ERRATUM TO “A UNIFIED APPROACH TO THE PLUS-CONSTRUCTION, BOUSFIELD LOCALIZATION, MOORE SPACES AND ZERO-IN-THE-SPECTRUM EXAMPLES”

BY

SHENGKUI YE

*Department of Mathematics, National University of Singapore
 2 Science Drive 2, 117543 Singapore
 e-mail: yeshengkui@nus.edu.sg*

In Corollary 4.10 of [2], we have to assume that the group G is finitely presented. This is due to a gap in the proof of Lemma 2.2 when claiming that the real reduced group C^* -algebra $C_{\mathbb{R}}^*(G)$ is G -dense. The proof only works for finitely generated free modules. We need to modify Definition 2.1 as follows.

Definition 1 (Definition 2.1): A G -dense (resp. **finitely G -dense**) **ring** (R, ϕ) is a unital ring R together with a ring homomorphism $\phi : \mathbb{Z}[G] \rightarrow R$ such that, when R is regarded as a left $\mathbb{Z}[G]$ -module via ϕ , then, for any right $\mathbb{Z}[G]$ -module M , free (resp. finitely generated free) right R -module F and R -module surjection $f : M \otimes_{\mathbb{Z}[G]} R \twoheadrightarrow F$, the module F has an R -basis in $f(M \otimes 1)$.

It is not hard to see that a G -dense ring is finitely G -dense. Lemma 2.2 should read as follows.

LEMMA 2 (Lemma 2.2): *The set of finitely G -dense rings contains the real group C^* -algebra $C_{\mathbb{R}}^*(G)$, the real group von Neumann algebra $\mathcal{N}_{\mathbb{R}}(G)$, the real Banach algebra $l_{\mathbb{R}}^1(G)$. The set of G -dense rings contains the rings $\mathbb{Z}/p$ for any prime p , $k \subseteq \mathbb{Q}$ a subring of rationals (with trivial G -actions), and the group rings $k[G]$.*

The main result Theorem 1.1 should read as follows, where we add the case of finitely G -dense rings.

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THEOREM 3 (Theorem 1.1): Assume that G is a group and (R, ϕ) is a G -dense ring. Let X be a CW complex with fundamental group $\pi = \pi_1(X)$. Suppose that $\alpha : \pi \rightarrow G$ is a group homomorphism such that

$$\begin{aligned} H_1(\alpha) : H_1(\pi; R) &\rightarrow H_1(G; R) \text{ is injective, and} \\ H_2(\alpha) : H_2(\pi; R) &\rightarrow H_2(G; R) \text{ is surjective.} \end{aligned}$$

Assume either that R is a principal ideal domain or that the relative homology group $H_1(G, \pi; R)$ is a stably free R -module. Then there exist a CW complex Y and an inclusion $g : X \rightarrow Y$ with the following properties:

- (i) Y is obtained from X by adding 1-cells, 2-cells and 3-cells, such that
- (ii) $\pi_1(Y) = G$ and $\pi_1(g) = \alpha : \pi_1(X) \rightarrow \pi_1(Y)$, and
- (iii) for any $q \geq 2$ the map g induces an isomorphism

$$(1) \quad g_* : H_q(X; R) \xrightarrow{\cong} H_q(Y; R).$$

If X is finite, G is finitely presented and R is finitely G -dense, then Y is also finite.

In Corollary 4.10, we have to assume that the group G is finitely presented, which is the same assumption as that of Higson, Roe and Schick [1].

COROLLARY 4 (Corollary 4.10): For a finitely presented group G with

$$H_0(G; C_r^*(G)) = H_1(G; C_r^*(G)) = H_2(G; C_r^*(G)) = 0,$$

there is a finite CW complex Y such that $\pi_1(Y) = G$ and for each integer $n \geq 0$, the homology group $H_n(Y; C_r^*(G)) = 0$.

All other results are still true.

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References

- [1] N. Higson, J. Roe and T. Schick, *Spaces with vanishing l^2 -homology and their fundamental groups (after Farber and Weinberger)*, *Geometriae Dedicata* **87** (2001), 335–343.
- [2] S. Ye, *A unified approach to the plus-construction, Bousfield localization, Moore spaces and zero-in-the-spectrum examples*, *Israel Journal of Mathematics* **192** (2012), 699–717. DOI: 10.1007/s11856-012-0051-y.