On the Current Error Based Sampled-data Iterative Learning Control with Reduced Memory Capacity

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Abstract: The design of iterative learning controller (ILC) requires to store the system input, output or control parameters of previous trials for generating the input of the current trial. In order to apply the iterative learning controller for a real application and reduce the memory size for implementation, a current error based sampled-data proportional-derivative (PD) type iterative learning controller is proposed for control systems with initial resetting error, input disturbance and output measurement noise in this paper. The proposed iterative learning controller is simple and effective. The first contribution in this paper is to prove the learning error convergence via a rigorous technical analysis. It is shown that the learning error will converge to a residual set if a forgetting factor is introduced in the controller. All the theoretical results are also shown by computer simulations. The second main contribution is to realize the iterative learning controller by a digital circuit using a field programmable gate array (FPGA) chip applied to repetitive position tracking control of direct current (DC) motors. The feasibility and effectiveness of the proposed current error based sampled-data iterative learning controller are demonstrated by the experiment results. Finally, the relationship between learning performance and design parameters are also discussed extensively.

Keywords: Iterative learning control, current error, sampled-data system, memory capacity, field programmable gate array (FPGA) chip.

1 Introduction

For repetitive tracking control of industrial applications, iterative learning control (ILC)^[1,2] has been known as one of the most effective strategies. To begin with, the proportional-integral-derivative (PID) type iterative learning controllers^[3–6] were developed with the control input updated using the data of input and error obtained in the previous iteration. The learning performance is then improved by this simple self-tuning process without using the knowledge of accurate system model. Later, in order to extend the ILC concept to other theoretical researches or practical applications, several ILC frameworks, such as robust ILC^[7,8], adaptive ILC^[9–11], adaptive fuzzy ILC^[12] and optimal ILC^[13,14] were developed in the past two decades. Most of the ILC algorithms were presented with elegant theoretical analysis. However, some of them required more information of plant's model for the controller design and most of them became more and more complicated. In general, it is hard to implement these ILC algorithms using digital control based approaches for real industrial applications.

Actually, PID-type ILC was widely used in real industrial

applications because it is simple, effective and easy for realization using any digital controller. In the beginning, most of the PID-type ILCs^[3–6] were designed and analyzed in continuous-time domain. Since the ILC is required to store the related control data in memory for generating the control input at the next trial, it is more practical to design and analyze the iterative learning controller in discrete-time domain^[15–18] with the controlled plants also being modeled as discrete-time systems. As most of the controlled plants are continuous-time systems, a more interesting issue called sampled-data iterative learning control was studied^[19–21]. For sampled-data ILC, the plant is modeled as a continuous-time system and the controller is implemented by a discrete-time algorithm. Although the technical analysis is similar between pure discrete-time ILC and sampled-data ILC, it is in general more difficult in dealing with the derivation of error equation for convergence analysis of sampled-data ILC.

For realization of ILC algorithms, another interesting issue is the consideration of memory size. For example, it is required to store the control input and tracking error of the previous trial in the memory when implementing a typical PID-type ILC. Suppose the number of data for the desired trajectory is N . Then, we need to store $3N$ data in the controller, i.e., the data of the desired trajectory, control input and tracking error. As the digital integrated circuit (IC) has become a main tool in the implementation of controller nowadays, more memory needed implies more cost of the controller. One of the possible ways to reduce the size of memory is to choose some specified tracking data that are

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desired points at certain time instants^[22]. Another way is to utilize the technique of terminal ILC where only the terminal point at the end of every trial is used for ILC algorithm^[23, 24]. However, terminal ILC is used for some special processes where only terminal output can be measured. How to reduce the memory size for the case to track the entire desired trajectory is still an interesting issue for ILC design.

To meet the requirements that the iterative learning controller can be implemented by a digital circuit and memory size can be as small as possible, a current error based sampled-data ILC is presented in this paper. Actually, using current error for the design of PID-type ILC is a well known approach in the related area. For example, the ILC which combines feedforward and feedback design can be found in [25–28]. Because the learning error is likely to grow quite significantly before it converges to zero in the process of learning and the rate of convergence is often slow in the pure feedforward iterative learning control, the current error based feedback design is often introduced to improve the learning performance. It is noted that the storage of previous error is still required in this feedback-feedforward configuration. Some other PID-type ILC designs using only current error can be found in [29, 30]. However, the PID-type ILC proposed in [29] is designed and analyzed in continuous-time domain. In [30], although the PID type ILC is designed in discrete-time domain, a more complicated controller is required to predict the current error at the next time instant. And this introduces some prediction error.

Based on our previous work^[31], we investigate the design, analysis and implementation of a simple and practical current error based sampled-data ILC in this paper. The proposed ILC algorithm is easily implemented and will use less memory with only $2N$ data to be stored if the number of data for desired output trajectory is N . The current error based learning mechanism is a simple PD-type algorithm. A forgetting factor is introduced to guarantee the convergence of the learning error. The first main contribution of this paper is to provide a new theoretical analysis for stability and convergence. We show that the tracking error will converge to a residual set even the system has initial resetting error, input disturbance and output measurement noise. The second main contribution is to implement the ILC algorithm by a digital circuit and apply it to repetitive position tracking control of direct current (DC) motors. The digital circuit is realized by using very high speed hardware description language (VHDL) code and then downloaded into a Xilinx field programmable gate array (FPGA) chip. According to the experimental results, we prove that the theoretical derivation is correct and the digital circuit is feasible.

This paper is organized as follows. In Section 2, the problem formulation is given and the current error based sampled-data ILC is proposed. A detailed and rigorous technical proof is shown in Section 3 to guarantee the stability and convergence. A simulation example is provided in Section 4 to demonstrate the correctness of the given

theoretical results. In Section 5, the ILC algorithm is implemented by a digital circuit and an experiment is executed for repetitive position tracking control of DC motors. Finally, conclusions are made in Section 6.

2 Problem formulation and ILC design

In this paper, we consider a class of linear systems which can perform a given task repetitively over a finite time interval $[0, T]$ as

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + \omega_i(t) \\ y_i(t) &= Cx_i(t) + \xi_i(t)\end{aligned}\quad (1)$$

where $x_i(t) \in \mathbf{R}^{n \times 1}$, $y_i(t) \in \mathbf{R}^{m \times 1}$, $u_i(t) \in \mathbf{R}^{m \times 1}$, $\omega_i(t) \in \mathbf{R}^{n \times 1}$ and $\xi_i(t) \in \mathbf{R}^{m \times 1}$ are state vector, output vector, input vector, input disturbance vector and output measurement noise vector, respectively. Here, $A \in \mathbf{R}^{n \times n}$ is the state matrix, $B \in \mathbf{R}^{n \times m}$ is the input gain matrix, $C \in \mathbf{R}^{m \times n}$ is the output gain matrix, and i denotes the iteration number and $t \in [0, T]$. Let ΔT be the sampling period of the digital control system with the sampling instant $n = 0, 1, 2, \dots, N, N\Delta T = T$, where N is a positive integer. The control input $u_i(t)$ is assumed to satisfy $u_i(t) = u_i(n\Delta T)$ for $n\Delta T \leq t < (n+1)\Delta T$. Now, given a realizable desired output $y_d(t) \in \mathbf{R}^{m \times 1}$ and an initial state x_0 which satisfies the following equation without input disturbance and output measurement noise,

$$\begin{aligned}\dot{x}_d(t) &= Ax_d(t) + Bu_d(t) \\ y_d(t) &= Cx_d(t)\end{aligned}\quad (2)$$

where $u_d(t)$ is an unknown desired input satisfying $u_d(t) = u_d(n\Delta T)$ and $n\Delta T \leq t < (n+1)\Delta T$, the control objective is to design a sampled-data ILC $u_i(n\Delta T)$ such that the output error at each sampling instant satisfies

$$\lim_{i \rightarrow \infty} \|y_d(n\Delta T) - y_i(n\Delta T)\| < \sigma, \quad \forall n \in \{0, 1, 2, \dots, N\}$$

where σ denotes a certain error tolerance bound. The followings are the required assumptions for a realizable controller to achieve the control objective.

Assumption 1. The initial state $x_i(0)$, input disturbance $\omega_i(t)$ and output measurement noise $\xi_i(t)$ satisfy $\|x_d(0) - x_i(0)\| \leq \varepsilon_1$, $\|\omega_i(t)\| \leq \varepsilon_2$ and $\|\xi_i(t)\| \leq \varepsilon_3$, for $i = 0, 1, 2, \dots, \infty$, $t \in [0, T]$, where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are positive constants.

Assumption 2. The desired output $y_d(n\Delta T)$ and desired input $u_d(n\Delta T)$ are bounded with the bound d defined as

$$d = \sup_{n \in \{0, 1, 2, \dots, N\}} \|u_d(n\Delta T)\|.$$

A current error based sampled-data PD-type iterative learning controller is now proposed for system (1) as

$$\begin{aligned}u_i(n\Delta T) &= (1 - \alpha)u_{i-1}(n\Delta T) + \alpha u_0(n\Delta T) + \\ &K_P e_i(n\Delta T) + K_D(e_i(n\Delta T) - e_i((n-1)\Delta T))\end{aligned}\quad (3)$$

where $u_i(n\Delta T)$ is the iterative learning control input at sampling instant $n\Delta T$ of the i -th iteration (current iteration), $u_{i-1}(n\Delta T)$ is the iterative learning control input at sampling instant $n\Delta T$ of the $(i-1)$ -th iteration (previous iteration), $e_i(n\Delta T)$ is the tracking error at sampling instant $n\Delta T$ of the i -th iteration, i.e., $e_i(n\Delta T) = y_d(n\Delta T) - y_i(n\Delta T)$, $n = 0, 1, 2, \dots, N$. In addition, $0 < \alpha < 1$ is the forgetting factor, K_P and K_D are the learning gains of the PD-type learning mechanism and $u_0(n\Delta T)$ is the initial bounded learning input. In general, we will set $u_0(n\Delta T)$ as close as to $u_d(n\Delta T)$ if $u_d(n\Delta T)$ can be predicted. Otherwise, we simply set $u_0(n\Delta T) = 0$ if $u_d(n\Delta T)$ is totally unknown.

3 Analysis of stability and convergence

Firstly, we define the state error and input error as $\delta x_i(t) \equiv x_d(t) - x_i(t)$ and $\delta u_i(t) \equiv u_d(t) - u_i(t)$. Then, we can derive the following result for $t \in [(n-1)\Delta T, n\Delta T]$

$$\begin{aligned} x_d(t) - x_i(t) = & x_d((n-1)\Delta T) - x_i((n-1)\Delta T) + \\ & \int_{(n-1)\Delta T}^t (\dot{x}_d(\tau) - \dot{x}_i(\tau)) d\tau = \\ & \delta x_i((n-1)\Delta T) + \\ & \int_{(n-1)\Delta T}^t (A\delta x_i(\tau) + B\delta u_i(\tau) - \omega_i(\tau)) d\tau. \end{aligned}$$

Since $\delta u_i(t) = \delta u_i((n-1)\Delta T)$, $t \in [(n-1)\Delta T, n\Delta T]$, we have

$$\begin{aligned} \|\delta x_i(t)\| \leq & \|\delta x_i((n-1)\Delta T)\| + \\ & \int_{(n-1)\Delta T}^t (a \|\delta x_i(\tau)\| + b \|\delta u_i(\tau)\| + \varepsilon_2) d\tau \leq \\ & \|\delta x_i((n-1)\Delta T)\| + a \int_{(n-1)\Delta T}^t \|\delta x_i(\tau)\| d\tau + \\ & b\Delta T \|\delta u_i((n-1)\Delta T)\| + \varepsilon_2\Delta T \end{aligned} \quad (4)$$

where $\|A\| = a$ and $\|B\| = b$. By using Bellman-Gronwall Lemma, (4) can be rewritten as

$$\begin{aligned} \|\delta x_i(t)\| \leq & e^{at} (\|\delta x_i((n-1)\Delta T)\| + \\ & b\Delta T \|\delta u_i((n-1)\Delta T)\| + \varepsilon_2\Delta T) \leq \\ & a_1 \|\delta x_i((n-1)\Delta T)\| + \\ & a_2\Delta T \|\delta u_i((n-1)\Delta T)\| + a_1\varepsilon_2\Delta T \end{aligned} \quad (5)$$

where $a_1 = e^{aT}$. Finally, let the time $t = n\Delta T$, we have

$$\begin{aligned} \|\delta x_i(n\Delta T)\| \leq & a_1 \|\delta x_i((n-1)\Delta T)\| + a_2\Delta T \|\delta u_i((n-1)\Delta T)\| + \\ & a_1\varepsilon_2\Delta T. \end{aligned} \quad (6)$$

Solving the difference equation (6) iteratively, we can get

$$\begin{aligned} \|\delta x_i(n\Delta T)\| \leq & a_1^n \|\delta x_i(0)\| + \sum_{j=0}^{n-1} a_1^{n-1-j} a_2\Delta T \|\delta u_i(j\Delta T)\| + \\ & \sum_{j=0}^{n-1} a_1^{n-j} \varepsilon_2\Delta T. \end{aligned} \quad (7)$$

Now, we state the main result in the following theorem.

Theorem 1. Let the system (1) satisfy Assumption 1 and the desired input as well as desired output satisfy Assumption 2. If we apply the current error based sampled-data iterative learning controller (3) to system (1), then we can guarantee that

$$\lim_{i \rightarrow \infty} \|y_d(n\Delta T) - y_i(n\Delta T)\| \leq \sigma, \quad \forall n \in \{0, 1, \dots, N\}$$

for some positive constant σ depending on $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and α .

Proof. To simplify the analysis, we use n to replace $n\Delta T$ in the following derivation. First of all, we derive the equation for output error as

$$\begin{aligned} e_i(n) = & y_d(n) - y_i(n) - \xi_i(n) = \\ & y_d(n) - y_i(n-1) + \int_{n-1}^n (\dot{y}_d(\tau) - \dot{y}_i(\tau)) d\tau - \xi_i(n) = \\ & e_i(n-1) + C \int_{n-1}^n (A(x_d(\tau) - x_i(\tau)) + \\ & B(u_d(\tau) - u_i(\tau)) - \omega_i(\tau)) d\tau - \xi_i(n) = \\ & e_i(n-1) + C \int_{n-1}^n (A\delta x_i(\tau) - \omega_i(\tau)) d\tau + \\ & CB\Delta T \delta u_i(n-1) - \xi_i(n). \end{aligned} \quad (8)$$

The input error $\delta u_i(n) = u_d(n) - u_i(n)$ is derived by using (8) as

$$\begin{aligned} \delta u_i(n) = & u_d(n) - (1 - \alpha)u_{i-1}(n) - \alpha u_0(n) - \\ & K_P e_i(n) - K_D (e_i(n) - e_i(n-1)) = \\ & (1 - \alpha)\delta u_{i-1}(n) - K_P e_i(n) - \\ & K_D (e_i(n) - e_i(n-1)) + \alpha\delta u_0(n) = \\ & (1 - \alpha)\delta u_{i-1}(n) - K_P C \delta x_i(n) - K_P C \xi_i(n) - \\ & K_D C \int_{n-1}^n (A\delta x_i(\tau) - \omega_i(\tau)) d\tau - \\ & K_D CB\Delta T \delta u_i(n-1) + K_D \xi_i(n) + \alpha\delta u_0(n). \end{aligned} \quad (9)$$

Let $|K_P| = k_P$ and $|K_D| = k_d$. Taking norms on (9) and

using (5), we have

$$\begin{aligned} \|\delta u_i(n)\| \leq & (1-\alpha) \|\delta u_{i-1}(n)\| + k_p \|\delta x_i(n)\| + k_p c \varepsilon_3 + \\ & k_d c a \int_{n-1}^n \|\delta x_i(\tau)\| d\tau + k_d c \varepsilon_2 \Delta T + \\ & k_d c b \Delta T \|\delta u_i(n-1)\| + k_d \varepsilon_3 + \alpha \|\delta u_0(n)\| \leq \\ & (1-\alpha) \|\delta u_{i-1}(n)\| + k_p \|\delta x_i(n)\| + k_p c \varepsilon_3 + \\ & k_d c a \Delta T (a_1 \|\delta x_i(n-1)\| + a_2 \Delta T \|\delta u_i(n-1)\| + \\ & a_1 \varepsilon_2 \Delta T) + k_d c \varepsilon_2 \Delta T + k_d c b \Delta T \|\delta u_i(n-1)\| + \\ & k_d \varepsilon_3 + \alpha \|\delta u_0(n)\| \leq (1-\alpha) \|\delta u_{i-1}(n)\| + \\ & k_1 \|\delta x_i(n)\| + k_2 \|\delta x_i(n-1)\| + k_3 \|\delta u_i(n-1)\| + k_4 \end{aligned} \quad (10)$$

where k_1, k_2, k_3, k_4 are suitably defined positive constants and k_4 is a constant depending on $\Delta T, \varepsilon_2, \varepsilon_3$ and α . By using the result of (7), (10) can be rewritten as

$$\|\delta u_i(n)\| \leq (1-\alpha) \|\delta u_{i-1}(n)\| + \beta \sum_{j=0}^{n-1} \|\delta u_i(j)\| + \gamma \quad (11)$$

where β and γ are some suitably defined positive constants and γ is a constant depending on $\Delta T, \varepsilon_1, \varepsilon_2, \varepsilon_3$ and α . It is noted that $\gamma = 0$ if $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \alpha = 0$. Now, we expand (11) for $n = 0, 1, 2, \dots, N$ as

$$\begin{aligned} \|\delta u_i(0)\| & \leq (1-\alpha) \|\delta u_{i-1}(0)\| + \gamma, \\ \|\delta u_i(1)\| - \beta \|\delta u_i(0)\| & \leq (1-\alpha) \|\delta u_{i-1}(1)\| + \gamma, \\ \|\delta u_i(2)\| - \beta \|\delta u_i(1)\| - \beta \|\delta u_i(0)\| & \leq \\ (1-\alpha) \|\delta u_{i-1}(2)\| + \gamma, \\ & \dots \\ \|\delta u_i(N)\| - \beta \|\delta u_i(N-1)\| - \dots - \beta \|\delta u_i(0)\| & \leq \\ (1-\alpha) \|\delta u_{i-1}(N)\| + \gamma. \end{aligned}$$

The above inequalities can be rearranged into the matrix form as

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ -\beta & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ -\beta & \dots & -\beta & 1 \end{bmatrix} \begin{bmatrix} \|\delta u_i(0)\| \\ \|\delta u_i(1)\| \\ \vdots \\ \|\delta u_i(N)\| \end{bmatrix} \leq (1-\alpha) \begin{bmatrix} \|\delta u_{i-1}^{(0)}\| \\ \|\delta u_{i-1}^{(1)}\| \\ \vdots \\ \|\delta u_{i-1}^{(N)}\| \end{bmatrix} + \gamma \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}. \quad (12)$$

Define $\delta U_i = [\|\delta u_i(0)\|, \dots, \|\delta u_i(N)\|]^T$, $I_1 = [1, \dots, 1]^T$

and note that

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ -\beta & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ -\beta & \dots & -\beta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \beta & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \beta(\beta+1)^{N-1} & \dots & \beta & 1 \end{bmatrix} \equiv \Lambda.$$

We can now rewrite (12) in the matrix form as

$$\delta U_i \leq (1-\alpha) \Lambda \delta U_{i-1} + \gamma \Lambda I_1. \quad (13)$$

The solution of (13) can be easily derived as

$$\delta U_i \leq ((1-\alpha)\Lambda)^i \delta U_0 + \sum_{j=0}^{i-1} \gamma ((1-\alpha)\Lambda)^j \Lambda I_1. \quad (14)$$

Since matrix $(1-\alpha)\Lambda$ is a triangular matrix and all the eigenvalues are less than 1, we conclude that

$$\lim_{i \rightarrow \infty} \delta U_i \leq \lim_{i \rightarrow \infty} \sum_{j=0}^{i-1} \gamma ((1-\alpha)\Lambda)^j \Lambda I_1. \quad (15)$$

This implies that the input error $\delta u_i(n)$ is bounded, $\forall i \geq 1$ and $\forall n \in \{0, 1, 2, \dots, N\}$. Furthermore, the norm of input error will converge to a residual set whose level of magnitude depends on the uncertainty bounds $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and the value of forgetting factor α .

Based on the result of (15), we have

$$\lim_{i \rightarrow \infty} \|\delta u_i(n)\| = \lim_{i \rightarrow \infty} \|u_d(n) - u_i(n)\| \leq \gamma h$$

for some $h > 0$. Furthermore, according to (7), we can find $h_1, h_2 > 0$ such that

$$\lim_{i \rightarrow \infty} \|\delta x_i(n)\| \leq \gamma h_1 + h_2(\varepsilon_1 + \varepsilon_2).$$

This implies that

$$\begin{aligned} \lim_{i \rightarrow \infty} \|y_d(n) - y_i(n)\| & \leq \\ \lim_{i \rightarrow \infty} \|Cx_d(n) - Cx_i(n) - \xi_i(n)\| & \leq \\ c\gamma h_1 + ch_2(\varepsilon_1 + \varepsilon_2) + \varepsilon_3 & \equiv \sigma. \end{aligned} \quad (16)$$

□

Remark 1. It is noted that the converged tracking error will only depend on α only if $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$. In this case, σ in (16) will be smaller if the value of α is smaller from theoretical point of view. We will demonstrate this observation via some experimental results.

Remark 2. In Theorem 1, the convergence is ensured by the existence of the forgetting factor α . However, the choice of the PD gains is also important for this ILC algorithm. Actually, the PD learning mechanism plays a role of feed-back controller if iterative learning process is not executed.

The choice of PD learning gains should stabilize the closed loop system in order to compensate for the output error to a small region. Then, the iterative learning controller can drive the output error to a more reasonable value around zero.

4 Simulation example

In this section, we use the following second order system for numerical simulation

$$\begin{aligned} \dot{x}_i(t) &= \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t) + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} \text{randn}(t) \\ y_i(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x_i(t) + 0.01 \text{randn}(t) \end{aligned}$$

where $\text{randn}(t)$ is a generator of random number with normal distribution, i.e., mean = 0 and variance = 1. The sampling period and desired trajectory are chosen as 0.01 s and $y_d(t) = 5t(2-t)$, $t \in [0, 1]$. We assume that the initial output at each iteration satisfies

$$y_i(0) = x_{i,2}(0) + 0.01 \text{randn}(0) \equiv 0.1 + 0.01 \text{randn}(0).$$

This implies that there will exist an initial output error at the beginning of each trial. The current error based sampled-data PD-type ILC is given as

$$\begin{aligned} u_i(n\Delta T) &= (1 - \alpha)u_{i-1}(n\Delta T) + \\ &K_P e_i(n\Delta T) + K_D(e_i(n\Delta T) - e_i((n-1)\Delta T)). \end{aligned} \quad (17)$$

It is noted that we set $u_0(n\Delta T) = 0$ since we assume that we don't have any prior information about the ideal desired input $u_d(n\Delta T)$. The current error based PD-type ILC is designed with learning gains as $K_P = 5$, $K_D = 5$ and forgetting factor $\alpha = 0.01$. In order to make some comparisons between the traditional previous error based sampled-data PD-type ILC, we use the following algorithm for another simulation,

$$\begin{aligned} u_i(n\Delta T) &= u_{i-1}(n\Delta T) + K_{P1}e_{i-1}(n\Delta T) + \\ &K_{D1}(e_{i-1}((n+1)\Delta T) - e_{i-1}(n\Delta T)) \end{aligned} \quad (18)$$

with $K_{P1} = 2$ and $K_{D1} = 10$. Furthermore, a sampled-data PD-type ILC using both previous error and current error design is also given as

$$\begin{aligned} u_i(n\Delta T) &= \\ &u_{i-1}(n\Delta T) + K_{P1}e_{i-1}(n\Delta T) + K_{D1}(e_{i-1}((n+1)\Delta T) - \\ &e_{i-1}(n\Delta T)) + K_{P2}e_i(n\Delta T) + K_{D2}(e_i(n\Delta T) - \\ &e_i((n-1)\Delta T)) \end{aligned} \quad (19)$$

with $K_{P1} = 2$, $K_{D1} = 10$, $K_{P2} = 5$ and $K_{D2} = 5$. It is noted that the previous error based PD gains of (19) are the same as those in (18), and the current error based PD gains of (19) are the same as those in (17). The maximum absolute output error is defined as

$$e_{i,\max} = \max_{n \in \{1, 2, \dots, 100\}} |y_d(n\Delta T) - y_i(n\Delta T)|.$$

In Fig. 1, we show the maximum absolute output error versus the iteration number. The solid line, dashed line and dotted line are the simulation results using ILC algorithms of (17), (18) and (19) respectively. It is clear that the previous error based sampled-data PD-type ILC can ensure the learning error convergence (dashed line) if the PD learning gains are suitably chosen. The learning performance can be further improved (dotted line) if the current error based PD-type ILC is added. This implies that the learning performance is in general better for ILC algorithm using both previous error and current error design. However, it is interesting to note that the ILC algorithm using only current error design (17) can almost have the same performance (solid line) as that obtained by (19).

Fig. 1 Maximum absolute output error $e_{i,\max}$ versus the iteration number i , solid line for ILC (17), dashed line ILC (18) and dotted line for ILC (19)

In order to study the relationship between forgetting factor and residual learning error as proved in Theorem 1, we use the current error based sampled-data PD-type ILC (17) for simulation with three different values of α . Fig. 2 shows the behavior of error convergence for $\alpha = 0.1$, 0.01 and 0.001, respectively. In general, the residual learning error will be small if α is small. But there is no significant difference if α is small enough.

Fig. 2 Maximum absolute output error $e_{i,\max}$ versus the iteration number i , solid line for $\alpha = 0.001$, dotted line for $\alpha = 0.01$, dashed line for $\alpha = 0.1$

5 Application to position tracking control of DC motors

In order to demonstrate the feasibility and effectiveness of the proposed current error based sampled-data iterative learning controller, we apply this controller to execute a repetitive position tracking control task of DC motor. The iterative learning controller is implemented by a digital circuit using VHDL code. The circuit code is compiled by Xilinx ISE and then downloaded into a Xilinx FPGA chip. Fig. 3 shows the hardware setup for the experiment in this paper. The experiment system includes a DC motor, an FPGA control board, a digital input output (DIO) interface card and a personal computer (PC). The experimental data is transmitted to PC by the FPGA control board through the DIO interface card. Then a Matlab program in PC will store the data and show the output position trajectory.

Fig. 3 The hardware setup for experiment

In this experiment, the desired position trajectory is given as that in Fig. 4. The desired position trajectory is designed to rotate the motor clockwise from an initial position of 0° to a final position of 90° for 2 s and then rotate the motor anticlockwise to the initial position for another 2 s. In other word, the period of one iteration is 4 s. The

sampling period ΔT is set to be 0.02 s so that there are 200 sampling data in one iteration. Due to the current error based design, only 400 registers instead of 600 registers are required for the memory to store the required data.

Fig. 4 Desired position trajectory for one iteration

The block diagram of the experiment system is shown in Fig. 5. The current error based sampled-data PD-type ILC (17) is realized with learning gains of $K_P = 0.5$ and $K_D = 0.5$. The proposed iterative learning control circuit includes a memory (register) which stores the input signal of the previous iteration and a current error learning mechanism circuit which generates the equation of the algorithm $K_P e_i(n\Delta T) + K_D(e_i(n\Delta T) - e_i((n-1)\Delta T))$. Another memory is utilized to store the initial input $u_0(n\Delta T)$ if required. However, it is omitted in this experiment. In Fig. 5, main control circuit is designed to control the circuit timing, to generate the pulse width modulation (PWM) signal, to decode the motor position signal Y and to serve as the I/O interface between the control circuit and DC motor or PC.

In Fig. 6 (a), we show the experiment of motor position trajectory at the first iteration using

$$u_1(n\Delta T) = K_P e_1(n\Delta T) + K_D(e_1(n\Delta T) - e_1((n-1)\Delta T)).$$

Fig. 5 Block diagram of the experiment system

In other word, only the PD-type feedback controller is used for the initial trial. Even though we assume that the dynamic model of the DC motor is unknown before controller design, we can still easily find suitable PD gains of $K_P = 0.5$ and $K_D = 0.5$ such that the closed loop is stable and the output tracking performance is acceptable. However, there exists an apparent steady state error which is about 1.6199° . Next, we show the experiment of motor position trajectories at the 5th iteration and 10th iteration respectively in Figs. 6(b) and (c) with $\alpha = 0.1$. It can be found from the experimental results that the tracking performance is improved under the proposed iterative learning controller since the steady state errors are reduced to be 0.54° in Fig. 6(b) and 0.0899° in Fig. 6(c), respectively. Furthermore, the root mean square (RMS) values of learning errors at the 1st, 5th and 10th iteration are 0.7156, 0.3057 and 0.2862, respectively. Based on these results, we can conclude the effectiveness and feasibility of the proposed current error based sampled-data PD-type iterative learning controller.

In addition to the results given in Fig. 6, several experiments were also conducted for the study of relationship between the value of α and the learning performance. Under the same learning gains of $K_P = 0.5$ and $K_D = 0.5$, we choose three different values of α and repeat the experiments for 10 iterations. The RMS values of the learning errors at the 10th iteration for the three experiments are 0.2862 ($\alpha = 0.1$), 0.4017 ($\alpha = 0.2$) and 0.5698 ($\alpha = 0.4$), respectively. In general, the RMS value of learning error decreases and the rate of convergence is a little faster if α is smaller.

6 Conclusions

In this paper, we propose a current error based sampled-data iterative learning controller and give a theoretical analysis as well as a real application. A PD-type learning mechanism is designed by using current error data so that the memory size can be reduced. A new theoretical proof is given to show the convergence of the learning control system. The PD-type learning mechanism which can be treated as a feedback controller compensates for the output error to a small region so that the iterative learning controller can drive the output error further to a more reasonable value around zero. In order to demonstrate the feasibility and effectiveness, some simulation results are presented. Furthermore, the proposed iterative learning controller is also implemented by a digital circuit for experiment. We use VHDL as a design tool to realize the digital circuit and download the code into an FPGA chip as the controller for repetitive position tracking control of DC motors. According to the simulation and experimental results, we prove that the theoretical analysis is correct and the corresponding digital circuit is feasible.

A future work is to extend the proposed design to more general class of nonlinear systems.

(a) Motor position trajectory at the 1st iteration

(b) Motor position trajectory at the 5th iteration

(c) Motor position trajectory at the 10th iteration

Fig. 6 Motor position trajectory

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