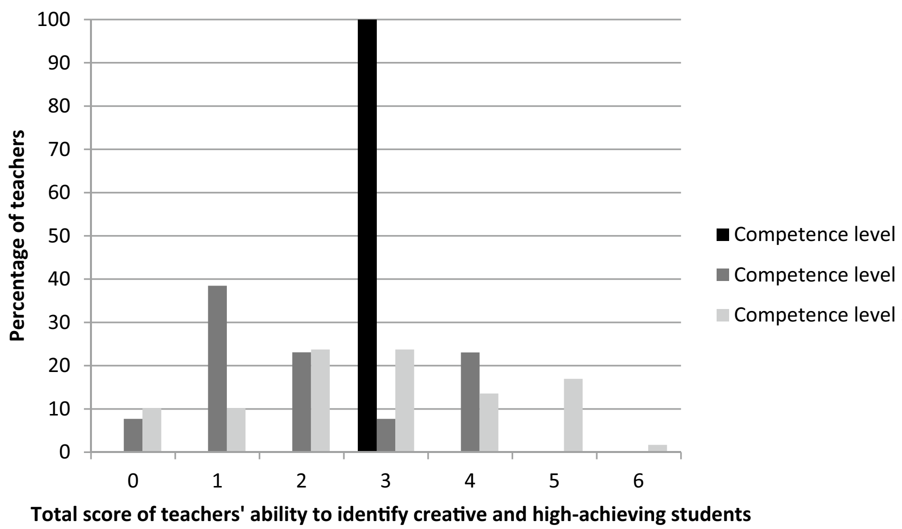

Fig. 6 Contingency analysis between the primary teachers' MCK and their ability to identify creative and high-achieving students

Figure 6 shows the relation between the primary school teachers' MCK at the end of their professional education and their ability to identify high-achieving and creative students. The figure indicates that only teachers of competence level 3 are able to achieve the highest scores (5 or 6) in tasks that require the identification of high-achieving and creative students' responses during class. About 77 % of those teachers with mathematical knowledge that is classified as competence level 2 were able to achieve only half or less of the maximal attainable score. These results indicate that teachers need profound content knowledge in order to identify complex and creative students' solutions.

Figure 7 shows the contingency analysis between the teachers' MCK and their ability to support creative and high-achieving students. Teachers who were able to achieve the highest possible score in supporting creative and high-achieving students exclusively belonged to the competence levels 2 or 3. In addition, about two-third of the teachers at competence level 3 reached a score of 1 or 2, while this result applied only to one-third of the teachers whose MPCK was classified as competence level 2. Then again, no teacher in competence level 1 reached a full score, but 50 % reached a score of 1.

This contingency analysis again indicates that there is a relation between the MCK of teachers and their ability to support creative and high-achieving students.

Figure 8 shows the connection between the primary teachers' MPCK at the end of their professional education and their ability to identify high-achieving and creative students in a classroom-like situation.

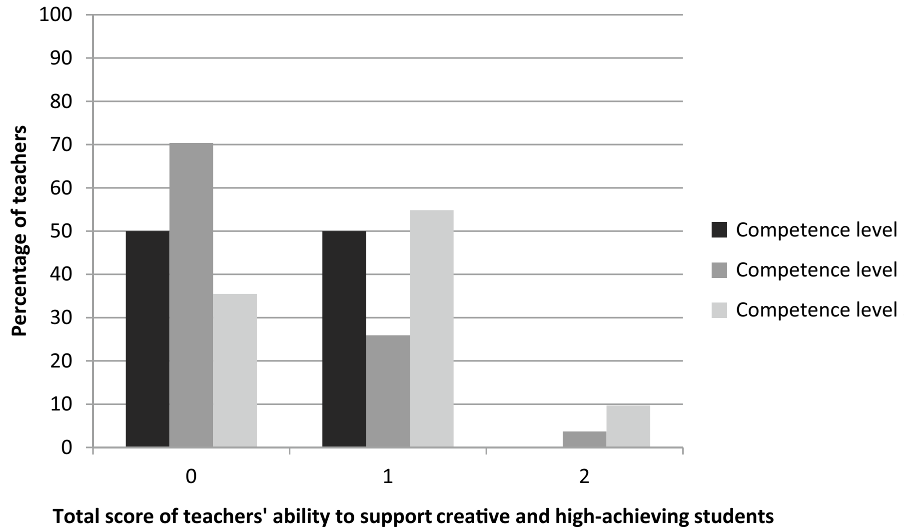

Fig. 7 Contingency analysis between the primary teachers' MCK and their ability to support creative and high-achieving students

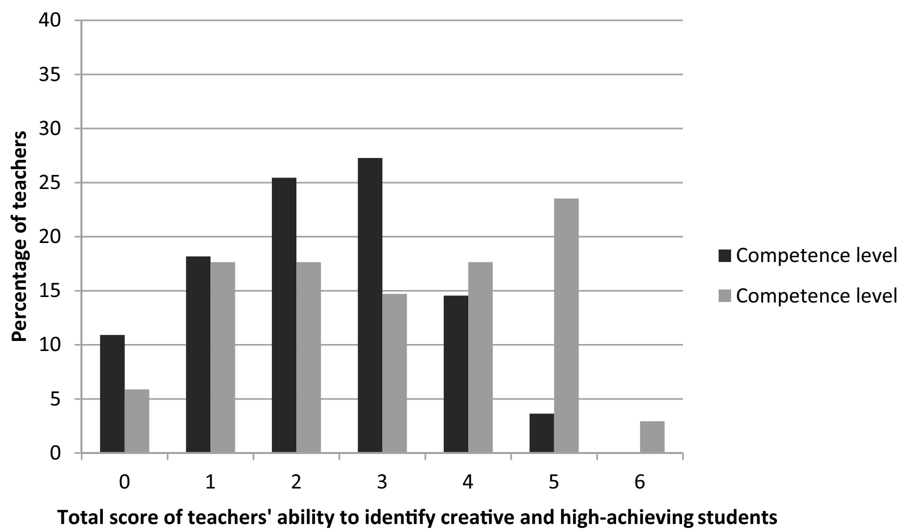

Fig. 8 Contingency analysis between the primary teachers' MPCK and their ability to identify creative and high-achieving students

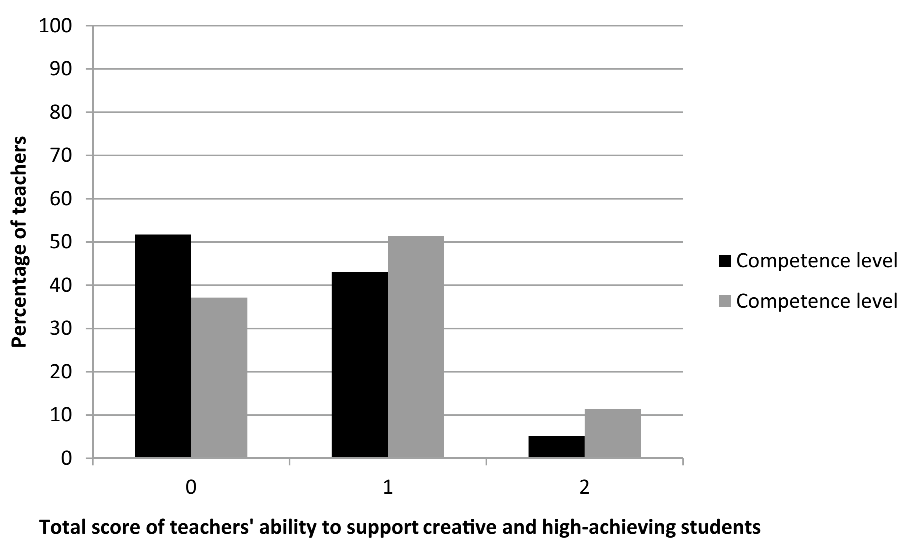

Fig. 9 Contingency analysis between the primary teachers' MPCK and their ability to support creative and high-achieving students

The figure also shows that about 82 % of those teachers whose MPCK was classified as competence level 1 were able to achieve only half or less of the score that indicates their ability to identify high-achieving and creative students (about 30 % of the teachers reached only a score of 0 or 1), whereas this result is true for only about 56 % of those teachers in competence level 2. Therefore, the results indicate that the primary school teachers' MPCK is relevant to identify high-achieving and creative students during class.

With regard to teachers' ability to support creative and high-achieving students during class (see Fig. 9), the figure shows that 52 % of the teachers in competence level 1 were not able to give adequate support (score 0) while this is true only for 37 % of the teachers in competence level 2. Only about 5 % of the teachers with MPCK at the competence level 1 reached a score of 2. Then again, this score was reached by about 11 % of the teachers at competence level 2. These results again indicate a relation between teachers' MPCK and their ability to support creative and high-achieving students during class.

5 Summary and discussion

Teachers must be equipped with professional competences to meet the various requirements that they encounter during class. Due to aspects of students' heterogeneity, one main challenge that they face is to teach the high-achieving and creative students as well as learners with great difficulties. In order to analyze whether primary mathematics teachers are able to identify and support high-achieving and creative students in the mathematics classroom and to analyze whether they were prepared to do that, data from the TEDS-M study and its follow-up (TEDS-FU) were analyzed. The TEDS-M study assessed mathematics teachers at the end of their professional education and provided information about the teachers' MCK as well as the MPCK that they bring from their teacher education. The TEDS-FU study is a German longitudinal Follow-Up that reassessed those teachers after about three years of teaching experience. One of the instruments of the study—a video analysis instrument—assessed the teachers' situation-specific skills. For the purpose of analyzing the teachers' ability to identify and support creative and high-achieving students in class, six items were selected from the video analysis test instrument that require the teachers to identify and further creative and high-achieving students' responses.

The data of about 131 primary school teachers who participated in TEDS-M gave insight into their knowledge to identify and support creative and high-achieving students, while their data from the TEDS-Follow-Up study gave insight into their ability to identify and support creative and high-achieving students during classroom activities. The longitudinal design of the studies allowed for contingency analyses between both data sets. However, it may be noted at this point that the study presented in this article is a secondary analysis of the data of the TEDS-M and TEDS-FU study. Therefore, the selected items were not developed with the aim of assessing teachers' knowledge and ability to identify and support creative and high-achieving students. This ability is represented in the present study by a quantitative score that results from the TEDS-FU coding process. A first summarizing approach is provided by the total score that described the number of aspects answered correctly for the purpose of the TEDS-FU study. Qualitative analyses of the teachers' individual responses might give additional and in depth information about the teachers' dealing with creative and high-achieving students but would exceed the scope of the present study at this point. The same applies to the scores resulting from the TEDS-M proficiency tests. These knowledge scores are generated from items spanning different content and cognitive subdomains. The scores, therefore, give comprehensive but also broad information about the teachers' professional knowledge. With regard to the intended research objective, it might be useful additionally to analyze the given requirements in detail.

The results from the TEDS-M proficiency tests regarding the teachers' MCK and their MPCK indicate that there are weaknesses in future teachers' content-specific knowledge concerning structural aspects of mathematics, logical reasoning and their analysis of students' answers. About half of the German future primary school mathematics teachers who participated in TEDS-M were likely to have more difficulty answering problems requiring more complex reasoning in applied or non-routine situations. However, this ability is essential in order to teach high-achieving students and to foster their creativity. This is in accordance with the results regarding the solution frequencies of the selected tasks from the TEDS-FU video analysis instrument. The teachers especially showed difficulties in naming quality features of a student's solution and formulating a question continuing a student's presented mathematical discovery. However, these two requirements are of special relevance, when teaching creative and high-achieving students in mathematics.

The contingency analyses between the teachers' content-specific knowledge at the end of their teacher education and their ability to identify, interpret and support creative and high-achieving students in the mathematics classroom, indicate a connection between the two components. Thus, teachers who are prepared with high content-specific knowledge at the end of their teacher education more often identify creative and high-achieving students' solution during class and they also offer more adequate learning possibilities for these creative learners. Therefore, teacher education programs as well as in-service teacher training should improve and take greater account of the mathematical and didactical aspects given that in class, teachers need to understand a variety of possible student responses to given mathematical problems. In our case, teachers would especially need mathematical knowledge to classify the responses and their mathematical value for the main mathematical idea which is in focus. Then again, they require MPCK to work with the given responses, value their representation and if necessary offer other forms of representation, etc. Then again, mathematical knowledge is needed to continue optimally the students' mathematical ideas and discoveries, but also for the purpose of guiding the main mathematical idea and goal of each teaching sequence.

These findings are in accordance with many demands formulated in literature concerning the education of high-achieving and creative students (see for example, Mann 2006; Nadjafikhah et al. 2012). As Diezmann and Watters (2000, 2002) point out, teachers require the ability to offer challenging tasks to creative and high-achieving students in order to support effective learning processes. This ability is especially focused in the two selected tasks from the TEDS-FU study that assess the teachers' ability to support the achievements of creative and high-achieving students. The results regarding these questions show that the primary school mathematics teachers had great difficulties formulating continuing questions and homework that challenge creative and high-achieving students mathematically. Therefore, Nadjafikhah et al.'s (2012) demand to "pay deeper attention to train teachers especially improving teachers' ability to design and implement educational environments that promote creativity in mathematics" (p. 289) is emphasized by the present findings.

In conclusion, the results show a great necessity for supporting teachers' professional dealing with creative and high-achieving students since many future and early career teachers seem to have strong deficiencies in providing an adequate mathematical education for creative and high-achieving students. In addition, the results show that teachers' ability to support these students is closely connected with their professional knowledge. This result also implies that teacher education needs to impart extensive professional knowledge.

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