A TV- l_1 Based Nonrigid Image Registration by Coupling Parametric and Non-parametric Transformation

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Abstract: To overcome the conflict between the global robustness and the local accuracy in the dense nonrigid image registration, we propose a union registration approach using a l_1 -norm based term to couple the parametric transformation and the non-parametric transformation. On one hand, we take the parametric deformation field as a constraint for the non-parametric registration, which is a strong constraint that guarantees the robustness of the non-parametric registration result. On the other hand, the non-parametric deformation field is taken as a force to improve the accuracy of the parametric registration. Then, an alternating optimization scheme is carried out to improve the accuracy of both the parametric registration and the non-parametric registration. Moreover, accounting for the effect of spatially-varying intensity distortions and the sparse gradient prior of the deformation field, we adopt the residual complexity (RC) as the similarity metric and the total variation (TV) as the regularization. Because of the TV- l_1 - l_2 composite property of the objective function in our union image registration, we resort to the split Bregman iteration for the complex problem solving. Experiments with both synthetic and real images in different domains illustrate that this approach outperforms the separately performed parametric registration or non-parametric registration.

Keywords: Nonrigid registration, free-form deformation, non-parametric transformation, total variation, residual complexity.

1 Introduction

Image registration aims to geometrically match two or more images of the same scene, taken at different times, from different viewpoints, or by different sensors, with the purpose of target localization, difference detection, and many other purposes^[1]. It is widely used in medical imaging^[2], remote sensing^[3], finger print or face recognition^[4], image compression^[5], video enhancement^[6], and etc. Existing image registration methods are either feature-based or intensity-based^[7]. Because identification and extraction of image features is often a challenging and time-consuming process in feature-based registration^[8–10], intensity-based image registration (IBIR)^[11–15] has received much attention recently.

Most existing IBIR procedures estimate the geometrical transformation or displacement field ϕ globally by optimizing a minimization problem, such as

$$\phi^* = \arg \min_{\phi \in \Phi} D(f, u(\phi)) + R(\phi) \quad (1)$$

where $D(\cdot)$ and $R(\cdot)$ are the distance metric and the regularization term, respectively, Φ is a specific transformation space. The entire set of images f (reference or fixed image) and u (floating or moving image) are involved in the optimization. The choice of transformation model is of great

importance for the registration process as it entails an important compromise between computational efficiency and richness of description. It also reflects the class of transformations that are desirable or acceptable, and therefore limits the solution to a large extent.

In the literature, there are two methods to model the transformation ϕ ^[12, 13]. The first method is parametric, which models ϕ in a parametric space Ψ . The number of parameters corresponds to the degrees of freedom of the transformation model and varies greatly, from 6 in the case of rigid transformations^[13] to tens of thousands when the nonrigid or deformable transformations^[16, 17] are considered. The second method is non-parametric, which models ϕ directly in the transformation space Φ . The non-parametric image registration, e.g., the physical model-based methods^[12, 13], the demons algorithm^[18–20], the optical flow^[21–23], has its transformation field directly optimized through the registration process. This type of registration needs an explicit deformation vector for every pixel. When dense nonrigid transformations are considered, the degrees of freedom of the deformation model are tremendous.

Due to the relatively simple computation and other salient properties, the parametric free form deformation (FFD) techniques based on B-spline basis are getting popular to deal with the nonrigid transformation problem in image registration community^[11, 24–28]. Based on the B-spline interpolation theory, the FFD registration estimates the underlying displacement field ϕ approximately rather than taking exactly the same value. In the classic FFD registration^[11], a 2D nonrigid deformation $\phi = [X \ Y]^T$ is

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parameterized using a set of 2D control points $\psi = [U \ V]^T$, such that

$$\phi = K\psi, \quad K = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix}, \quad \psi \in \Psi \quad (2)$$

where B denotes the matrix of the B-spline basis functions and Ψ is the B-spline parametric space. One main difficulty of the FFD approach is to cope with the conflict between global robustness and local accuracy^[29, 30]. Shi et al.^[30] defined the robustness as the ability to recover the deformation in the presence of noise, and the accuracy as the ability to reconstruct the highly-localized and potentially discontinuous deformation with as little error as possible. The trade-off between accuracy and robustness stems from the fact that the FFD approach uses a smooth B-spline basis to model the contribution of each control point to the deformation. To model global and smooth deformations, a coarse control point grid spacing is typically used. To allow very localized deformations, a finer control point spacing is required. However, this renders the FFD registration less robust as the model has far more degrees of freedom which must be optimized.

Compared to the parametric FFD method, the non-parametric registration has a stronger descriptive power of the transformation due to the high degrees of freedom of transformation space Φ . However, this model enrichment may be accompanied by the model's complexity which in turn, results in a challenging and computationally demanding inference. Usually, to reduce the risk of being trapped in local minima, a physical model based regularization term for ϕ is needed and a multi-level optimization strategy is used^[12, 13]. When dense nonrigid deformations are considered, the non-parametric registration has the ability to model highly-localized deformations but lacks the robustness to reconstruct such deformations in the case of noisy images.

Many modified parametric or non-parametric approaches have been proposed to overcome the conflict between robustness and accuracy in estimation of the deformation field ϕ . For the parametric FFD registration, we can find some research focused on the adaptive parameterization of the B-spline control point grid^[31, 32]. Fujiwara et al.^[33] proposed a dual-grid FFD framework for the locally rigid but globally non-rigid surface registration. More recently, Shi et al.^[29, 30] assumed that the deformation was sparse in the parametric space Ψ and then introduced a sparse regularization term for ψ on the basis of the classical FFD registration. The sparse FFD (SFFD) method reduced the conflict between global smoothness and the local detail of the transformation to some extent, but the assumption of sparsity in SFFD was limited in the scenario without densely-localized transformation. For non-parametric registration approach, in particular the optical flow algorithm, Wedel et al.^[34] employed the total variation (TV) regularization to preserve discontinuities in the flow field and applied the robust l_1 norm in the data fidelity term to eliminate outliers. More recently, sparse representation has been proposed to evalu-

ate the patch similarity between two images^[35] and to constrain the transformation^[36].

In this paper, to overcome the conflict between robustness and accuracy, we propose a union nonrigid image registration approach using a l_1 -based term $|\phi - K\psi|$ to couple the parametric transformation and the non-parametric transformation. On one hand, we take the parametric deformation field $K\psi$ as a constraint for the non-parametric registration, which is a strong constraint that guarantees the robustness of the non-parametric result. On the other hand, the non-parametric deformation field ϕ is taken as a force to improve the accuracy of the parametric registration. Then, an alternating optimization scheme is carried out for both parametric registration and non-parametric registration. Due to the combination of robustness and accuracy, our union registration outperforms the separately performed parametric registration or non-parametric registration.

Moreover, suitable similarity measure and regularization are crucial for the intensity-based registration. For similarity measure, considering that real-world images often have spatially-varying intensity distortions, which is caused by inhomogeneities from staining, illumination or attenuation^[28, 37], we adopt the residual complexity (RC)^[28] as the similarity metric. Compared to other similarity measures, image registration by minimizing the residual complexity is simple in terms of both computational complexity and implementation, and meanwhile produces accurate registration results on problems with spatially-varying intensity distortions. For the regularization, inspired by the work in [34, 36], we introduce a total variation (TV) regularization term for ϕ to the proposed union registration, which preserves the densely and highly localized deformation of ϕ . Because of the TV- l_1 - l_2 composite property of the objective function in our union image registration, we resort to the split Bregman iteration^[38] for the complex problem solving.

This paper is organized as follows. Section 2 describes the proposed registration approach in detail. Experimental results are given in Section 3 to verify our approach and show the performance as compared with other methods. Finally, we discuss and conclude the algorithm in Section 4.

2 Proposed method

2.1 The registration model

Consider two images f and $u(\phi)$ to be aligned, assuming the following intensity relationship^[28]

$$f = u(\phi) + s + \eta \quad (3)$$

where $\phi = [X \ Y]^T$ is a 2D deformation field, η is zero mean Gaussian noise, and s is an intensity correction field which accounts for complex spatially-varying intensity distortions in mono-modal settings^[28]. Assuming s and ϕ are independent, the maximum a posteriori (MAP) approach

to estimate s and ϕ is to maximize the probability

$$P(\phi, s \mid f, u) \propto P(f, u \mid \phi, s)P(\phi)P(s) \quad (4)$$

where $P(f, u \mid \phi, s) \propto e^{-\|f - u(\phi) - s\|^2}$ is a joint likelihood of the images, $P(\phi)$ is a prior used to regularize the transformation, $P(s) \propto e^{-\beta\|\Delta s\|^2}$ is a prior on s that reflects the assumption on spatial intensity interactions^[28], and Δ before s is the Laplacian operator.

Choosing a suitable regularizer for ϕ is important to the well-posedness of the registration problem. In most cases, assuming the deformation is smooth (see Figs. 1 (a) to 1 (c)), the l_2 -norm based constraints are used. However, in practice, the smoothness assumptions are violated at motion boundaries^[34, 36] when motion discontinuities exist (see Figs. 1 (d) to 1 (f)). Frequently, the deformation field ϕ has some local structures or homogeneous regions. When there are multiple motions in an image patch, motion boundaries and motion discontinuities exist, and thus the corresponding deformation field is neither smooth nor reflecting a homogeneous motion. In each structure, the deformation tends to change smoothly, and the gradients of the deformation are also small. At the discontinuities between structures, the gradients are large. Therefore, only a small number of gradients should have large values, and the deformation gradient field has to be sparse in the image domain. In other words, the MAP estimator in (4) should use sparse gradient priors to describe $P(\phi)$. Hence, we choose the total variation of ϕ (i.e., $J_{TV}(\phi) = \|\nabla\phi\|$) as regularization, which is a heavy-tailed gradient prior and describes motion discontinuities between local structures.

Fig. 1 Transformations. (c) is the smooth motion field corresponding to the transform (a) and (b). (f) is the discontinuous motion field corresponding to the transform (d) and (e).

Taking into account the spatially-varying intensity distortions and the motion discontinuities, we can formulate

the problem as

$$\min_{\phi, s} J_{TV}(\phi), \quad \text{s.t.} \quad f = u(\phi) + s + \eta \quad (5)$$

where the transformation field ϕ is non-parametric, which directly represents the displacement of every image element. The high degrees of freedom of ϕ results in a strong descriptive power of the transformation. However, this model enrichment results in lowering of robust to noise. On the contrary, the B-spline based parametric registration, FFD, uses a set of control points $\psi = [U \ V]^T$ to parameterize the transformation field ϕ (See (2)). Compared to the non-parametric registration, the FFD requires fewer degrees of freedom to describe the deformation, and the implicit smoothness of the B-spline basis leads to FFD registration results with smooth deformation, which are robust to the noise.

It is natural to consider how to combine the flexibility of the non-parametric registration and the robustness of the FFD registration. In the simulated experiments without the distortion of noise, we found that the difference between the non-parametric displacement field ϕ and the FFD parametric displacement field $K\psi$ is sparse. In other words, the statistics of the displacement difference satisfies the Laplace distribution, which is shown in Fig. 2. Therefore, we use l_1 norm to measure the mapping error between the parameter space of ψ and the latent transformation space of ϕ , and the final physical model can be formulated as

$$\min_{\phi, s} J_{TV}(\phi), \quad \text{s.t.} \quad f = u(\phi) + s + \eta, \quad |\phi - K\psi| \leq \varepsilon \quad (6)$$

where $\varepsilon > 0$ is the maximum mapping error, which is determined by the extent of the localized and discontinuous deformations. From an MAP angle, we can revise (4) into

$$P(\phi, \psi, s \mid f, u) \propto P(f, u \mid \phi, \psi, s)P(\phi, \psi)P(s) \quad (7)$$

$$\propto P(f, u \mid \phi, \psi, s)P(\phi \mid \psi)P(\psi)P(s) \quad (8)$$

where $P(\phi \mid \psi) \propto e^{-\gamma\|\phi - K\psi\|}$ is a global smoothness constraints, which guarantees the robustness of image registration, $P(\psi) \propto e^{-\beta\|\Delta\psi\|^2}$ is a regularization prior on ψ ^[11], and $P(f, u \mid \phi, \psi, s)$ and $P(s)$ are as same as the terms in (4).

2.2 TV- l_1 based union image registration

Since s is independent from ϕ and ψ , leaving out the regularization of ϕ and ψ , maximization of the posterior probability in (8) is equivalent to minimization of the following objective function:

$$E(\phi, s) = \|f - u(\phi) - s\|^2 + \beta\|\Delta s\|^2. \quad (9)$$

Fig. 2 Statistics of difference between ϕ and $K\psi$ for X field (a) and Y field (b)

By analytically solving for the intensity correction field s , Myronenko and Song^[28] proposed the residual complexity (RC) similarity measure

$$E_{rc}(\phi) = \log\left(\frac{\|Q^T(f - u(\phi))\|^2}{\alpha + 1}\right) \quad (10)$$

where Q^T is the discrete cosine transform (DCT) basis^[39] and α is a trade-off parameter to measure the correlation of the noise.

The RC measure penalizes small DCT coefficients stronger than the large ones. Thus, it encourages the presence of few large coefficients opposed to many small ones. Let $x = Q^T r$ be the DCT coefficients of the residual image. The term $\sum \log(x^2 + 1)$ indeed forces sparseness of the DCT coefficients of the residual image, because the logarithm function decreases quickly to zero compared to its increase for larger values of the DCT coefficients. The RC similarity measure is minimized when the residual image can be sparsely represented using only a few basis functions (e.g., favoring spatially smooth or piece-wise smooth residual). Such properties of RC allow robust alignment of the images distorted by the spatially-varying intensity distortions. In contrast to the RC measure, both of the sum of square difference (SSD) and the entropy of the difference (EOD) take no account of the spatial positions of

misaligned structures^[28].

We use RC as the similarity measure, then maximizing the posterior probability $P(\phi, \psi, s | f, u)$ in (7) is equivalent to minimizing the following objective function

$$E(\phi, \psi) = J_{TV}(\phi) + \lambda E_{rc}(\phi) + \gamma |\phi - K\psi| + \frac{\beta}{2} \|\Delta\psi\|^2 \quad (11)$$

where λ , γ and β are free parameters. To solve the composite problem with total variance terms, l_1 terms and l_2 terms, we apply an alternating strategy as

$$\begin{cases} \psi^* = \min_{\psi} \lambda E_{rc}(\psi) + \gamma |\phi - K\psi| + \frac{\beta}{2} \|\Delta\psi\|^2 & (12a) \\ \phi^* = \min_{\phi} J_{TV}(\phi) + \lambda E_{rc}(\phi) + \gamma |\phi - K\psi| & (12b) \end{cases}$$

where (12a) is the classic FFD method with the constraint of non-parametric transformation field ϕ , which is taken as a force to improve the accuracy of the FFD method, and (12b) is the TV-based non-parametric registration with the constraint of the parametric transformation field $K\psi$, which guarantees its global robustness. Both of them are composed of smooth terms and non-smooth terms, and we resort to split Bregman method^[38] to optimize the composite convex problems due to its rapid convergence especially in dealing with the TV-based optimization problem.

2.3 Implementation of the proposed algorithm

2.3.1 Optimization of the parametric image registration

To “de-couple” the l_1 and l_2 portions of (12a), we introduce a new variable and get the following equivalent problem

$$\begin{aligned} \min_{\psi, d} \quad & \lambda E_{rc}(\psi) + \gamma |d| + \frac{\beta}{2} \|\Delta\psi\|^2 \\ \text{s.t.} \quad & d = \phi - K\psi. \end{aligned} \quad (13)$$

As shown in [38], the above iteration is equivalent to the simple version of the split Bregman iteration:

$$\begin{aligned} (\psi^{k+1}, d^{k+1}) = \min_{\psi, d} \quad & \lambda E_{rc}(\psi) + \gamma |d| + \\ & \frac{\beta}{2} \|\Delta\psi\|^2 + \frac{\mu\gamma}{2} \|H(\psi, d, b^k)\|^2 \end{aligned} \quad (14)$$

where μ is the free parameter and

$$H(\psi, d, b^k) = d - (\phi - K\psi) - b^k \quad (15)$$

and

$$b^{k+1} = b^k + (\phi - K\psi^{k+1}) - d^{k+1}. \quad (16)$$

We can perform the minimization of (14) efficiently by iteratively minimizing with respect to ψ and d separately.

The two steps we must perform are

$$\text{Step 1. } \psi^{k+1} = \min_{\psi} \lambda E_{rc}(\psi) + \frac{\beta}{2} \|\triangle \psi\|^2 + \frac{\mu\gamma}{2} \|H(\psi, d^k, b^k)\|^2. \quad (17)$$

$$\text{Step 2. } d^{k+1} = \min_d |d| + \frac{\mu}{2} \|H(\psi^{k+1}, d, b^k)\|^2. \quad (18)$$

Because we have “de-coupled” ψ from the l_1 portion of the problem, the optimization problem that we must solve for ψ^{k+1} in Step 1 is now differentiable. We can use a time-marching fix-point iteration scheme^[24] to solve this problem.

For Step 2, there is no coupling between elements of d . The optimal value of d can be explicitly computed by soft shrinkage operators

$$d^{k+1} = \text{shrink}(\phi - K\psi^{k+1} + b^k, \frac{1}{\mu}) \quad (19)$$

where the shrink operator is

$$\text{shrink}(x, \frac{1}{\mu}) = \frac{x}{|x|} \times \max(|x| - \frac{1}{\mu}, 0) = \begin{cases} x - \frac{1}{\mu}, & \text{if } x > \frac{1}{\mu} \\ 0, & \text{if } -\frac{1}{\mu} \leq x \leq \frac{1}{\mu} \\ x + \frac{1}{\mu}, & \text{if } x < -\frac{1}{\mu}. \end{cases} \quad (20)$$

We summarize the above-mentioned algorithm for the parametric registration in Algorithm 1, where “PIR” stands for “parametric image registration” and E in the stopping criterion is the energy of cost function in (12a).

Algorithm 1. Parametric image registration

Input: Fixed image f , floating image u , deformation field ϕ , and initialized parameter ψ_0 .

Output: Deformation parameter ψ .

Initialize: $\psi^0 = \psi_0, d = b = 0$.

while $\|E^{k+1} - E^k\|^2 / \|E^k\|^2 > \text{tol}$

Solve ψ^{k+1} in (17)

Solve d^{k+1} using (19)

Solve b^{k+1} using (16)

end

Return ψ

2.3.2 Optimization of the nonparametric image registration

We also use the scheme combining split Bregman iteration and time-marching fix-point iteration to solve (12b). To “de-couple” the TV, l_1 and l_2 portions, we introduce auxiliary variables and get the equivalent problem as

$$\begin{aligned} \min_{\phi, d_{\nabla}, d} & |d_{\nabla}| + \gamma |d| + \lambda E_{rc}(\phi) \\ \text{s.t. } & d_{\nabla} = \nabla \phi, d = \phi - K\psi. \end{aligned} \quad (21)$$

Then, the split Bregman iteration is

$$\begin{aligned} \min_{\phi, d_{\nabla}, d} & |d_{\nabla}| + \lambda E_{rc}(\phi) + \frac{\mu}{2} \|H_{\nabla}(\nabla \phi, d_{\nabla}, b_{\nabla}^k)\|^2 + \\ & \gamma |d| + \frac{\mu\gamma}{2} \|H(\phi, d, b^k)\|^2 \end{aligned} \quad (22)$$

where $H(\phi, d, b^k)$ is same as in (15) but is function of ϕ , and

$$H_{\nabla}(\phi, d_{\nabla}, b_{\nabla}^k) = d_{\nabla} - \nabla \phi - b_{\nabla}^k \quad (23)$$

$$b_{\nabla}^{k+1} = b_{\nabla}^{k+1} + \nabla \phi^{k+1} - d_{\nabla}^{k+1} \quad (24)$$

$$b^{k+1} = b^k + \phi^{k+1} - K\psi - d^{k+1}. \quad (25)$$

By separating ϕ and d in (22), we can get

$$\begin{aligned} \text{Step 1. } \phi^{k+1} = \min_{\phi} & \lambda E_{rc}(\phi) + \frac{\mu}{2} \|H_{\nabla}(\phi, d_{\nabla}^k, b_{\nabla}^k)\|^2 + \\ & \frac{\mu\gamma}{2} \|H(\phi, d^k, b^k)\|^2 \end{aligned} \quad (26)$$

$$\text{Step 2. } d_{\nabla}^{k+1} = \min_{d_{\nabla}} |d_{\nabla}| + \frac{\mu}{2} \|H_{\nabla}(\phi^{k+1}, d_{\nabla}, b_{\nabla}^k)\|^2 \quad (27)$$

$$d^{k+1} = \min_d |d| + \frac{\mu}{2} \|H(\phi^{k+1}, d, b^k)\|^2. \quad (28)$$

We can also use the time-marching fix-point iteration scheme to solve (26) for Step 1. For Step 2, the optimal values of d_{∇} and d can be explicitly computed by soft shrinkage operators

$$d_{\nabla}^{k+1} = \text{shrink}(\nabla \phi^{k+1} + b_{\nabla}^k, \frac{1}{\mu}) \quad (29)$$

$$d^{k+1} = \text{shrink}(\phi^{k+1} - K\psi + b^k, \frac{1}{\mu}). \quad (30)$$

We summarize the above-mentioned optimization algorithm for the non-parametric registration in Algorithm 2, where “NPIR” stands for “non-parametric image registration”.

Algorithm 2. TV- l_1 based NPIR

Input: Fixed image f , floating image u , deformation parameter ψ , and initialized deformation field ϕ_0 .

Output: Deformation field ϕ .

Initialize: $\phi^0 = \phi_0, d^0 = b^0 = 0, d_{\nabla}^0 = b_{\nabla}^0 = 0$.

while $\|E^{k+1} - E^k\|^2 / \|E^k\|^2 > \text{tol}$

Solve ϕ^{k+1} in (26)

Solve d_{∇}^{k+1} and d^{k+1} using (29) and (30)

Solve b_{∇}^{k+1} and b^{k+1} using (24) and (25)

end

Return ϕ

2.3.3 The union image registration

By uniting Algorithms 1 and 2, we get the TV- l_1 based union image registration which is summarized in Algorithm 3, where “UIR” stands for “union image registration” and “CFFD” means “classical FFD” method. The union method in Algorithm 3 is divided into two stages. In the first stage, the classical FFD method using RC as similarity measure is applied to get the robust initial solutions for ϕ and ψ . In the second stage, the accuracy of ϕ and ψ is improved by alternating optimization using Algorithms 1 and 2.

Next, we will analyze the computational complexity of our union image registration. For each iteration in Algorithms 1 and 2, there exist basic operators, such as, the discrete cosine transform, the gradient-based operator, and the shrink operator. Because the discrete cosine transform

can be computed through FFT in $O(N \log N)$ ^[39] while both of the gradient-based operator and the shrink operator can be computed in $O(N)$, where N is the image size, the time complexity for each iteration in Algorithms 1 and 2 is $O(N \log N)$. On the other hand, the Bregman iteration converges quickly for l_1 -based problems, which has been proved in [38] and 2 or 3 iterations are enough to achieve desired registration results. Therefore, our union method is efficient and just takes a little more time than the classical FFD method.

Algorithm 3. TV- l_1 UIR

Input: Fixed image f , floating image u .

Output: Deformation field ϕ and parametric field ψ .

Stage 1: Initial solution

Solve ψ_0 using CFFD

Stage 2: Union image registration

Initialize: $\psi = \psi_0$, $\phi = K\psi_0$

for $n = 1$ to N

$\phi_0 = \phi$, solve ϕ using Algorithm 2

$\psi_0 = \psi$, solve ψ using Algorithm 1

end

Return ϕ and ψ

3 Experiments

In this section, we will illustrate the performance of the proposed approach using both synthetic and real images. Several methods or toolboxes, such as FFD (RC)^[28], SFFD (RC)^[29, 30], Markov random field (MRF) based Drop (NMI)^[40, 41], and Fair (SSD)^[13] were used as comparisons¹. The first 3 comparison methods are B-spline based parametric image registration using residual complexity (RC) or normalized mutual information (NMI) as similarity measure, and the Fair is non-parametric image registration using sum of square differences (SSD) as similarity measure. We ran all the tests on an Intel Core i7-4700MQ CPU 2.4 GHz Windows 64-bit operating system with 8 GB RAM. Both mutual information (MI) and root mean square error (RMSE) indices were adopted to evaluate the quality of the registered images.

3.1 Parameter settings

We used 3 hierarchical levels of image resolution for all methods. Each level was achieved by down-sampling the original image. For all B-spline based registration methods, including FFD(RC)^[28], SFFD(RC)^[29, 30], Drop(NMI)^[40, 41] and the parametric registration part in our algorithm, the spacing of the control points was set 6 pixels. We set parameter $\alpha = 0.05$ in (10) for RC measure. For our union registration, we set parameter $\lambda = 500$, which penalizes the RC-based fidelity term, parameter $\gamma = 0.05$, which

penalizes the l_1 -based error between the non-parametric deformation field ϕ and the parametric deformation field $K\psi$, parameter $\beta = 0.005$, which penalizes the l_2 -based regularizer for ψ , and the auxiliary parameter $\mu = 0.05$, which is used in the split Bregman iteration. We set the number of alternating optimization $N = 2$ for our union image registration in Algorithm 3. Note that the parameter settings mentioned above for our method were empirically determined to give the best performance and were kept fixed in all tests.

The image registration software toolbox, Drop, provides a broad range of intensity-based registration options and settings. We selected NMI as the similarity measure for it, which performed better than other metrics in the following tests. In the Drop implementation, FastPD^[40] was used as the optimizer, and we selected approximated curvature penalty^[42] as the regularization function, which penalizes the approximated second-order derivatives of the displacement field. Moreover, we have empirically found that 500 iterations were sufficient to achieve a registration result with no further improvements and the suitable value for the registration parameter was 10. For the non-parametric image registration method, Fair^[13], we set the Lamé constants $\lambda = 1$ and $\mu = 0$ in the elastic regularizer and empirically set regularization parameter $\alpha = 100$ in tests without intensity distortion and 300 in tests with intensity distortion. We refer interested readers to [13] for details.

3.2 Synthetic experiments

In this subsection, we illustrate the performance of the proposed approach using synthetic deformation images with noise or without noise. We designed 3 different warping ways and used different warping parameters to simulate the free-form deformation. The first 2 ways are based on the B-spline interpolation, while the third way is based on the thin-plate interpolation. Three images from different domains were used as test images.

3.2.1 Registration without intensity distortion

Fig. 3 illustrates the first warping way, where the image Lena (256×256) was used as test image. The left-center part in black box is a 3-level cascade manual warp in the original image resolution. We first moved the bilateral control points in the coarse level with 32 pixels spacing to generate global deformations, which warped the background. Next, we moved the control points at the neighborhood of the nose of Lena in the middle level with 16 pixels spacing, which generated middle-extent deformations. Then, we moved the control points at the neighborhood of the left eye of Lena (considering the coordinate of the picture) in the refined level with 8 pixels spacing, which generated smaller deformations. The right-center part in red box is a random warp. The grid of control points with 4 pixels spacing was randomly perturbed to generate highly-localized or discontinuous deformations, and the random perturbation was drawn from a zero-mean Gaussian with 2 pixels standard deviation. The right-most image in Fig. 3 is the final warped result with different extent deformations.

¹In this section, the outputs of FFD (RC), Drop (NMI) and Fair (SSD) are generated using the original code. The outputs of SFFD are produced by our own implementation following [29] but using RC as similarity measure.

Fig. 3 The first warping way

Fig. 4 shows the registration results of the warped Lena image using different methods. Fig. 5 shows the absolute difference with the reference image (Fig. 4 (a)). The brighter is the difference image, the more accurate is the registration. It can be seen that all the B-spline based methods (including the parametric registration part in our

Fig. 4 Registration results of Lena. (c) and (d) are the non-parametric output and parametric output of our union image registration after 2 alternations.

Fig. 5 Difference of registration results of Lena

algorithm) failed to correct the random highly-localized deformations (see the wavy geometrical distortions). Our method and the non-parametric method Fair achieved the desired results, which illustrate that the non-parametric transformation model has a strong descriptive power for transformation. However, due to the constraint of the parametric registration result, our method performed more robustly than Fair. Fig. 6 shows the difference in some details between our result and the Fair's one, and we can see that there existed some excessive localized alignments in the Fair's result while our registration result kept consistent with the reference image.

Fig. 7 illustrates the second warping way which replaces the manual warp used in Fig. 3 with a zero-mean Gaussian random warp whose standard deviation is 4 pixels. A pair of images, Army, with multiple independently moving rigid objects, from the Middlebury benchmarks' training dataset^[43] were used as test images. Fig. 8 shows the corresponding registration results using different methods, and

Fig. 6 The proposed versus the Fair. The small images in the bottom row correspond to the magnifications of patches in the big images.

Fig. 7 The second warping way

Fig. 8 Registration results of Army. (c) and (d) are the non-parametric output and parametric output of our union image registration after 2 alternations.

Fig. 9 shows the absolute difference with the reference image (Fig. 8 (a)). Also, our method and Fair performed better than the other methods, but in some details, our registration result remained more consistent with the reference image than the Fair's one.

Fig. 9 Difference of registration results of Army

The third warping way is to randomly perturb a uniform grid of points followed by the thin-plate spline (TPS) interpolation of the reference image. The grid size was 7×7 and its random perturbation was drawn from a zero-mean Gaussian with 3 pixels standard deviation (std). We selected a 2-D slice (216×180) of the brain from the BrainWeb T1-weighted magnetic resonance imaging (MRI) images^[44] for the third simulation. Fig. 10 shows the registration results of the deformed brain image using different methods and Fig. 11 shows the difference images with respect to the reference image (Fig. 11 (a)). We can see that the non-parametric method in our union image registration outperformed the other ones. This is because our non-parametric method searches the optimal solution constrained by the global robust solution of the parametric method, which makes the final solution robust and accurate. On the contrary, due to the large solution space, the solution of Fair was easily trapped in the local optimum, and its registration result revealed excessive deformations which broke the structure consistency with the reference image. There is no obvious difference among the outputs of 4 parametric

Fig. 10 Registration results of Brain. (c) and (d) are the non-parametric output and parametric output of our union image registration after two alternations.

methods (see Figs. 10(d) to 10(g)), and all of them achieved robust performance.

Table 1 gives the RMSE values and MI values for all registration results with the 5 registration methods after rescaling the images to $[0, 255]$. Accounting for the boundary effect, we removed the boundary with 5 pixels spacing before calculating the values. It can be seen that the proposed approach outperformed the other methods in terms of RMSE and MI. Table 2 gives the computing time in seconds. We can see that Drop ran the longest time due to the time-consuming calculation of normalized mutual information (NMI). Our method took less time but more than classical FFD method, which illustrates the efficiency of split Bregman iteration in dealing with the l_1 -based optimization problem.

3.2.2 Registration with intensity distortion

To illustrate the robustness to the intensity nonstationarities and complex spatially-varying intensity distortions,

following [28], we simulated the additive and multiplicative spatially varying intensity distortion fields. The formula of the additive distortion field is

$$u(x, y) = u(x, y) + \left(\frac{1}{K}\right) \sum_{k=1}^K e^{-\frac{||[x; y] - \mu_k||^2}{2(30)^2}} \quad (31)$$

where the image u is rescaled to $[0, 1]$. The last term models locally-varying intensity field with mixture of K randomly centered Gaussians. Fig. 12 shows registration results for $K = 2$ (two Gaussians). It can be seen that the RC-based methods, such as FFD (RC), SFFD (RC) and our method performed robustly, while the other methods performed poorly with significant misalignment. In addition, the non-parametric registration in our method outperformed the other RC-based method in the local structures (see the local magnification in green rectangle in Fig. 12 (c)).

Fig. 11 Difference of registration results of Brain

Table 1 Summary of RMSE and MI of the Lena, Army and Brain. The images are rescaled to $[0, 255]$

Methods	Lena		Army		Brain	
	MI	RMSE	MI	RMSE	MI	RMSE
Proposed (ϕ)	3.771 1	9.346 5	3.813 5	4.993 2	3.689 5	4.244 8
Proposed (ψ)	2.951 2	13.027 2	3.455 3	6.978 1	3.107 5	5.267 0
FFD (RC)	2.860 0	13.950 4	3.386 6	7.442 9	3.029 7	5.469 8
SFFD (RC)	2.860 6	13.892 3	3.346 3	7.478 1	3.021 9	5.507 4
Drop (NMI)	3.120 7	12.688 3	3.531 4	6.721 0	3.228 5	5.044 8
Fair (SSD)	3.726 0	9.620 3	3.789 2	5.114 1	3.651 6	4.977 1

Fig. 12 Registration results of Brain with multiplicative distortion. (f) and (g) are intensity distortion fields which are multiplied to the reference image (a) and floating image (b), respectively. The local magnification in green rectangle is presented on the lower-right corner in each example.

Fig. 13 Registration results of Brain with multiplicative distortion. (f) and (g) are intensity distortion fields which are multiplied to the reference image (a) and floating image (b), respectively. The local magnification in green rectangle is presented on the lower-right corner in each example.

Table 2 Computing time (s)

Methods	Lena (256 × 256)	Army (256 × 256)	Brain (216 × 180)
Proposed	95	88	79
FFD (RC)	55	53	43
SFFD (RC)	51	46	37
Drop (NMI)	136	127	118
Fain (SSD)	45	28	21

Next, we corrupted both the reference and floating images with the multiplicative intensity field according to

$$u(x, y) = u(x, y) \cdot \left(0.2 + \left(\frac{1}{K}\right) \sum_{k=1}^K e^{-\frac{\| [x; y] - \mu_k \|^2}{2(30)^2}}\right) \quad (32)$$

where the image u is rescaled to $[0, 1]$. Fig. 13 shows registration results for $K = 2$. We can see that the proposed method outperformed the other methods.

3.3 Real image experiments

In this subsection, we register 3 groups of images from different domains, i.e., the MR images Cardiac B (180 × 180) with discontinuous motion from [29, 30]², the standard underwater images, Small Fonts (256 × 256) from [44], which are geometrically distorted by water fluctuation, and Chimney images (237 × 237) with severe turbulent motion from [6, 45]. We first used the images of Cardiac B as an example to illustrate the effect of our union image registration step by step. Then, we used all of the 5 methods to correct the geometrical distortions caused by water fluctuation or atmospheric turbulence and compared the results qualitatively and quantitatively.

3.3.1 Cardiac B images

To illustrate the effect of the l_1 -based coupling term $|\phi - K\psi|$ in (12a) and (12b), we compared the results of our union image registration with the non-parametric registration result without the constraint of the coupling term.

²[The objective function of non-coupling registration is](http://The discontinuous motion of Cardiac B images is synthetic in [29,30], but in this subsection, we consider it as real because we do not have the ground truth of motion field.</p>
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$$E(\phi, \psi) = J_{TV}(\phi) + \lambda E_{rc}(\phi) \quad (33)$$

where the result of the FFD (RC) is the initial solution. Fig. 14 shows the registration result using (33) and the ones of our union method after alternating optimization once. Fig. 15 shows the difference. Compared to the non-coupling result (Fig. 14 (d)), the non-parametric output of our method (Fig. 14 (f)) kept a better contour of the image content inside the green rectangle, which illustrates that the coupling term is contributing to get the global optimum of the objective function. In other words, the rough registration result of FFD (RC) (Fig. 14 (c)) offers a robust initial guess of the global optimum in most cases, and the coupling term constrains the searching process for the optimum near the initial solution. Therefore, the final solution with the constraint of the coupling term is global and accurate.

Fig. 14 Registration results of Cardiac B after alternating once. (d) is the non-coupling registration result. (e) and (f) are results of our union registration after alternating once.

To illustrate the effect of the alternating optimization, we computed MI values of 10 outputs in 10 alternations. Fig. 16 shows the evolution process of MI value, where the blue bar denotes the parametric output and the red bar de-

notes the non-parametric output. For sharp contrast, we rescaled the MI values using

$$\text{Bar}(\psi) = \text{MI}(\psi) - \text{MI}(\psi_1) + 0.1 \quad (34)$$

$$\text{Bar}(\phi) = \text{MI}(\phi) - \text{MI}(\psi) + \text{Bar}(\psi) \quad (35)$$

Fig. 15 Difference of registration results of Cardiac B

Fig. 16 MI bars

where ψ_1 is the first output of parametric registration, namely, the output of FFD (RC). We can see that the MI value increases in the previous 5 alternations but stabilizes in the following alternations. This is because the solution of our union registration is very close to the global optimum after a few alternations, and the accuracy would not

be improved even more alternations were taken. In fact, we usually ran 2 alternations to get the desired registration results. Fig. 17 shows partial registration results in 10 alternations. Fig. 18 shows registration results using 5 different methods and we can see that our method outperformed the others.

3.3.2 Images geometrically distorted by water or turbulence

We corrected the geometrical distortions of images when seeing through fluctuating water or atmosphere with severe turbulent motion. Figs. 19 and 20, where only one observed distorted image was used as an example, show the registration results using 5 methods. Table 3 gives the RMSE and MI values for all the registration results with the 5 registration methods after rescaling the images to $[0, 255]$. It can be seen that the proposed method and the Fair method performed better than the other methods. However, in Fig. 19, there existed some discontinuities in the results of Fair, while our ones kept well consistent with the reference image.

4 Discussion and conclusions

In this paper, we have developed a TV- l_1 based union image registration approach which addresses some of the most important short-comings of FFD-based registration and the non-parametric registration. The proposed method combines the robustness of the FFD-based method and the flexibility of the non-parametric method by a l_1 -norm based coupling term. In addition, considering the sparse gradient prior of the transformation field ϕ , we use total variance (TV) as the regularizer for ϕ . Due to the effect of the

Fig. 17 Partial registration results of Cardiac B in ten alternations

Fig. 18 Comparison of registration results of Cardiac B

Fig. 19 Registration results of Waterfont. The local magnification in green rectangle is presented on the lower-right corner in each example. (c) and (d) are the non-parametric output and parametric output of our union image registration after 2 alternations.

Fig. 20 Registration results of Chimney. The local magnification in green rectangle is presented on the lower-right corner in each example. (c) and (d) are the non-parametric output and parametric output of our union image registration after 2 alternations.

Table 3 Summary of RMSE and MI of Cardiac B, Small Fonts and Chimney (The images are rescaled to [0, 255])

Methods	Cardiac B		Small Fonts		Chimney	
	MI	RMSE	MI	RMSE	MI	RMSE
Proposed (ϕ)	2.414 8	19.218 7	3.453 9	35.291 5	1.753 2	14.414 3
Proposed (ψ)	2.272 3	22.171 2	2.716 3	36.248 6	1.658 2	14.531 0
FFD (RC)	2.251 5	23.194 3	2.677 1	36.659 1	1.671 5	14.610 5
SFFD (RC)	2.241 4	23.236 5	2.701 7	36.211 2	1.659 6	15.005 8
Drop (NMI)	2.207 4	23.600 0	2.628 7	37.160 5	1.512 4	15.718 6
Fair (SSD)	2.233 5	21.461 4	3.160 5	35.218 4	1.715 4	14.468 5

spatially-varying intensity distortions in real word, we adopt the RC as the similarity measure. To solve the TV- l_1 - l_2 based composite optimization problem, we proposed alternating optimization scheme which uses the advanced split Bregman iteration. By updating either ϕ or ψ in every iteration, the accuracy of both the parametric registration and the non-parametric registration is improved progressively. Experiments using both simulated and real images illustrate the accuracy and robustness of our method.

In our union image registration, there are 2 outputs, namely, the parametric registration result $K\psi$ and the non-parametric registration result ϕ . Because of the strong descriptive power of the non-parametric transformation, the non-parametric output is always more accurate than the parametric output. But in the situation that the reference image is very blurred, e.g., the reference image is the average of the geometrical distorted sequence^[6, 44, 45], a more accurate registration means a more blurred registration result. How to deal with this problem will be the focus of our further study. In addition, it is important to extend a registration method to handle 3D data. The only challenge of the 3D extension for our method is how to solve the TV- l_1 based optimization problem using split Bregman iteration in 3D scene, and we plan to investigate it in future research.

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