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N4SID and MOESP Algorithms to Highlight the Ill-conditioning into Subspace Identification

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Abstract: In this paper, an analysis for ill conditioning problem in subspace identification method is provided. The subspace identification technique presents a satisfactory robustness in the parameter estimation of process model which performs control. As a first step, the main geometric and mathematical tools used in subspace identification are briefly presented. In the second step, the problem of analyzing ill-conditioning matrices in the subspace identification method is considered. To illustrate this situation, a simulation study of an example is introduced to show the ill-conditioning in subspace identification. Algorithms numerical subspace state space system identification (N4SID) and multivariable output error state space model identification (MOESP) are considered to study, the parameters estimation while using the induction motor model, in simulation (Matlab environment). Finally, we show the inadequacy of the oblique projection and validate the effectiveness of the orthogonal projection approach which is needed in ill-conditioning; a real application dealing with induction motor parameters estimation has been experimented. The obtained results proved that the algorithm based on orthogonal projection MOESP, overcomes the situation of ill-conditioning in the Hankel′s block, and thereby improving the estimation of parameters.

Keywords: Subspace identification, ill-conditioning, oblique projection, orthogonal projection, algorithms numerical subspace state space system identification (N4SID), multivariable output error state space model identification (MOESP), induction motor.

1 Introduction

The determination of control laws for any system requires the knowledge of its dynamic model. The modelling involves then, for the system to be controlled, to obtain a mathematical model that well describes its dynamic behaviour[1]. The system identification is the main powerful technique for building accurate mathematical models of complex systems from noisy data. Indeed, many identification approaches are studied in literature such as those called predictor error methods[2, 3] and those based on the approximate least absolute deviation criteria[4]. The iterative nature of optimization algorithms used in these methods can lead to some problems like numerical instability or lack of convergence. The subspace identification techniques[5−8] constitute a good alternative to these classical methods, and especially for multiple-input and multiple-output (MIMO) linear systems.

During the last decades, subspace techniques have been well used for systems identification. The main advantage in these new identification methods is that some nonlinear optimization algorithm is required[7]. Based on geometric and mathematical procedures, they use only robust tools of linear algebra, such as the QR decomposition (Q: orthogonal matrice; R: upper triangular matrice), the singular value decomposition (SVD) singular value decomposition, the matrices projection and the angles of subspaces[7]. The purpose of the subspace identification method is to estimate a linear time invariant state space model, directly from input-output measures. Recently, many research works on subspace identification algorithms have been developed to estimate model parameters of industrial processes. Among these algorithms, we can cite the algorithms numerical subspace state space system identification (N4SID) and the multivariable output error state space model identification (MOESP)[6, 9, 10]. The main difference between these two algorithms is the kind of the employed subspaces projection in the Hankel matrices which are constituted of the system input/output data. In fact, the N4SID uses the oblique projection but the MOESP is particularly based on the orthogonal one. In literature, it has been proved that at some cases the N4SID algorithm leads to a biased estimation due to the use of correlated Hankel blocks matrices and noisy data and these are the main reasons of ill-conditioning problem in subspace identification[11].

The main objective of this paper is to validate with simulations and experiment, the improvement of estimation quality in ill-conditioning context, by using orthogonal projection instead of oblique projection in subspace identification. A simulation study, using a practical case of ill-conditioning in subspace identification algorithms (N4SID and MOESP), is presented. Our choice is focused on induction motor described by the Park model[12, 13]. The ill-conditioning problem has been, also, illustrated in experiment environment to estimate its parameters.

This paper is organised as follows: Section 2 deals with subspace identification approach. We present briefly the most important geometric and mathematical tools in linear algebra used in subspace identification such as oblique and orthogonal projections. In the third section, we highlight subspace identification in ill-conditioning context. For this purpose, the N4SID and MOESP algorithms are described. Then, through an example, we illustrate the ill-conditioning problem in subspace identification. Section 3 leads with a study of ill-conditioning phenomenon according to an in-

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duction motor model using N4SID and MOESP algorithms in Bode magnitude tracing and root locus for the estimated model. This study is, also, carried out in experiment environment in Section 4. Discussions are mainly focused on the obtained results to compare the two algorithms in the literature (N4SID and MOESP). These results show clearly the performance of orthogonal projection (MOESP) in front of the oblique one (N4SID) used in subspace identification. Finally, we conclude on the main results obtained in this work.

2 Subspace identification

Subspace techniques have been successfully handled for MIMO linear systems identification. These new identification methods did not involve nonlinear optimization algorithm. In fact, they use only robust mathematical and geometric tools of linear algebra, such as the QR decomposition, the SVD singular value decomposition, and the matrices projection and the angles of subspaces. The main objective of the subspace identification methods is to estimate a linear time invariant state space model, directly from data obtained from input-output measurements. Their application to estimate model parameters of industrial processes became recently highly developed. The first main implemented algorithms used are the N4SID and the MOESP. These two algorithms will be highlighted in the next sections of our contribution, especially, in context of ill-conditioning. In this section, we are interested in the presentation of the mathematical and geometric tools used in subspace identification and we emphasize on the oblique projection.

2.1 Problem formulation

Given q measurements of the input $u_k \in \mathbf{R}^m$ and the output $y_k \in \mathbf{R}^l$ generated by an unknown deterministic system of order n and described by the following discrete equations:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k + Du_k. \end{cases} \tag{1}$$

One should determine:

1) the order n of the unknown system,

2) the realization (A, B, C, D) within a similarity transformation; with $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, $C \in \mathbf{R}^{l\times n}$ and $D \in \mathbf{R}^{l\times m}$.

2.2 Geometric tools

In this section, we present, the mathematical tools of linear algebra used in subspace method[7]. We assume that the matrices $A \in \mathbf{R}^{p\times j}$, $B \in \mathbf{R}^{q\times j}$ and $C \in \mathbf{R}^{r\times j}$ are given.

1) $A^\dagger$ denotes the pseudo inverse Moore-Penrose of matrix A;

2) We define the operator of orthogonal projection Π_B which projects the row space of a matrix onto the row space of the matrix $B \in \mathbf{R}^{q\times j}$.

$$\Pi_B = B^{\mathrm{T}}(BB^{\mathrm{T}})^\dagger B. \tag{2}$$

3) The projection of the row space of the matrix $A \in \mathbf{R}^{p\times j}$ onto the row space of the matrix $B \in \mathbf{R}^{q\times j}$ is defined by:

$$\frac{A}{B} = A\Pi_B = AB^{\mathrm{T}}(BB^{\mathrm{T}})^\dagger B. \tag{3}$$

4) $B^\perp$ denotes a base of the orthogonal space to the row space of B.

5) $\Pi_{B^\perp}$ defines the geometric operator that projects the row space of a matrix onto the orthogonal complement to the row space of the matrix B.

$$\Pi_{B^\perp} = I_j - \Pi_B. \tag{4}$$

It results in:

$$\frac{A}{B^\perp} = A\Pi_{B^\perp} = A\left(I_j - \Pi_B\right). \tag{5}$$

While taking account of (2) and (4), one can easily show that matrix A can be decomposed into two matrices where their row spaces are orthogonal such as

$$A = A\Pi_B + A\Pi_{B^\perp}. \tag{6}$$

We present in Fig. 1, the geometric interpretation of (5). One defines the oblique projection of a matrix $A \in \mathbf{R}^{p\times j}$ along the row space of the matrix $B \in \mathbf{R}^{q\times j}$ onto the row space of the matrix $C \in \mathbf{R}^{l\times n}$:

$$\begin{aligned} A/_B C \quad = \quad & \left[\frac{A}{B^\perp}\right]\left[\frac{C}{B^\perp}\right]^\dagger C = \\ & A\Pi_{B^\perp} + (C\Pi_{B^\perp})^\dagger C. \end{aligned} \tag{7}$$

A geometric interpretation of (5) is illustrated, Fig. 1.

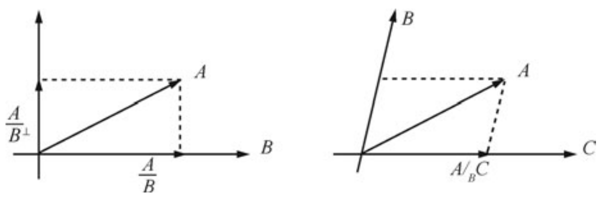


Fig. 1 Orthogonal and oblique projection in two dimensional space

In the next subsection, the study of the mathematical tools used in subspace identification will be given.

2.3 Mathematical tools

Let U be a vector such as $U = \begin{pmatrix} u_0 & u_1 & \cdots & u_{q-1} \end{pmatrix}^{\mathrm{T}}$. The matrix of Hankel associated to the vector U is given by

$$U_{0|2i-1} = \begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \\ u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{bmatrix} \tag{8}$$

with $q = 2i + j - 2$ and $i < j$.

One define respectively the matrices of Hankel for past and future inputs.

$$U_p = \begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{bmatrix} \tag{9}$$

and

$$U_f = \begin{bmatrix} u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{bmatrix}. \tag{10}$$

In the same way one may define the Hankel matrices of past and future outputs as Y_p and Y_f, respectively.

The instrumental variable matrix (or Hankel matrix of past data) is given by

$$Z_P = \begin{pmatrix} U_p & Y_p \end{pmatrix}^{\mathrm{T}}. \tag{11}$$

The matrix related to the sequence defining past:

$$X_p = \begin{pmatrix} x_0 & x_1 & \cdots & x_{j-1} \end{pmatrix} \in \mathbf{R}^{n\times j}. \tag{12}$$

And the matrix for future is

$$X_f = \begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-1} \end{pmatrix} \in \mathbf{R}^{n\times j}. \tag{13}$$

The extended observability matrix (we extend the observability to an order that is superior to the one of the system):

$$\Gamma_i = \begin{pmatrix} C & CA & \cdots & CA^{i-1} \end{pmatrix}^{\mathrm{T}} \in \mathbf{R}^{li\times n} \tag{14}$$

and the reversed extended controllability matrix is

$$\Delta_i = \begin{pmatrix} A^{i-1}B & A^{i-2}B & \cdots & B \end{pmatrix} \in \mathbf{R}^{n\times mi} \tag{15}$$

where H_i is the block Toeplitz matrix.

$$H_i = \begin{bmatrix} D & 0 & \cdots & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & CB & D \end{bmatrix}. \tag{16}$$

3 N4SID and MOESP algorithms

We describe, in this subsection, the steps of N4SID and MOESP algorithms.

3.1 N4SID

N4SID algorithm has recently attributed a special attention in the system identification field. It is presented to be a powerful alternative to the classical system identification method based on iterative approaches. The key step of this method is the oblique projection of subspaces generated by the block Hankel matrices formed by input/output data of system. Other geometric and mathematics tools of linear algebra like SVD are used to extract the order of the system and the observability matrix which contain the parameters of the estimated model.

We present, in this subsection, the N4SID algorithm.

From the system equation described above (1) we can easily obtain as follows the matrix equation using Hankel block:

Input-output matrix equations

$$X_f = A^i X_p + \Delta_i U_p \tag{17}$$

$$Y_p = \Gamma_i X_p + H_i U_p \tag{18}$$

$$Y_f = \Gamma_i X_f + H_i U_f. \tag{19}$$

Deterministic identification

While assuming that:

1) The inputs excitation is persistent of order $2i$, (rank $\left(U_{0|2i-1} U_{0|2i-1}\right) = 2mi$);

2) Intersection between the row space of matrix U_f and the one of matrix X_p;

3) The weighting matrices $W_1 \in \mathbf{R}^{li\times li}$ and $W_2 \in \mathbf{R}^{j\times j}$ are such as W_1 is of full rank and W_2 satisfies:

$$\mathrm{rank}(Z_p) = \mathrm{rank}(Z_p W_2) \tag{20}$$

with O_i as the oblique projection:

$$O_i = Y_f /_{U_f} Z_p \tag{21}$$

and

$$W_1 O_i W_2 = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^{\mathrm{T}} \\ V_2^{\mathrm{T}} \end{bmatrix} \tag{22}$$

$$W_1 O_i W_2 = U_1 S_1 V_1^{\mathrm{T}}. \tag{23}$$

It results:

1)

$$O_i = \Gamma_i X_f. \tag{24}$$

2) The order of the model defined by the system (22) is equal to the number of singular values different from zero, given by the matrix S_1.

3) The extended observability matrix Γ_i is given as

$$\Gamma_i = W_1^{-1} U_1 S_1^{\frac{1}{2}} T. \tag{25}$$

4)

$$X_f W_2 = T^{-1} S_1^{\frac{1}{2}} V_1^{\mathrm{T}}. \tag{26}$$

5)

$$X_f = \Gamma_i^{\dagger} O_i. \tag{27}$$

$T \in \mathbf{R}^{n\times n}$ is a non-singular similarity transformation matrix.

System order determination

1) From a finite number q of input-output data (u_k, y_k), we form the following Hankel matrices $(Y_p, Y_f)^{\mathrm{T}}$ and $(U_p, U_f)^{\mathrm{T}}$, then, we deduce the instrumental variable matrix $Z_p = (Y_p, U_f)^{\mathrm{T}}$. We compute the oblique projection as:

$$O_i = Y_f /_{U_f} Z_p. \tag{28}$$

2) We multiply O_i at left and at right, respectively, by the matrices W_1 and W_2.

$$W_1 O_i W_2 = U_1 S_1 V_1^{\mathrm{T}} \tag{29}$$

where W_1 and W_2 are the weighting matrices which are used for improving the estimation of $\Gamma_i X_f$. Their choice defines the computation algorithm[14].

3) We compute the SVD of $W_1 O_i W_2$.

S_1 is a diagonal matrix formed by n singular values different from zero. The order of the system is then n.

Determination of matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ through $\boldsymbol{\Gamma_i}$

Once $\Gamma_i = \begin{pmatrix} C & CA & \cdots & CA^{i-1} \end{pmatrix}^{\mathrm{T}}$ is computed, the matrix C is extracted directly from the first l rows of Γ_i. The matrix A is determined from the shift structure of Γ_i. Denoting

$$\underline{\Gamma}_i A = \overline{\Gamma}_i \tag{30}$$

where $\underline{\Gamma}_i$ is the matrix Γ_i without the last l rows:

$$\underline{\Gamma}_i = \begin{pmatrix} C & CA & \cdots & CA^{i-2} \end{pmatrix} \tag{31}$$

and $\overline{\Gamma}_i$ is Γ_i without the first l rows:

$$\overline{\Gamma}_i = \begin{pmatrix} CA & CA^2 & \cdots & CA^{i-1} \end{pmatrix}. \tag{32}$$

The matrix A is such as

$$A = \underline{\Gamma}_i^{\dagger} \overline{\Gamma}_i. \tag{33}$$

Determination of matrices $\boldsymbol{B}$ and $\boldsymbol{D}$

While multiplying (19) at left by $\Gamma_i^{\perp}$ and at right by $U_f^{\dagger}$ one obtains then:

$$\Gamma_i^{\perp} Y_f U_f^{\dagger} = \Gamma_i^{\perp} \Gamma_i X_f U_f^{\dagger} + \Gamma_i^{\perp} H_i U_f U_f^{\dagger} \tag{34}$$

and knowing that the product $\Gamma_i^{\perp}\Gamma_i$ is null, it results in

$$\Gamma_i^{\perp} Y_f U_f^{\dagger} = \Gamma_i^{\perp} H_i. \tag{35}$$

Denote

$$\begin{cases} L = \Gamma_i^{\perp} \\ M = \Gamma_i^{\perp} Y_f U_f^{\dagger}. \end{cases} \tag{36}$$

The equations (35) and (36) become

$$M = L H_i \tag{37}$$

with

$$L = \begin{pmatrix} L_1 & L_2 & \cdots & L_i \end{pmatrix} \tag{38}$$

and

$$M = \begin{pmatrix} M_1 & M_2 & \cdots & M_i \end{pmatrix}. \tag{39}$$

A system of equations which are function of B and D is resolved by a linear regression algorithm.

Experiments have shown that N4SID algorithm can lead in some cases to ambiguous estimation due to the use of oblique projection. So it is interesting to study the influence of the orthogonal projection on the estimation performances. For this reason we will present, in the next section, the MOESP algorithm which is especially founded on orthogonal projection.

3.2 MOESP

The MOESP algorithm, which needs the knowledge of the Hankel matrices Y_f and U_f, is particularly based on the orthogonal projection. From it, we are able to disclose matrices with specially structured row or columns. In this work, we focused on the case called ordinary MOESP. It uses the RQ factorization (representation of a matrix in simpler form via orthogonal transformations) of the composed matrix $\begin{pmatrix} U_f & Y_f \end{pmatrix}^{\mathrm{T}}$ to an another matrix O_i with orthogonal rows as a matrix with a row space that is equal to the column space.

Algorithm steps

In the MOESP algorithm, the first step is to compute the following RQ factorization with the knowledge of the Hankel matrices Y_f and U_f. So, we should

1) Construct the Hankel matrices Y_f and U_f: considering the system (1), and from the input/output measuring data, we generate the block Hankel matrices U_f and Y_f.

Let the instrumental variable be

$$Z_f = \begin{pmatrix} U_f & Y_f \end{pmatrix}^{\mathrm{T}}. \tag{40}$$

2) Execute a data compression via RQ factorization:

We apply a QR factorization to the matrix Z_f. The QR decomposition represents an orthogonal projection of the row space of Y_f on to the row space of the U_f matrix.

We obtain:

$$\begin{bmatrix} U_f \\ Y_f \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tag{41}$$

where R_{ii} are lower triangular, with R_{11} and R_{22} square invertible matrices.

3) Compute the SVD of R_{22} as

$$R_{22} = \begin{bmatrix} U_N & /U_N^{\perp} \end{bmatrix} \begin{bmatrix} \Sigma_n & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_n^{\mathrm{T}} \\ V_b^{\perp} \end{bmatrix}. \tag{42}$$

We extract the R_{22} matrix to which is then applied the SVD decomposition.

For a regular square matrix T, the relationship between the column spaces can be expressed as

$$O_k = U_N T. \tag{43}$$

4) Calculate the solutions for the set of equations:

$$U_N^{(1)} A_T = U_N^{(2)} \tag{44}$$

where $U_N^{(1)}$ and $U_N^{(2)}$ denote respectively the first and the last $l_{(i-1)}$ rows of U_N. The equation (44) expresses the property of shift invariance satisfied by the U_H or O_k matrices column spaces.

We obtain:

$$R_{22} = USV^{-1}. \tag{45}$$

The singular values contained in the S matrix are inspected to estimate n, the order of the system, which is considered as the number of singular values different from zero.

Then we can easily obtain:

$$\Gamma_i = U_1 S_1^{\frac{1}{2}}. \tag{46}$$

Finally the matrix C can be obtained directly from the first l rows of Γ_i. So, the matrix A is given by

$$A = \underline{\Gamma}_i^{\dagger} \overline{\Gamma}_i. \tag{47}$$

The matrix, by the same A system of equations which are function of B and D, is solved by a linear regression algorithm.

The matrices B and D are determined by solving the equation (37) with the same linear regression algorithm used for N4SID method.

The problem of ill-conditioning in system of equations can be encountered while computing N4SID and MOESP algorithms. In fact, for some cases, it can appear a poor quality for estimated parameters and this is principally due to the fact that the system operates sometimes with ill-conditioned matrices. In addition, experimentation with noisy environment leads to increasingly bad estimation performance.

The following section deals with the study of ill-conditioning.

4 Ill-conditioning problem

One can ask the following question: what does ill-conditioned and well-conditioned system of equations mean?

Indeed, a system of equations is considered to be well-conditioned[15] if a small change in the coefficient matrix or a small change in the right hand side results in a small change in the solution vector. In the opposite case, a system of equations is considered to be ill-conditioned if a small change in the coefficient matrix or a small change in the right hand side results in a large change in the solution vector[16, 17].

Firstly, we attempt to highlight through a numerical example to illustrate the problem of ill-conditioning, and secondly by experiments on a real process: induction motor (IM) parameters estimation[18, 19].

4.1 Numerical example

To explain the ill-conditioning problem of matrices, we consider the following matrix equation.

$$AX = B \tag{48}$$

with $A = \begin{bmatrix} 1 & 2 \\ 2 & 3.999 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 7.999 \end{bmatrix}$.

The solution of the set of this equation is $X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

When we make a small change to the matrix B such as $B = \begin{bmatrix} 4.001 \\ 7.998 \end{bmatrix}$.

The solution of this set becomes $X = \begin{bmatrix} -3.999 \\ 4 \end{bmatrix}$.

From these results, we can conclude that a small variation on the coefficient matrix A leads to a very large variation on the result. The matrix A is called an ill-conditioned matrix. So, the conditioning of the non-singular A matrix is defined by

$$\text{cond}(A) = \|A\| \times \left\|A^{-1}\right\| = 24992 \gg 1. \tag{49}$$

A matrix is called well-conditioned, when the quantity $\|A\| \times \left\|A^{-1}\right\|$ tends toward 1.

In literature, two main reasons are often cited when ill-conditioning problem is evoked while using the N4SID algorithm. The first one is the strong correlation between the past and future data present in Hankel block matrices which generate subspaces where the main step of N4SID algorithm focuses on their oblique projection. The second, relates to stochastic contributions to the output which are correlated with system inputs. This leads to unreliable estimates[20]. In the following two dimensional subspace example we illustrate in Fig. 2, a typical case when a small variation of the output data can give a large effect on estimates when data are projected obliquely onto the past input-output data along the future input.

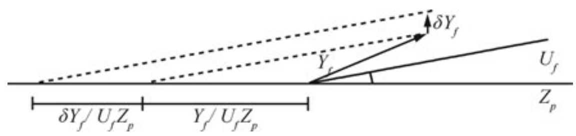


Fig. 2 Two dimension subspace in oblique projection

To illustrate the improvement in the estimation by the use of the orthogonal projection, obtained from the QR (Q: orthogonal matrice and R: upper triangular matrice) decomposition instead of the oblique projection proposed in N4SID algorithm, a real parameters estimation has been done on an IM platform[18, 19]. This is the aim of the next section.

4.2 Parameters estimation of IM in ill-conditioned situation

In this section, we propose the induction motor parameters estimation in ill-conditioning context. This study is, firstly, done in simulation based on a park model (PM) in the stator referential[13, 18]. Our main objective is to emphasize the improvement of the estimated parameters while using the orthogonal projection applied in MOESP algorithm instead of the oblique projection in N4SID algorithm. Secondly, a similar study will be treated on an experimental platform of induction motors in Section 5.

4.3 Modelling of the IM

The induction motor has been described in literature by several kinds of models which differ according to the intended application. In our case we will propose the complex formed park model in the stator referential[12, 18].

The used model is given by

$$\begin{cases} \dfrac{\mathrm{d}X}{\mathrm{d}t} = A_cX + B_cU \\ Y = C_cX + D_cU \end{cases} \tag{50}$$

with

$$X = \begin{bmatrix} \overline{I} \\ \Phi \end{bmatrix}, \quad U = \overline{V}, \quad Y = \overline{I} \tag{51}$$

and

$$A_c = \begin{pmatrix} -(\dfrac{1}{\sigma\tau_s} + \dfrac{1-\sigma}{\sigma\tau_r}) & \dfrac{L_m}{\sigma L_s L_r \tau_r} + \mathrm{j}\dfrac{L_m}{\sigma L_s L_r} p\omega_r \\ \dfrac{L_m}{\tau_r} & -\dfrac{1}{\tau_r} + \mathrm{j}p\omega_r \end{pmatrix} \tag{52}$$

$$B_c = \begin{pmatrix} \dfrac{1}{\sigma L_s} \\ 0 \end{pmatrix} \tag{53}$$

$$C_c = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{54}$$

and

$$D_c = 0 \tag{55}$$

$$\sigma = 1 - \frac{L_m^2}{L_r L_s}, \quad \tau_s = \frac{L_s}{R_s}, \quad \tau_r = \frac{L_r}{R_r}. \tag{56}$$

4.4 Use of N4SID and MOESP: Simulation results

We have simulated, using Matlab environment, the tracing of Bode magnitude plot associated to transfer functions of IM park model under white Gaussian noise (SNR=30 dB) which is added, simultaneously, to input and output datasets. The same simulation conditions are adopted to carry out, also, the tracing of Bode magnitude plot for N4SID and MOESP estimates according to generated datasets using 100 trials (1000 data per trial).

The IM model parameters, used in simulation, are as follow: $R_s = 10.00\,\Omega$, $R_r = 6.58\,\Omega$, $L_r = 0.31\,\mathrm{H}$, $L_s = 0.31\,\mathrm{H}$, $\omega_r = 157.08\,\mathrm{rad/s}$.

The adopted sampling period is equal to 10^{-4} s.

These parameters lead to the following matrix:

$$A_c = \begin{pmatrix} -200.32 & 247.02 + \mathrm{j}3656.16 \\ 5.73 & -21.23 + \mathrm{j}314.16 \end{pmatrix} \tag{57}$$

$$\mathrm{cond}(A_c) = 322.47. \tag{58}$$

The computed condition number of this matrix is very greater than one; and consequently the matrix is ill-conditioned. For this reason, we opted for the choice of IM model to illustrate the ill-conditioning phenomenon.

We illustrate in Fig. 3, the Bode magnitude tracing of IM model (solid line) as well as N4SID (dotted line) and MOESP (dashed line) estimates. It is to notice, in such ill-conditioning situation of matrix, that the MOESP estimates are better than N4SID ones; and that are due to the use of orthogonal projection in MOESP algorithm instead of oblique projection in N4SID.

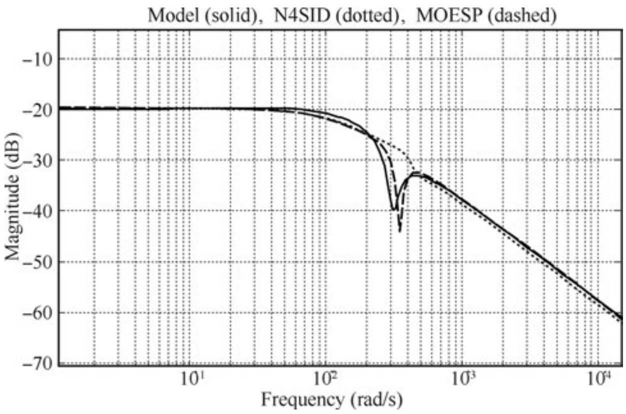


Fig. 3 Magnitude curve of Bode locus for transfer functions associated to IM model, N4SID and MOESP estimates

In order to confirm the obtained results in Bode magnitude tracing, we have, also, simulated the root locus of the estimated model using N4SID and MOESP algorithms. The simulation is carried out from 100 trials based on datasets generated according to IM model (1000 data per trial). The gotten curves, shown in Fig. 4, lead to a similar conclusion validated by Bode tracing.

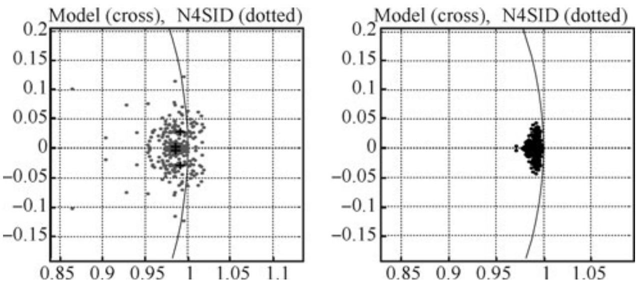


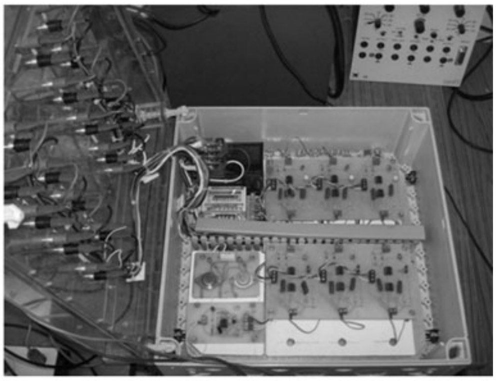
Fig. 4 Estimated poles (dots) with N4SID and MOESP for 100 trials using datasets generated according to IM model

5 Application to IM parameters estimation

It is very important to note that the proposed model is constituted by matrices using a complex and invariant time parameters. In constant frame the induction motor can be described by a linear time invariant (LTI) model. The parameters of the used induction motor bench are as follows: $R_s = 10.00\,\Omega$, $R_r = 6.58\,\Omega$, $L_r = 0.31\,\mathrm{H}$, $L_s = 0.31\,\mathrm{H}$, $\omega_r = 157.08\,\mathrm{rad/s}$.

Input-output data are acquired from motors experiment platform of a 1 kW power, mounted around an acquirement and control card DSPACE 1104 and an inverter, at the Laboratory of Sciences and Techniques of Automatic Control and Computer Engineering (Lab-STA) of National Engineering School of Sfax. The currents and the voltages have been measured using a sampling frequency of 10^4 Hz. The voltage applied to the motor is 230 V with frequency 50 Hz. 100 trials have been done with 1000 input/output data per trial. We present in Fig. 5, a general view of the experiment platform[21].

(a) View of the platform



(b) Electronic cards modules

Fig. 5 View of the platform and the electronic cards module

5.1 Experimental results based on N4SID and MOESP

As presented in Section 2.4, the order of the system is determined by applying SVD to the oblique projection of the data. When we examine the SVD results, we can easily estimate the order of the system. We illustrate in Fig. 6, the singular values of the projection matrix ($W_1 O_i W_2$) using N4SID and MOESP algorithms.

While inspecting the distance between two consecutive singular values given by MOESP and N4SID, we can notice the efficiency of the MOESP curve in comparison with the N4SID, which shows that there are approximately the two more dominant values. For this reason, we can conclude that the system is of second order. We present in Fig. 7, the Bode magnitude tracing of IM model as well as the estimate models obtained out from experimental data using N4SID and MOESP.

The tracings of Bode magnitude for IM model (solid line) as well as N4SID (dashed line) and MOESP (dashed-dotted line) estimates, are shown Fig. 7. In experiment environment, we can confirm the same conclusion predicted by the simulation results (see Section 4.4).

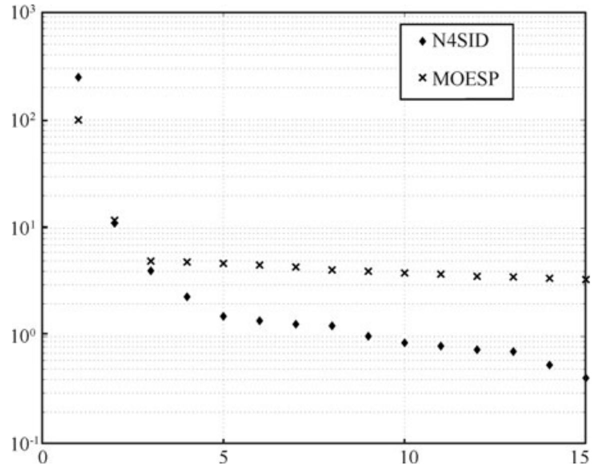


Fig. 6 Singular values obtained from N4SID and MOESP algorithms

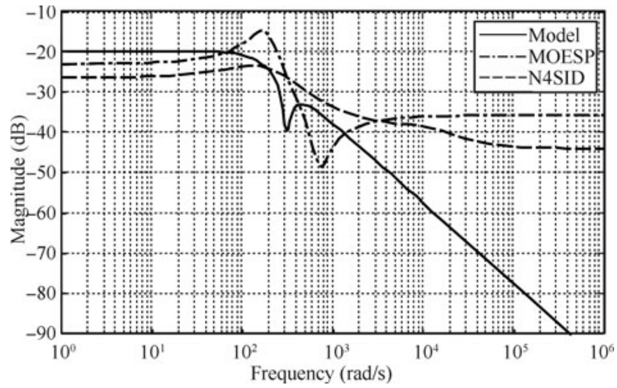


Fig. 7 Bode plot of the transfer function associated to the real system, N4SID and MOESP estimated models

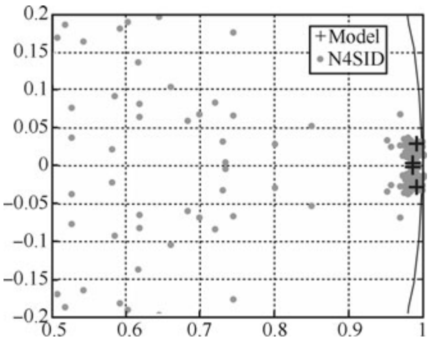


Fig. 8 Root locus plot from N4SID estimated model

We have also plotted from experimental data, the root locus of the estimated model using N4SID and MOESP. These plots have been carried out in the same simulation conditions adopted previously (100 trials and a measurement horizon of 1000 data). The obtained plots, shown in Figs. 8 and 9, prove that the estimated model using MOESP gives better results than N4SID. Also, it has to be noticed that in the experimental environment the estimated model using N4SID gives more poor plot in comparison with the one obtained from simulation environment.

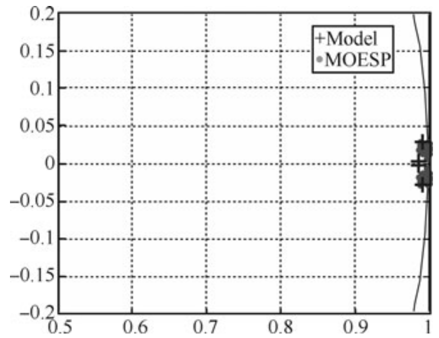


Fig. 9 Root locus plot from MOESP estimated model

6 Conclusions

In this contribution, the main work has been reserved to the study of ill conditioning problem in subspace identification method. In fact, it has been demonstrated that the performance of standard subspace algorithms from the literature (e.g., the N4SID method) may be surprisingly poor in certain experimental conditions.

In this paper, the analysis of ill conditioning problem in subspace identification method is considered. The subspace identification technique taking into account this problem presents a satisfying robustness in the parameters estimation of process model which allows achieving a performing control. As a first step, the main geometric and mathematical tools used in subspace identification are briefly summarized. In the second step, the problem of ill-conditioned matrices in the subspace identification method is analyzed. To illustrate this situation, a simulation study of an academic example is introduced.

In order to illustrate the ill-conditioning phenomenon, we have described a study in simulation and experimentation on an induction motor described by the park model. For this purpose, both Bode magnitude and root locus plots have been built with the use of N4SID and MOESP algorithms to estimate the model parameters. Both in simulation and in experiments, it has been demonstrated that the MOESP gives better results.

Finally, this study has shown the inadequacy of the oblique projection and validates the effectiveness of the orthogonal projection approach which is needed in ill-conditioning. The obtained results have proved that the algorithm based on orthogonal projection MOESP allowed, simultaneously, overcoming the situation of ill-conditioning in the Hankel′s block, and improving the estimation of parameters.

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