A new fourth order embedded RKHeM(4,4) method with error control on single layer/raster cellular neural network

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Following corrections are in order for the above mentioned paper:

In Abstract, page no. 1, left column, line no. 9: Multi-layer should be read as Single layer.

In Abstract, page no. 1, left column, line no. 17, 18: RK-Embedded Centroidal Mean and RK-Embedded Contra-Harmonic Mean should be ignored.

In Keywords, page no. 1, left column, line no. 21: Multilayer Raster, should be read as Single layer/Raster.

In page no. 2, left column, line no. 2: Multilayer Raster, should be read as Single layer/Raster.

In page no. 2, right column, line no. 8, 9: RK-Embedded Centroidal Mean and RK-Embedded Contra-Harmonic Mean should be ignored.

In page no. 5, left column, Eq. 13 should be read as:

$$|LTE_{AM} - LTE_{HeM}| = \frac{h^5}{1658880} \left[33583 f f_y^4 + 40662 f^2 f_y^2 f_{yy} + 33120 f^3 f_{yy}^2 + 14444 f^3 f_y f_{yyy} \right]$$

In page no. 5, right column, Eq. 16 should be read as:

$$LTE_{AM} - LTE_{HeM} \leq \left(\frac{121809}{1658880} \right) P^4 \cdot Qh^5$$

In page no. 5, right column, Eq. 17 should be read as:

$$\text{Hence, } |y_{n+1}^{AM} - y_{n+1}^{HeM}| \leq \frac{121809}{1658880} P^4 \cdot Qh^5$$

In page no. 5, right column, Eq. 18 should be read as:

$$\frac{121809}{1658880} P^4 Qh^5 < TOL \quad \text{or} \quad h < \left[\frac{13.618698 \times TOL}{P^4 Q} \right]^{1/5}$$

In page no. 6, right column, line no. 3, 4: RK-Embedded Centroidal Mean and RK-Embedded Contra-Harmonic Mean should be ignored.

In page no. 6, right column, Eq. 23 should be

$$ERREST = |Y_{AM} - Y_{HeM}| \times \frac{121809}{1658880}$$

In page no. 7, right column, line no. 29, Sect. 2.1, Step 2 should be read with the following sub step:

Sub Step 2.1 /*Calculating by RK-Embedded Heronian Mean Method*/

$$k_1 = f(x_n, y_n) = k_1^*, \quad k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{hk_1}{2}\right) = k_2^*,$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{hk_2}{2}\right), \quad k_4 = f(x_n + h, y_n + hk_3),$$

$$k_3 = f\left(x_n + \frac{1}{2}h, y_n - \frac{1}{48}hk_1 + \frac{25}{48}hk_2\right) = k_3^*,$$

$$k_4 = f\left(x_n + h, y_n - \frac{1}{24}hk_1 + \frac{47}{600}hk_2 + \frac{289}{300}hk_3\right) = k_4^*,$$

$$y_{n+1} = y_n + h \left[\frac{k_2}{6} + \frac{k_3}{6} + \frac{2}{3} \left(\frac{k_1 k_2}{k_1 + k_2} \right) + \frac{2}{3} \left(\frac{k_3 k_4}{k_3 + k_4} \right) \right]$$

$$y_{n+1}^* = y_n + \frac{h}{9} \left[k_1^* + 2(k_2^* + k_3^*) + k_4^* + \sqrt{|k_1^* + k_2^*|} \right.$$

$$\left. + \sqrt{|k_2^* + k_3^*|} + \sqrt{|k_3^* + k_4^*|} \right]$$

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Sub Step 2.2 /*Calculating by RK-Embedded Harmonic Mean Method*/

$$\begin{aligned}
 k_1 &= f(x_n, y_n) = k_1^*, \quad k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{hk_1}{2}\right) = k_2^*, \\
 k_3 &= f\left(x_n + \frac{h}{2}, y_n + \frac{hk_2}{2}\right), \quad k_4 = f(x_n + h, y_n + hk_3), \\
 k_3^* &= f\left(x_n + \frac{h}{2}, y_n - \frac{hk_1}{8} + \frac{5hk_2}{8}\right), \\
 k_4^* &= f\left(x_n + h, y_n - \frac{hk_1}{4} + \frac{7hk_2}{20} + \frac{9hk_3}{10}\right), \\
 y_{n+1} &= y_n + h \left[\frac{k_2}{6} + \frac{k_3}{6} + \frac{2}{3} \left(\frac{k_1 k_2}{k_1 + k_2} \right) + \frac{2}{3} \left(\frac{k_3 k_4}{k_3 + k_4} \right) \right]
 \end{aligned}$$

Fig. 2 **a** Original image.
b After averaging and edge detection templates by employing RK-Heronian Mean Algorithm and **c** after averaging and edge detection templates by employing RK-Harmonic Mean Algorithm

Fig. 3 Maximum step-size (ΔT) yields the convergence for three different templates using two different numerical integration techniques

In page no. 8, right column, Sub-Sect.6.1.1 should be ignored.

In page no. 9, right column, Sub-Sect.6.1.3 should be ignored.

From page no. 9 to page 10 (before Conclusion), Sect.7 should contain as follows:

Using high power workstation, all the simulated outputs are presented below. It is observed that the actual CPU time used is equal to the simulation time. Figures 2b and c illustrate the results of the raster simulator obtained from a complex image of 256 * 256 pixels. The results of the raster simulator obtained from a complex image of 256 * 256 pixels are

Fig. 4 Comparison of four numerical integration techniques using the edge detection template

depicted respectively in figures 2b and c using RK-embedded Heronian Mean and RK-embedded Harmonic Mean. On the original image in order to get the images shown in figures 2b and c respectively, an averaging template followed by an edge detection template is applied. It is observed from figures 2b and c that the edges obtained by RK-embedded Heronian Mean is better than obtained by RK-embedded Harmonic Mean. Furthermore by raster CNN simulation the simulation took 189.56 s. It is seen from Fig. 3 that RK-embedded Heronian Mean allows us to select a maximum step-size (ΔT) as compared to RK-embedded Harmonic Mean method irrespective of the selection of templates.

Using edge detection template the results of Fig. 4 is obtained by simulating a small image. It is of importance to see from Fig. 4 that for a larger step-size (ΔT), RK-embedded Heronian Mean takes lesser simulation time in comparison with RK-embedded Harmonic Mean. Moreover, in specific from Fig. 4 if the step size (ΔT), is chosen too small, it might

take many iterations, hence longer time, to achieve convergence. But, on the other hand, if the step-size (ΔT), taken is too large, it might not converge at all or it would converge to erroneous steady state values for the cases of adapting Explicit Euler and RK-Gill Methods. One of the notable results of the present investigation is that even for a larger time step-size (ΔT); the RK-embedded Heronian Mean gives better simulation results. It is seen from Table 1 that in views of Local Truncation Error, Global Truncation Error and Error Estimation the presently proposed RK-embedded Heronian Mean method is superior than the RK-embedded Harmonic Mean method.

In page no. 10, right column, Conclusion: RK-embedded Centroidal Mean and RK-Embedded Contra-Harmonic Mean should be ignored.

In page no. 11, the corrected version of Table 1 is as follows.

Table 1 Comparison of LTE, GTE, error estimation for different fourth order four stage RK-embedded methods

Sl. no.	RK-embedded methods	Local truncation error [LTE]	Global truncation error [GTE]	Error estimation
1	RK-Embedded Heronian Mean (Present Paper)	$LTE_{AM} - LTE_{HeM} \leq \left(\frac{121809}{1658880} \right) P^4 \cdot Qh^5 = y_{n+1}^{AM} - y_{n+1}^{HeM} $ $\leq \frac{121809}{1658880} P^4 \cdot Qh^5$	$ \varepsilon_n \leq \left(\frac{h^4}{1658880LD} \right) M(e^{DL(x_n - x_0)} - 1)$	$ERREST = Y_{AM} - Y_{HeM} \times \frac{121809}{1658880}$
2	RK-Embedded Harmonic Mean	$LTE_{AM} - LTE_{HM} \leq \left(\frac{5469}{69120} \right) P^4 \cdot Qh^5 = y_{n+1}^{AM} - y_{n+1}^{HM} $ $\leq \frac{5469}{69120} P^4 \cdot Qh^5$	$ \varepsilon_n \leq \left(\frac{h^4}{69120LD} \right) M(e^{DL(x_n - x_0)} - 1)$	$ERREST = Y_{AM} - Y_{HM} \times \frac{5469}{69120}$