An Improved H_∞ Filter Design for Nonlinear Systems Described by T-S Fuzzy Models with Time-varying Delay

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Abstract: This paper addresses a robust H_∞ filter design problem for nonlinear systems with time-varying delay through Takagi-Sugeno (T-S) fuzzy model approach. Firstly, by introducing free-weighting matrix method combined with a matrix decoupling approach and adopting an improved integral inequality method without ignoring any integral term, less conservative results are achieved. Next, based on the model, new delay-dependent sufficient conditions are derived, which are less conservative than the existing ones via solving the linear matrix inequalities (LMIs). Lastly, simulations show a significant improvement over the previous results.

Keywords: Takagi-Sugeno (T-S) fuzzy model H_∞ filter, nonlinear systems, linear matrix inequality (LMI), time-varying delays.

1 Introduction

It is well known that the conventional Kalman filter (KF)^[1] has been proven to be the most popular and widely applied for practical systems with disturbances. However, it lacks the robustness and the approach is not applicable when the statistics of the external disturbance are not exactly known. Compared to the traditional Kalman filtering, the H_∞ filtering approach can deal with this adverse factor, and it is shown to be insensitive to the uncertainties in the exogenous signal statistics as well as in dynamic models, which is earlier introduced in [2]. Based on these advantages, the H_∞ filtering issues have been extensively investigated by more and some notable results which have been obtained in the last two decades, see for example [3–8].

On the other hand, delay phenomena are often encountered in various practical systems, such as aircraft stabilization, manual control, nuclear reactors, mechanical systems, chemical process systems and networked control systems^[9–13]. It is proved that time delay is usually the source of instability, chaotic mode, and degradation of the systems. In light of this reason, increasing attention has been drawn to the study of the H_∞ filtering design problem for time delay systems^[4–8, 14–24]. For example, H_∞ filtering for systems with a time delay in the measurement was discussed based on a Riccati equation approach in [19]. Further, Souza^[20] showed that the robust H_∞ filtering for systems with time-varying delays was addressed by employing a linear matrix inequality (LMI) approach. However, the existence of time delay

often brings about poor performance^[6, 15–18]. Some nice delay-independent results^[20] and newly proposed delay-dependent results^[11, 12, 21–26] have been obtained. At the same time, it is worth noting that delay-dependent results are usually less conservative than delay-independent ones, especially when the time delay is small.

It is obvious that the Takagi-Sugeno (T-S) fuzzy model has been extended to deal with nonlinear systems with time delay since first proposed in [27]. Consequently, more and more attention has been focused on T-S fuzzy systems and fruitful results have been achieved, especially in recent ten years. Owing to that, some good results have been obtained in [28–30] by adopting the technique of free-weighting matrix method combined with a matrix decoupling approach. Moreover, a better result is acquired in [31] in contrast to those mentioned before. However, all those aforementioned researches omit some useful terms, which lead to inevitable conservativeness. Thus, how to keep the necessary terms and reduce conservativeness has been our great motivation.

For the sake of solving the above problem, an improved integral inequality method without ignoring any integral term is applied to convert the integral inequalities presented in [32], which definitely leads to less conservativeness in this paper than those in [28–31]. Besides, [33–36] played an important role in our work. Finally, one example is used to demonstrate its merits.

Notations. $R > 0$ means that R is symmetric positive definite, I denotes the identity matrix, and the symbol “ T ” in a matrix $X \in \mathbf{R}^n$ stands for the transposed elements in the symmetrical position. Also, the superscript “ T ” and “ -1 ” denote matrix transpose and inverse, respectively. Furthermore, $\text{diag}\{\cdots\}$ stands for a diagonal matrix. Besides, $\|X\|$ denotes the Euclidean norm and $L_2[0, \infty)$ is the space of square integrable vectors.

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2 Problem formulation

Consider the following continuous-time T-S model with time-varying delay. It is expressed by a linear system and the i -th rule of the T-S fuzzy model is written as follows:

Plant rule i :

IF $\sigma_1(t)$ is $M_{i1}, \dots$, and $\sigma_p(t)$ is M_{ip} , THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_i w(t), & t > 0 \\ y(t) = C_i x(t) + C_{\tau i} x(t - \tau(t)) + D_i w(t) \\ z(t) = E_i x(t) + E_{\tau i} x(t - \tau(t)) \\ x(t) = \phi(t), \quad \forall t \in [-\tau_0, 0] \end{cases} \quad (1)$$

where M_{ij} is the fuzzy set, $x(t) \in \mathbf{R}^n$ is the state vector, $y(t) \in \mathbf{R}^m$ is the measurement vector, $z(t) \in \mathbf{R}^p$ stands for the signal vector to be estimated, $w(t) \in \mathbf{R}^q$ is the disturbance variable which belongs to $L_2[0, \infty)$; $A_i, A_{\tau i}, B_i, C_i, C_{\tau i}, D_i, E_i$, and $E_{\tau i}$ are some constant real matrices of appropriate dimensions, where $i = 1, 2, \dots, r$ and r is the number of IF-THEN rules; $\sigma_1(t) \dots \sigma_p(t)$ are the premise variables; $\phi(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau_0, 0]$, and $\tau(t)$ is a time-varying delay term satisfying

$$0 \leq \tau(t) \leq \tau_0, \quad \dot{\tau}(t) \leq d \quad (2)$$

where τ_0 and d are two scalars. The final transform form of the fuzzy system is inferred below

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r \mu_i [A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_i w(t)], & t > 0 \\ y(t) = \sum_{i=1}^r \mu_i [C_i x(t) + C_{\tau i} x(t - \tau(t)) + D_i w(t)] \\ z(t) = \sum_{i=1}^r \mu_i [E_i x(t) + E_{\tau i} x(t - \tau(t))] \\ x(t) = \phi(t), \quad \forall t \in [-\tau_0, 0] \end{cases} \quad (3)$$

where

$$\beta_i(\sigma(t)) = \prod_{j=1}^p M_{ij}(\sigma_j(t))$$

$$\sigma(t) = (\sigma_1(t), \sigma_2(t), \dots, \sigma_p(t))$$

$$\mu_i = \mu_i(\sigma(t)) = \frac{\beta_i(\sigma(t))}{\sum_{j=1}^r \beta_j(\sigma(t))}$$

where $M_{ij}(\sigma_j(t))$ is the grade of membership of $\sigma_j(t)$ in M_{ij} . It is easy to find that $\beta_i(\sigma(t)) \geq 0$, $i = 1, 2, \dots, r$, and $\sum_{j=1}^r \beta_j(\sigma(t)) > 0, \forall t$. Then, $\mu_i(\sigma(t)) \geq 0$, $i = 1, 2, \dots, r$ and $\sum_{j=1}^r \mu_j(\sigma(t)) = 1, \forall t$.

A T-S fuzzy H_∞ filter is constructed as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^r \mu_i [A_{fi} \hat{x}(t) + B_{fi} y(t)], \quad \hat{x}(0) = \hat{x}_0 \\ \hat{z}(t) = \sum_{i=1}^r \mu_i C_{fi} \hat{x}(t) \end{cases} \quad (4)$$

where $A_{fi}, B_{fi}, C_{fi}, i = 1, 2, \dots, r$ are the filter parameters to be determined. Thus, through (3) and (4), the filter error system is written in the following form:

$$\begin{cases} \dot{\xi}(t) = \hat{A}(t)\xi(t) + \hat{A}_\tau(t)\xi(t - \tau(t)) + \hat{B}(t)w(t) \\ e(t) = z(t) - \hat{z}(t) = \hat{E}(t)\xi(t) + \hat{E}_\tau(t)\xi(t - \tau(t)) \end{cases} \quad (5)$$

where

$$\xi(t) = [x^T(t), \hat{x}^T(t)], \text{ and } \xi(0) = [\phi^T(t), \hat{x}_0^T], \text{ for } \forall t \in [-\tau_0, 0]$$

$$\hat{A}(t) = \sum_{i,j=1}^r \mu_i \mu_j \begin{bmatrix} A_j & 0 \\ B_{fi} C_j & A_{fi} \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ B_f(t) C(t) & A_f(t) \end{bmatrix}$$

$$\hat{A}_\tau(t) = \sum_{i,j=1}^r \mu_i \mu_j \begin{bmatrix} A_{\tau j} & 0 \\ B_{fi} C_{\tau j} & 0 \end{bmatrix} = \begin{bmatrix} A_\tau(t) & 0 \\ B_f(t) C_\tau(t) & 0 \end{bmatrix}$$

$$\hat{B}(t) = \sum_{i,j=1}^r \mu_i \mu_j \begin{bmatrix} B_j \\ B_{fi} D_j \end{bmatrix} = \begin{bmatrix} B(t) \\ B_f(t) D(t) \end{bmatrix}$$

$$\hat{E}(t) = \sum_{i,j=1}^r \mu_i \mu_j \begin{bmatrix} E_j & C_{fi} \end{bmatrix} = \begin{bmatrix} E(t) & C_f(t) \end{bmatrix}$$

$$\hat{E}_\tau(t) = \sum_{i=1}^r \mu_i \begin{bmatrix} E_{\tau j} & 0 \end{bmatrix} = \begin{bmatrix} E_\tau(t) & 0 \end{bmatrix}.$$

So the fuzzy H_∞ filtering problem for system (3) can be expressed as follows.

1) The filter error system (6) with $w = 0$ is asymptotically stable;

2) The following H_∞ performance is satisfied:

$$\int_0^L \|e(t)\|^2 dt \leq \gamma^2 \int_0^L \|w(t)\|^2 dt \quad (6)$$

for all $L > 0$ and $w(t) \in L_2[0, \infty)$ under zero initial conditions.

Lemma 1. For $\tau_0 > 0$, suppose that $\zeta^T(t) = [x^T(t), x^T(t - \tau(t)), x^T(t - \tau_0), w^T(t)]$, $0 \leq \tau(t) \leq \tau_0$, $R > 0$ and Q is a symmetric matrix of appropriate dimensions. Then,

$$\zeta^T(t) Q \zeta(t) - \int_{t-\tau_0}^t \dot{x}^T(s) R \dot{x}(s) ds \leq 0$$

holds, if there exist free weighting matrices M and N of appropriate dimensions such that

$$Q - \Psi - \Psi^T + \tau_0 M R^{-1} M^T \leq 0$$

and

$$Q - \Psi - \Psi^T + \tau_0 N R^{-1} N^T \leq 0$$

where

$$\Psi = \begin{bmatrix} M_1 & N_1 - M_1 & -N_1 & 0 \\ M_2 & N_2 - M_2 & -N_2 & 0 \\ M_3 & N_3 - M_3 & -N_3 & 0 \\ M_4 & N_4 - M_4 & -N_4 & 0 \end{bmatrix}$$

$$M = [M_1^T, M_2^T, M_3^T, M_4^T]^T$$

$$N = [N_1^T, N_2^T, N_3^T, N_4^T]^T.$$

Proof. For $\tau_0 > 0$ and $0 \leq \tau(t) \leq \tau_0$, by Leibniz-Newton

formula, one has

$$\begin{aligned} & \zeta^T(t)Q\zeta(t) - \int_{t-\tau_0}^t \dot{x}^T(s)R\dot{x}(s)ds = \\ & \zeta^T(t)Q\zeta(t) - \\ & 2\zeta^T(t)M[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^t \dot{x}(s)ds] - \\ & 2\zeta^T(t)N[x(t-\tau(t)) - x(t-\tau_0) - \int_{t-\tau_0}^{t-\tau(t)} \dot{x}(s)ds] - \\ & \int_{t-\tau(t)}^t \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_0}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)ds \leq \\ & \frac{1}{\tau_0} \int_{t-\tau(t)}^t \begin{bmatrix} \zeta(t) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q - \Psi - \Psi^T & \tau_0 M \\ * & -\tau_0 R \end{bmatrix} \begin{bmatrix} \zeta(t) \\ \dot{x}(s) \end{bmatrix} ds + \\ & \frac{1}{\tau_0} \int_{t-\tau_0}^{t-\tau(t)} \begin{bmatrix} \zeta(t) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q - \Psi - \Psi^T & \tau_0 N \\ * & -\tau_0 R \end{bmatrix} \begin{bmatrix} \zeta(t) \\ \dot{x}(s) \end{bmatrix} ds. \end{aligned}$$

On the other hand, if inequalities

$$Q - \Psi - \Psi^T + \tau_0 M R^{-1} M^T \leq 0$$

and

$$Q - \Psi - \Psi^T + \tau_0 N R^{-1} N^T \leq 0$$

hold, then by Schur complement, these are equivalent to

$$\begin{bmatrix} Q - \Psi - \Psi^T & \tau_0 M \\ * & -\tau_0 R \end{bmatrix} < 0$$

and

$$\begin{bmatrix} Q - \Psi - \Psi^T & \tau_0 N \\ * & -\tau_0 R \end{bmatrix} < 0.$$

That is, $\zeta^T(t)Q\zeta(t) - \int_{t-\tau_0}^t \dot{x}^T(s)R\dot{x}(s)ds \leq 0$ holds. $\square$

3 Fuzzy H_∞ filtering analysis

Lemma 2. For the system (3) with (2) and a prescribed real number $\gamma > 0$, if there exist matrices $\hat{P} > 0$, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$, $\hat{R} > 0$, and matrix function $A_f(t)$, $B_f(t)$, $C_f(t)$, $\hat{M}_1(t)$, $\hat{M}_2(t)$, $\hat{M}_3(t)$, $\hat{M}_4(t)$ as well as $\hat{N}_1(t)$, $\hat{N}_2(t)$, $\hat{N}_3(t)$, $\hat{N}_4(t)$ such that the matrix inequality (6) holds for given scalars $k > 0$ and $\delta > 0$, then the filtering error system (5) is asymptotically stable and satisfies the H_∞ performance index, where

$$\begin{bmatrix} \hat{\Xi}(t) & \tau_0 \hat{M}(t) & \sqrt{\tau_0} \hat{\Gamma}^T(t) & \hat{\Upsilon}^T(t) \\ * & -\tau_0 \hat{R} & 0 & 0 \\ * & * & \frac{-2\delta \hat{P} + \delta^2 \hat{R}}{k} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} \hat{\Xi}(t) & \tau_0 \hat{N}(t) & \sqrt{\tau_0} \hat{\Gamma}^T(t) & \hat{\Upsilon}^T(t) \\ * & -\tau_0 \hat{R} & 0 & 0 \\ * & * & \frac{-2\delta \hat{P} + \delta^2 \hat{R}}{k} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (8)$$

and

$$\hat{\Omega}(t) = \begin{bmatrix} \phi_{11} & \phi_{12} & 0 & \hat{P}\hat{B}(t) \\ * & \phi_{13} & 0 & 0 \\ * & * & -\hat{Q}_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \quad (9)$$

where

$$\hat{M}(t) = [\hat{M}_1^T(t), \hat{M}_2^T(t), \hat{M}_3^T(t), \hat{M}_4^T(t)]^T$$

$$\hat{N}(t) = [\hat{N}_1^T(t), \hat{N}_2^T(t), \hat{N}_3^T(t), \hat{N}_4^T(t)]^T$$

$$\hat{\Psi}(t) = \begin{bmatrix} \hat{M}_1(t) & \hat{N}_1(t) - \hat{M}_1(t) & -\hat{N}_1(t) & 0 \\ \hat{M}_2(t) & \hat{N}_2(t) - \hat{M}_2(t) & -\hat{N}_2(t) & 0 \\ \hat{M}_3(t) & \hat{N}_3(t) - \hat{M}_3(t) & -\hat{N}_3(t) & 0 \\ \hat{M}_4(t) & \hat{N}_4(t) - \hat{M}_4(t) & -\hat{N}_4(t) & 0 \end{bmatrix}$$

$$\hat{\Xi}(t) = \hat{\Omega}(t) - \hat{\Psi}(t) - \hat{\Psi}^T(t)$$

$$\phi_{11} = \hat{P}\hat{A}(t) + \hat{A}^T(t)\hat{P} + \hat{Q}_1 + \hat{Q}_2$$

$$\phi_{12} = \hat{P}\hat{A}_\tau(t), \quad \phi_{13} = -(1-d)\hat{Q}_1$$

$$\hat{\Gamma}(t) = [\hat{A}(t)\hat{P}, \hat{A}_\tau(t)\hat{P}, 0, \hat{B}(t)\hat{P}]$$

$$\hat{\Upsilon}(t) = [\hat{E}(t), \hat{E}_\tau(t), 0, 0].$$

Proof. The considered Lyapunov function candidate is chosen as follows:

$$\begin{aligned} V(\xi(t)) &= \xi^T(t)\hat{P}\xi(t) + \int_{t-\tau(t)}^t \xi^T(s)\hat{Q}_1\xi(s)ds + \\ & \int_{t-\tau_0}^t \xi^T(s)\hat{Q}_2\xi(s)ds + \\ & k \int_{-\tau_0}^0 \int_{t+\theta}^t \xi^T(s)\hat{R}\xi(s)dsd\theta \end{aligned} \quad (10)$$

where $\hat{P}$, $\hat{Q}_1$, $\hat{Q}_2$, and $\hat{R}$ are positive definite symmetric matrices. With the aforementioned equations, differentiating (10) along the trajectories of system (5) yields

$$\begin{aligned} \dot{V}(\xi(t)) &= 2\xi^T(t)\hat{P}\dot{\xi}(t) + \xi^T(t)(\hat{Q}_1 + \hat{Q}_2)\xi(t) - \\ & (1 - \dot{\tau}(t))\xi^T(t - \tau(t))\hat{Q}_1\xi(t - \tau(t)) - \\ & \xi^T(t - \tau_0)\hat{Q}_2\xi(t - \tau_0) + k\tau_0\xi^T(t)\hat{R}\dot{\xi}(t) - \\ & k \int_{t-\tau_0}^t \xi^T(s)\hat{R}\dot{\xi}(s)ds. \end{aligned}$$

From Lemma 1 and by Schur complement

$$\begin{aligned} \dot{V}(\xi(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) &\leq \\ & \frac{1}{\tau_0} \int_{t-\tau(t)}^t \begin{bmatrix} \zeta(t) \\ \dot{\xi}(s) \end{bmatrix}^T \begin{bmatrix} \Theta(t) & \tau_0 \hat{M}(t) \\ * & -\tau_0 \hat{R} \end{bmatrix} \begin{bmatrix} \zeta(t) \\ \dot{\xi}(s) \end{bmatrix} ds + \\ & \frac{1}{\tau_0} \int_{t-\tau_0}^{t-\tau(t)} \begin{bmatrix} \zeta(t) \\ \dot{\xi}(s) \end{bmatrix}^T \begin{bmatrix} \Theta(t) & \tau_0 \hat{N}(t) \\ * & -\tau_0 \hat{R} \end{bmatrix} \begin{bmatrix} \zeta(t) \\ \dot{\xi}(s) \end{bmatrix} ds \end{aligned} \quad (11)$$

where

$$\zeta^T(t) = [\xi^T(t), \xi^T(t - \tau(t)), \xi^T(t - \tau_0), w^T(t)]$$

$$\Theta(t) = \hat{\Xi}(t) + \tau_0 \hat{\Gamma}^T(t)[k\hat{R}]^{-1}\hat{\Gamma}(t) + \hat{\Upsilon}^T(t)\hat{\Upsilon}(t). \quad (12)$$

From $(\delta\hat{R} - \hat{P})\hat{R}^{-1}(\delta\hat{R} - \hat{P}) \geq 0$, one has that $-\hat{P}(k\hat{R})^{-1}\hat{P} \leq \frac{-2\delta\hat{P} + \delta^2\hat{R}}{k}$ for any scalars $k > 0$, $\delta > 0$. Hence, it can be verified that (6) results in $\Xi(t) < 0$, where $\Xi(t) < 0$ is a matrix derived from (6) through changing the $(3, 3)$ block $\frac{-2\delta\hat{P} + \delta^2\hat{R}}{k}$ to $-\hat{P}(k\hat{R})^{-1}\hat{P}$. Therefore, by the Schur complement, it is obvious that $\Xi(t) < 0$ is equivalent to $\Xi(t) + \tau_0\hat{\Gamma}^T(t)[k\hat{R}]^{-1}\hat{\Gamma}(t) + \hat{\Upsilon}^T(t)\hat{\Upsilon}(t) < 0$. Consequently, it follows from (11), $V(\xi(t))|_{t=0} = 0$ and $V(\xi(t))|_{t=L} \geq 0$ that

$$\int_0^L (\|e(t)\|^2 - \gamma^2 \|w(t)\|^2) dt + V(\xi(t))|_{t=L} - V(\xi(t))|_{t=0} \leq 0 \quad (13)$$

which implies that (6) holds. Thus, H_∞ performance is verified. In addition, it can be clearly seen from (6) and (7) that the time derivative of $V(\xi(t))$ along the solution of (5) when $w = 0$ satisfies $\dot{V}(\xi(t)) < 0$. As a result, the asymptotic stability of system (5) follows immediately when $w = 0$.

4 Main results

Lemma 1 provides a sufficient condition for the existence of an asymptotically stable filter with H_∞ performance. However, there exist some coupled matrix variables in the matrix inequality. In order to decouple the variables in (6) and (7), we will use decoupling technique similar to that in [28]. In such a way, inequality (6) and (7) can be equivalently transformed into another form as follows:

Lemma 3. There exist matrices $\hat{P} > 0$, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$ and $\hat{R} > 0$, and matrix functions $A_f(t)$, $B_f(t)$, $C_f(t)$, $\hat{M}_1(t)$, $\hat{M}_2(t)$, $\hat{M}_3(t)$, $\hat{M}_4(t)$ and $\hat{N}_1(t)$, $\hat{N}_2(t)$, $\hat{N}_3(t)$, $\hat{N}_4(t)$ such that the matrix inequality (6) and (7) hold if and only if there exist matrices $P > 0$, $F > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$ and matrix functions $\bar{A}_f(t)$, $\bar{B}_f(t)$, $\bar{C}_f(t)$, $M_1(t)$, $M_2(t)$, $M_3(t)$, $M_4(t)$ and $N_1(t)$, $N_2(t)$, $N_3(t)$, $N_4(t)$ such that the following inequality holds.

$$\begin{bmatrix} \Xi(t) & \tau_0 M(t) & \sqrt{\tau_0} \Gamma^T & \Upsilon \\ * & -\tau_0 R & 0 & 0 \\ * & * & \frac{-2\delta\varphi_{66} + \delta^2 R}{k} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (14)$$

$$\begin{bmatrix} \Xi(t) & \tau_0 N(t) & \sqrt{\tau_0} \Gamma^T & \Upsilon \\ * & -\tau_0 R & 0 & 0 \\ * & * & \frac{-2\delta\varphi_{66} + \delta^2 R}{k} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (15)$$

where

$$\Omega(t) = \begin{bmatrix} \theta_{11} & \theta_{12} & 0 & \varphi_{14} \\ * & \theta_{22} & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \quad (16)$$

and

$$\Psi(t) = \begin{bmatrix} M_1(t) & N_1(t) - M_1(t) & -N_1(t) & 0 \\ M_2(t) & N_2(t) - M_2(t) & -N_2(t) & 0 \\ M_3(t) & N_3(t) - M_3(t) & -N_3(t) & 0 \\ M_4(t) & N_4(t) - M_4(t) & -N_4(t) & 0 \end{bmatrix}$$

$$\Xi(t) = \Omega(t) - \Psi(t) - \Psi^T(t)$$

$$\theta_{11} = \varphi_{11} + \varphi_{11}^T + Q_1 + Q_2$$

$$\theta_{12} = \varphi_{12}, \quad \theta_{22} = -(1-d)Q_1$$

$$M(t) = [M_1^T(t), M_2^T(t), M_3^T(t), M_4^T(t)]^T$$

$$N(t) = [N_1^T(t), N_2^T(t), N_3^T(t), N_4^T(t)]^T$$

$$\Gamma = [\varphi_{11}, \varphi_{12}, 0, \varphi_{14}]$$

$$\Upsilon = [\varphi_{17}, \varphi_{27}, 0, 0]$$

$$\varphi_{11} = \begin{bmatrix} PA(t) + \bar{B}_f(t)C(t) & \bar{A}_f(t) \\ FA(t) + \bar{B}_f(t)C(t) & \bar{A}_f(t) \end{bmatrix}$$

$$\varphi_{12} = \begin{bmatrix} PA_\tau(t) + \bar{B}_f(t)C_\tau(t) & 0 \\ FA_\tau(t) + \bar{B}_f(t)C_\tau(t) & 0 \end{bmatrix}$$

$$\varphi_{14} = \begin{bmatrix} PB(t) + \bar{B}_f(t)D(t) \\ FB(t) + \bar{B}_f(t)D(t) \end{bmatrix}, \quad \varphi_{66} = \begin{bmatrix} P & F \\ F & F \end{bmatrix}$$

$$\varphi_{17} = \begin{bmatrix} E^T(t) \\ -\bar{C}_f^T(t) \end{bmatrix}, \quad \varphi_{27} = \begin{bmatrix} E_\tau^T(t) \\ 0 \end{bmatrix}.$$

Proof. (Necessity) Suppose that (6) and (7) hold for $\hat{P} > 0$, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$, $\hat{R} > 0$ as well as matrix functions $A_f(t)$, $B_f(t)$, $C_f(t)$, $\hat{M}_1(t)$, $\hat{M}_2(t)$, $\hat{M}_3(t)$, $\hat{M}_4(t)$ and $\hat{N}_1(t)$, $\hat{N}_2(t)$, $\hat{N}_3(t)$, $\hat{N}_4(t)$. Partition $\hat{P} > 0$ as

$$\hat{P} = \begin{bmatrix} P & S \\ S^T & W \end{bmatrix} \quad (17)$$

where $P > 0$, $W > 0$ and S is invertible. Let

$$H = \begin{bmatrix} I & 0 \\ 0 & SW^{-1} \end{bmatrix} \quad (18)$$

Pre- and post-multiplying (6) and (7) by $\text{diag}\{H, H, H, I, H, H, I\}$ and its transpose, respectively, produces (14) and (15) with the variable changing as

$$F = SW^{-1}S^T, Q_1 = H\hat{Q}_1H^T, Q_2 = H\hat{Q}_2H^T$$

$$R = H\hat{R}H^T, M_4(t) = \hat{M}_4(t)H^T, N_4(t) = \hat{N}_4(t)H^T$$

$$M_1(t) = H\hat{M}_1(t)H^T, M_2(t) = H\hat{M}_2(t)H^T$$

$$M_3(t) = H\hat{M}_3(t)H^T, N_1(t) = H\hat{N}_1(t)H^T$$

$$N_2(t) = H\hat{N}_2(t)H^T, N_3(t) = H\hat{N}_3(t)H^T$$

$$\bar{A}_f(t) = SA_f(t)W^{-1}S^T, \bar{B}_f(t) = SB_f(t)$$

$$\bar{C}_f(t) = C_f(t)W^{-1}S^T.$$

This proves necessity.

(Sufficiency) Suppose that (13) and (14) hold for $P > 0$, $F > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$, and matrix functions $\bar{A}_f(t)$, $\bar{B}_f(t)$, $\bar{C}_f(t)$, $M_1(t)$, $M_2(t)$, $M_3(t)$, $M_4(t)$, and $N_1(t)$, $N_2(t)$, $N_3(t)$, $N_4(t)$. Choose two matrices $W > 0$ and S invertible such that $F = SW^{-1}S^T$. Let $\hat{P}$ and H be defined as in (16) and (17), respectively. Then, $\hat{P} > 0$ due to $\varphi_{66} > 0$ as inferred by (14) and (15). Pre and post-multiplying (14) and (15) by $\text{diag}\{H^{-1}, H^{-1}, H^{-1}, I, H^{-1}, H^{-1}, I\}$ and its transpose yields (6) with the variable changing:

$$\hat{Q}_1 = H^{-1}Q_1H^{-T}, \hat{Q}_2 = H^{-1}Q_2H^{-T}$$

$$\hat{R} = H^{-1}RH^{-T}, \hat{M}_4(t) = M_4(t)H^{-T}, \hat{N}_4(t) = N_4(t)H^{-T}$$

$$\hat{M}_1(t) = H^{-1}M_1(t)H^{-T}, \hat{M}_2(t) = H^{-1}M_2(t)H^{-T}$$

$$\hat{M}_3(t) = H^{-1}M_3(t)H^{-T}, \hat{N}_1(t) = H^{-1}N_1(t)H^{-T}$$

$$\hat{N}_2(t) = H^{-1}N_2(t)H^{-T}, \hat{N}_3(t) = H^{-1}N_3(t)H^{-T}$$

$$B_f(t) = S^{-1}\bar{B}_f(t), C_f(t) = \bar{C}_f(t)S^{-T}W.$$

□

Theorem 1. For the given positive scalars: τ_0, d, δ, k and γ , the H_∞ filter system (5) is asymptotically stable and satisfies the H_∞ performance index if there exist matrices $P > 0$, $F > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$, $\bar{A}_{fi}$, $\bar{B}_{fi}$, $\bar{C}_{fi}$, M_{1i} , M_{2i} , M_{3i} , M_{4i} and N_{1i} , N_{2i} , N_{3i} , N_{4i} , for $i = 1, 2, \dots, N$ such that the following LMIs hold

$$\Phi_{ii} < 0, \quad i = 1, 2, \dots, r \quad (19)$$

$$\Phi_{ij} + \Phi_{ji} \leq 0, \quad 1 \leq i < j \leq r \quad (20)$$

$$\Pi_{ii} < 0, \quad i = 1, 2, \dots, r \quad (21)$$

$$\Pi_{ij} + \Pi_{ji} \leq 0, \quad 1 \leq i < j \leq r. \quad (22)$$

In this case, the filter parameters in (4) are given by

$$A_{fi} = F^{-1}\bar{A}_{fi}, B_{fi} = F^{-1}\bar{B}_{fi} \\ C_{fi} = \bar{C}_{fi}, i = 1, 2, \dots, r \quad (23)$$

where

$$\Phi_{ij} = \begin{bmatrix} \Xi_{ij} & \tau_0 M_i & \sqrt{\tau_0} \Gamma^T & \Upsilon \\ * & -\tau_0 R & 0 & 0 \\ * & * & \frac{-2\delta\varphi_{66} + \delta^2 R}{k} & 0 \\ * & * & * & -I \end{bmatrix} \quad (24)$$

$$\Pi_{ij} = \begin{bmatrix} \Xi_{ij} & \tau_0 N_i & \sqrt{\tau_0} \Gamma^T & \Upsilon \\ * & -\tau_0 R & 0 & 0 \\ * & * & \frac{-2\delta\varphi_{66} + \delta^2 R}{k} & 0 \\ * & * & * & -I \end{bmatrix} \quad (25)$$

$$\Xi_{ij} = \Omega_{ij} - \Psi_{ij} - \Psi_{ij}^T \quad (26)$$

and

$$\Omega_{ij} = \begin{bmatrix} \theta_{11} & \theta_{12} & 0 & \varphi_{14} \\ * & \theta_{22} & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Psi_{ij} = \begin{bmatrix} M_{1i} & N_{1i} - M_{1i} & -N_{1i} & 0 \\ M_{2i} & N_{2i} - M_{2i} & -N_{2i} & 0 \\ M_{3i} & N_{3i} - M_{3i} & -N_{3i} & 0 \\ M_{4i} & N_{4i} - M_{4i} & -N_{4i} & 0 \end{bmatrix}$$

where

$$M_i = [M_{1i}^T, M_{2i}^T, M_{3i}^T, M_{4i}^T]^T$$

$$N_i = [N_{1i}^T, N_{2i}^T, N_{3i}^T, N_{4i}^T]^T$$

$$\theta_{11} = \varphi_{11} + \varphi_{11}^T + Q_1 + Q_2$$

$$\theta_{12} = \varphi_{12}, \quad \theta_{22} = -(1-d)Q_1$$

$$\varphi_{11} = \begin{bmatrix} PA_j + \bar{B}_{fi}C_j & \bar{A}_{fi} \\ FA_j + \bar{B}_{fi}C_j & \bar{A}_{fi} \end{bmatrix}$$

$$\varphi_{12} = \begin{bmatrix} PA_{\tau j} + \bar{B}_{fi}C_{\tau j} & 0 \\ FA_{\tau j} + \bar{B}_{fi}C_{\tau j} & 0 \end{bmatrix}$$

$$\varphi_{14} = \begin{bmatrix} PB_j + \bar{B}_{fi}D_j \\ FB_j + \bar{B}_{fi}D_j \end{bmatrix}, \varphi_{27} = \begin{bmatrix} E_{\tau j}^T \\ 0 \end{bmatrix}$$

$$\varphi_{66} = \begin{bmatrix} P & F \\ F & F \end{bmatrix}, \varphi_{17} = \begin{bmatrix} E_{fi}^T \\ -\bar{C}_{fi}^T \end{bmatrix}.$$

Proof. Set

$$\bar{A}_f(t) = \sum_{i=1}^r \mu_i \bar{A}_{fi}, \quad \bar{B}_f(t) = \sum_{i=1}^r \mu_i \bar{B}_{fi}$$

$$\bar{C}_f(t) = \sum_{i=1}^r \mu_i \bar{C}_{fi}, \quad M_1(t) = \sum_{i=1}^r \mu_i M_{1i}$$

$$M_2(t) = \sum_{i=1}^r \mu_i M_{2i}, \quad M_3(t) = \sum_{i=1}^r \mu_i M_{3i}$$

$$M_4(t) = \sum_{i=1}^r \mu_i M_{4i}, \quad N_1(t) = \sum_{i=1}^r \mu_i N_{1i}$$

$$N_2(t) = \sum_{i=1}^r \mu_i N_{2i}, \quad N_3(t) = \sum_{i=1}^r \mu_i N_{3i}$$

$$N_4(t) = \sum_{i=1}^r \mu_i N_{4i}.$$

It further follows that

$$\sum_{i=1}^r \mu_i^2 \Phi_{ii} + \sum_{i=1}^r \sum_{i < j}^r \mu_i \mu_j (\Phi_{ij} + \Phi_{ji}) < 0 \quad (27)$$

and

$$\sum_{i=1}^r \mu_i^2 \Pi_{ii} + \sum_{i=1}^r \sum_{i < j}^r \mu_i \mu_j (\Pi_{ij} + \Pi_{ji}) < 0. \quad (28)$$

By virtue of Lemmas 1 and 2, the H_∞ filter design problem is easily solvable, and the filter matrix functions can be given by $A_f(t) = S^{-1}\bar{A}_f(t)S^{-T}W$, $B_f(t) = S^{-1}\bar{B}_f(t)$, and $C_f(t) = \bar{C}_f(t)S^{-T}W$, where matrices $W > 0$ and S are such that $F = SW^{-1}S^T$. Or equivalently, through transformation $S^{-T}W\hat{x}(t)$, the filter matrix functions would be of the following forms:

$$A_f(t) = S^{-T}W(S^{-1}\bar{A}_f(t)S^{-T}W)W^{-1}S^T = F^{-1}\bar{A}_f(t)$$

$$B_f(t) = S^{-T}W(S^{-1}\bar{B}_f(t)) = F^{-1}\bar{B}_f(t)$$

$$C_f(t) = (\bar{C}_f(t)S^{-T}W)W^{-1}S^T = \bar{C}_f(t).$$

Hence, the filter parameters in (4) are given by (23). $\square$

5 Simulation

In this section, a numerical example is used to test the effectiveness of the proposed method.

Example 1. Consider the following system with parameters^[28]:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^2 \mu_i(\sigma(t)) [A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_i w(t)] \\ y(t) &= \sum_{i=1}^2 \mu_i(\sigma(t)) [C_i x(t) + C_{\tau i} x(t - \tau(t)) + D_i w(t)] \\ z(t) &= \sum_{i=1}^2 \mu_i(\sigma(t)) [E_i x(t) + E_{\tau i} x(t - \tau(t))] \end{aligned}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix} \\ A_{\tau 1} &= \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad A_{\tau 2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix} \\ C_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix} \\ C_{\tau 1} &= \begin{bmatrix} -0.8 & 0.6 \end{bmatrix}, \quad C_{\tau 2} = \begin{bmatrix} -0.2 & 1 \end{bmatrix} \\ D_1 &= 0.3, \quad D_2 = -0.6 \\ E_1 &= \begin{bmatrix} 1 & -0.5 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix} \\ E_{\tau 1} &= \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad E_{\tau 2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix}. \end{aligned}$$

By means of Theorem 1, for given $d = 0.2$, $\tau_0 = 0.5$ and $\delta = 1$, and computing LMI (19)–(22), we can have the minimum attenuation level $\gamma = 0.229$ as well as a series of available solutions below:

$$\begin{aligned} F &= \begin{bmatrix} 0.1407 & -0.1313 \\ -0.1313 & 0.1760 \end{bmatrix} \\ \bar{A}_{f1} &= \begin{bmatrix} -0.7750 & 0.4227l \\ 0.8057 & -0.5735l \end{bmatrix} \\ \bar{A}_{f2} &= \begin{bmatrix} -0.3167 & 0.3477 \\ 0.2042 & -0.3251 \end{bmatrix} \\ \bar{B}_{f1} &= \begin{bmatrix} -0.3606 \\ 0.3309 \end{bmatrix}, \quad \bar{B}_{f2} = \begin{bmatrix} -0.2321 \\ 0.2233 \end{bmatrix} \\ \bar{C}_{f1} &= \begin{bmatrix} -0.9669 & 0.6275 \end{bmatrix} \\ \bar{C}_{f2} &= \begin{bmatrix} -0.1240 & -0.1528 \end{bmatrix}. \end{aligned}$$

Furthermore, the H_∞ filter parameter matrices are computed from (23) as

$$\begin{aligned} A_{f1} &= \begin{bmatrix} -4.0661 & -0.1163 \\ 1.5452 & -3.3449 \end{bmatrix} \\ A_{f2} &= \begin{bmatrix} -3.8384 & 2.4571 \\ -1.7022 & -0.0146 \end{bmatrix} \\ B_{f1} &= \begin{bmatrix} -2.6579 \\ -0.1023 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -1.5317 \\ 0.1262 \end{bmatrix} \\ C_{f1} &= \begin{bmatrix} -0.9669 & 0.6275 \end{bmatrix} \\ C_{f2} &= \begin{bmatrix} -0.1240 & -0.1528 \end{bmatrix}. \end{aligned}$$

To compare with the presented fuzzy $L_2 - L_\infty$ filter, we consider different (τ_0, δ) via choosing k to get the minimum index r for $d = 0.2$. The corresponding results are summarized in Tables 1–4.

By means of Theorem 1, we choose $d = 0.2$, and suitable scalar k , and then run the simulation for studying two aspects. On the one hand, we find the minimum index γ for the given different τ_0 and δ by choosing suitable scalar k from Tables 1–4, respectively (we let $k = 0.1$ when $\delta = 0.7, 1$; $k = 0.58$ when $\delta = 2, 5$; $k = 2$ when $\delta = 20$). When $\tau_0 = 1$, $\delta = 0.7$ or $\tau_0 = 1$, $\delta = 20$, we can obtain a practicable solution by Theorem 1, however, the methods given in [28–31] are not applicable for this case. In addition, a smaller γ than that in Tables 1–4 would be obtained by choosing more suitable scalar k .

Table 1 Minimal index γ for $d = 0.2$, $\tau_0 = 0.5$

Method	$\delta = 0.7$	$\delta = 1$	$\delta = 2$	$\delta = 5$	$\delta = 20$
[28]	0.59	0.38	0.35	0.34	0.37
[29]	0.42	0.27	0.25	0.24	0.26
[30]	0.4163	0.2661	0.2452	0.2371	0.2537
[31]	0.2575	0.2471	0.2379	0.2359	0.2406
TH.1	0.231	0.229	0.230	0.229	0.230

Table 2 Minimal index γ for $d = 0.2$, $\tau_0 = 0.6$

Method	$\delta = 0.7$	$\delta = 1$	$\delta = 2$	$\delta = 5$	$\delta = 20$
[28]	1.03	0.43	0.36	0.34	0.45
[29]	0.74	0.31	0.25	0.24	0.32
[30]	0.7264	0.3009	0.2495	0.2391	0.2769
[31]	0.2685	0.2531	0.2421	0.2388	0.2501
TH. 1	0.231	0.232	0.235	0.231	0.232

Table 3 Minimal index γ for $d = 0.2$, $\tau_0 = 0.8$

Method	$\delta = 0.7$	$\delta = 1$	$\delta = 2$	$\delta = 5$	$\delta = 20$
[28]	11.98	0.83	0.38	0.35	1.01
[29]	8.54	0.59	0.27	0.25	0.70
[30]	8.487	0.5821	0.2607	0.2445	0.4338
[31]	0.3613	0.2695	0.2481	0.2442	0.3449
TH. 1	0.237	0.240	0.241	0.238	0.242

Table 4 Minimal index γ for $d = 0.2$, $\tau_0 = 1$

Method	$\delta = 0.7$	$\delta = 1$	$\delta = 2$	$\delta = 5$	$\delta = 20$
[28]	-	2.22	0.41	0.37	-
[29]	-	1.57	0.29	0.26	-
[30]	-	1.564 4	0.282 0	0.253 3	-
[31]	0.667 5	0.310 5	0.258 5	0.253 1	0.448 5
TH. 1	0.224	0.247	0.250	0.245	0.247

On the other hand, the maximal index τ_0 for the given different γ and δ can be obtained. The simulation results are shown in Table 5. As such, it is established that our method generates less conservative results than the presented ones in [28–31].

Table 5 Maximal τ_0 for $d = 0.2$ by TH.1

Method	$\delta = 0.7$	$\delta = 1$	$\delta = 2$	$\delta = 5$
$\gamma = 0.3$	1.2	1.4	1.4	1.2
$\gamma = 0.4$	1.5	1.7	1.7	1.6
$\gamma = 0.5$	1.7	1.9	1.9	1.7

6 Conclusions

This paper deals with the problem of H_∞ filter design for nonlinear systems through T-S fuzzy models. An improved integral inequality method without ignoring any integral term is applied to convert the integral inequalities by constructing a new Lyapunov functional and using free-weighting matrix approach, thus the results turn out to be less conservative. In the mean time, a numerical example is used to illustrate the design procedure and the effectiveness of the proposed method. Further, the approach of H_∞ filter design in this paper can be extended to fuzzy time-delay systems with norm-bounded uncertainties and linear fractional uncertainties.

References

- [1] R. E. Kalman. A new approach to linear filtering and prediction problems. *Journal of Fluids Engineering*, vol. 82, no. 1, pp. 35–45, 1960.
- [2] A. Elsayed, M. J. Grimble. A new approach to the H_∞ design of optimal digital linear filters. *IMA Journal of Mathematical Control and Information*, vol. 6, no. 2, pp. 233–251, 1989.
- [3] L. H. Xie, C. E. de Souza, M. Y. Fu. H_∞ estimation for discrete-time linear uncertain systems. *International Journal of Robust and Nonlinear Control*, vol. 1, no. 2, pp. 111–123, 1991.
- [4] M. Basin, E. Sanchez, R. Martinez-Zuniga. Optimal linear filtering for systems with multiple state and observation delays. *International Journal of Innovative Computing, Information and Control*, vol. 3, no. 5, pp. 1309–1320, 2007.
- [5] H. J. Gao, C. H. Wang. A delay-dependent approach to robust H_∞ filtering for uncertain discrete-time state-delayed systems. *IEEE Transactions on Signal Processing*, vol. 52, no. 6, pp. 1631–1640, 2004.
- [6] H. J. Gao, J. Lam, L. H. Xie, C. H. Wang. New approach to mixed H_2/H_∞ filtering for polytopic discrete-time systems. *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 3183–3192, 2005.
- [7] X. M. Zhang, Q. L. Han. Robust H_∞ filtering for a class of uncertain linear systems with time-varying delay. *Automatica*, vol. 44, no. 1, pp. 157–166, 2008.
- [8] J. Sun, J. Chen, G. P. Liu, D. Rees. Delay-dependent robust H_∞ filter design for uncertain linear systems with time-varying delay. *Circuits, Systems & Signal Processing*, vol. 28, no. 5, pp. 763–779, 2009.
- [9] G. P. Liu, Y. Q. Xia, D. Rees, W. S. Hu. Design and stability criteria of networked predictive control systems with random network delay in the feedback channel. *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol. 37, no. 2, pp. 173–184, 2007.
- [10] D. Yue, Q. L. Han, J. Lam. Network-based robust H_∞ control of systems with uncertainty. *Automatica*, vol. 41, no. 6, pp. 999–1007, 2005.
- [11] C. Gong, B. K. Su. Robust $L_2 - L_\infty$ filtering of convex polyhedral uncertain time-delay fuzzy systems. *International Journal of Innovative Computing, Information and Control*, vol. 4, no. 4, pp. 793–802, 2008.
- [12] C. L. Zhang, J. M. Li. Adaptive iterative learning control for nonlinear time-delay systems with periodic disturbances using FSE-neural network. *International Journal of Automation and Computing*, vol. 8, no. 4, pp. 403–410, 2011.
- [13] D. Zhang, Q. G. Wang, L. Yu, Q. K. Shao. H_∞ filtering for networked systems with multiple time-varying transmissions and random packet dropouts. *IEEE Transactions on Industrial Informatics*, vol. 9, no. 3, pp. 1705–1716, 2013.
- [14] M. S. Mahmoud. Resilient L_2-L_∞ filtering of polytopic systems with state delays. *IET Control Theory & Applications*, vol. 1, no. 1, pp. 141–154, 2007.
- [15] Z. D. Wang, B. Huang, H. Unbehauen. Robust H_∞ observer design of linear time-delay systems with parametric uncertainty. *Systems & Control Letters*, vol. 42, no. 4, pp. 303–312, 2001.
- [16] Z. D. Wang, D. P. Goodall, K. J. Burnham. On designing observers for time-delay systems with non-linear disturbances. *International Journal of Control*, vol. 75, no. 11, pp. 803–811, 2002.
- [17] S. Y. Xu, T. W. Chen. An LMI approach to the H_∞ filter design for uncertain systems with distributed delays. *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 51, no. 4, pp. 195–201, 2004.
- [18] S. Y. Xu, J. Lam, T. W. Chen, Y. Zou. A delay-dependent approach to robust H_∞ filtering for uncertain distributed delay systems. *IEEE Transactions on Signal Processing*, vol. 53, no. 10, pp. 3764–3772, 2005.
- [19] A. Pila, U. Shaked, C. de Souza. H_∞ filtering for continuous-time linear systems with delay. *IEEE Transactions on Automatic Control*, vol. 44, no. 7, pp. 1412–1417, 1999.
- [20] C. E. de Souza, R. M. Palhares, P. L. D. Peres. Robust H_∞ filter design for uncertain linear systems with multiple time-varying state delays. *IEEE Transactions on Signal Processing*, vol. 49, no. 3, pp. 569–576, 2001.

- [21] X. H. Ge, Q. L. Han, X. F. Jiang. Sampled-data H_∞ filtering of Takagi-Sugeno fuzzy systems with interval time-varying delays. *Journal of the Franklin Institute*, vol. 351, no. 5, pp. 2515–2542, 2014.
- [22] J. H. Zhang, Y. Q. Xiao, R. Tao. New results on H_∞ filtering for fuzzy time-delay systems. *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 1, pp. 128–137, 2009.
- [23] W. G. Yu, Z. Gu, S. L. Hu. H_∞ filtering for time-delay systems with Markovian jumping parameters: Delay partitioning approach. *Journal of the Chinese Institute of Engineers*, vol. 33, no. 3, pp. 357–365, 2010.
- [24] P. Wang, W. W. Sun. Adaptive H_∞ control for nonlinear Hamiltonian systems with time delay and parametric uncertainties. *International Journal of Automation and Computing*, vol. 11, no. 4, pp. 368–376, 2014.
- [25] X. M. Zhang, Q. L. Han. Novel delay-derivative-dependent stability criteria using new bounding techniques. *International Journal of Robust and Nonlinear Control*, vol. 23, no. 13, pp. 1419–1432, 2013.
- [26] X. M. Zhang, Q. L. Han. New stability criterion using a matrix-based quadratic convex approach and some novel integral inequalities. *IET Control Theory & Applications*, vol. 8, no. 12, pp. 1054–1061, 2014.
- [27] T. Takagi, M. Sugeno. Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.
- [28] C. Lin, G. Q. Wang, T. H. Lee, B. Chen. H_∞ filter design for nonlinear systems with time-delay through T-S fuzzy model approach. *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 3, pp. 739–746, 2008.
- [29] Y. K. Su, B. Chen, C. Lin, H. G. Zhang. A new fuzzy H_∞ filter design for nonlinear continuous-time dynamic systems with time-varying delays. *Fuzzy Sets and Systems*, vol. 160, no. 24, pp. 3539–3549, 2009.
- [30] J. H. Zhang, Y. Q. Xiao, R. Tao. New results on H_∞ filtering for fuzzy time-delay systems. *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 1, pp. 128–137, 2009.
- [31] S. J. Huang, X. Q. He, N. N. Zhang. New results on H_∞ filter design for nonlinear systems with time delay via T-S fuzzy models. *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 1, pp. 193–199, 2011.
- [32] S. J. Huang, G. H. Yang. Fault tolerant controller design for T-S fuzzy systems with time-varying delay and actuator faults: A k -step fault-estimation approach. *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 6, pp. 1526–1540, 2014.
- [33] X. F. Jiang, Q. L. Han. On designing fuzzy controllers for a class of nonlinear networked control systems. *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 4, pp. 1050–1060, 2008.
- [34] X. F. Jiang, Q. L. Han. New stability criteria for linear systems with interval time-varying delay. *Automatica*, vol. 44, no. 10, pp. 2680–2685, 2008.
- [35] X. F. Jiang, Q. L. Han, S. R. Liu, A. K. Xue. A new H_∞ stabilization criterion for networked control systems. *IEEE Transactions on Automatic Control*, vol. 53, no. 4, pp. 1025–1032, 2008.
- [36] X. F. Jiang, Q. L. Han. Delay-dependent H_∞ filter design for linear systems with interval time-varying delay. *IET Control Theory & Applications*, vol. 1, no. 4, pp. 1131–1140, 2007.

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