Design of a Discrete-time Output-feedback Based Repetitive-control System

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Abstract: This paper deals with the problem of designing a robust discrete output-feedback based repetitive-control system for a class of linear plants with periodic uncertainties. The periodicity of the repetitive-control system is exploited to establish a two-dimensional (2D) model that converts the design problem into a robust stabilization problem for a discrete 2D system. By employing Lyapunov stability theory and the singular-value decomposition of the output matrix, a linear-matrix-inequality (LMI) based stability condition is derived. The condition can be used directly to design the gains of the repetitive controller. Two tuning parameters in the LMI enable the preferential adjustment of control and learning. A numerical example illustrates the design procedure and demonstrates the validity of the method.

Keywords: Repetitive control, discrete two-dimensional system, singular-value decomposition, output feedback, linear matrix inequality (LMI).

1 Introduction

By repeating the same operation, a person gradually becomes skilled, and finally can operate with great efficiency and precision. In fact, the repetition of operation is a process of learning and gradual progress. Based on the internal model principle^[1], Inoue et al.^[2] added a human-like learning capability to a control system and devised a new control strategy called repetitive control (RC). Fig. 1 shows the configuration of a discrete-time repetitive-control system (RCS).

Fig. 1 Configuration of a basic discrete-time repetitive-control system

In Fig. 1, $G(z)$ is a compensated plant, and z is a shift operator. The part enclosed by the dotted line is a repetitive controller that contains a pure delay with a positive-feedback loop. For a given periodic reference input $r(k)$ with period of N , a repetitive controller gradually improves the tracking precision through repeated learning actions, which involve adding the control input of the previous period $v(k - N)$ to that of the present period to regulate the control input. As a result, the tracking error is reduced step

by step. Finally, the output tracks the reference input without any steady-state error. RC is similar to iterative learning control (ILC)^[3], which is another well-known method that makes use of the previous control trials. However, the difference between RC and ILC regarding the initial state of every period makes the issues of interest (stabilization, control system design, etc.) for these two methods very different.

An RCS has its inherent two-dimensional (2D) structural characteristics. It actually involves two different actions: Control within each repetition period and learning between periods. However, most analysis and design methods for an RCS developed for one-dimensional (1D) space ignore the difference between these two actions and only consider their overall effect. Consequently, they are incapable of making fundamental improvements in transient performance^[4-6]. For the case of continuous RC, Zhou et al.^[7, 8] employed 2D system theory^[9, 10] to design a robust RCS. Mapping the RCS to 2D space yields a continuous-discrete 2D model. Unlike the 1D methods, the method in [7, 8] enables preferential adjustment of control and learning. The resulting control system exhibits both satisfactory robustness and good tracking performance.

RCS has been studied mainly in the continuous time domain. However, in practical control systems, digital implementation of repetitive controller is simpler than analog counterpart. So, developing a design method for a discrete-time robust RCS has practical significance. References [11,12] proposed a design method for discrete-time robust RCS, which is based on a discrete 2D model. It converted the system design to the robust stabilization problem for a discrete 2D system. However, the whole state of the plant is needed for the design of the controller, which is unfortunately unavailable in many practical applications. A design method that employs only the output of a plant is more practical.

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This paper presents a method of designing a discrete-time robust RCS based on static output-feedback for a class of linear plants with periodic uncertainties. First, applying the lifting technique, a new discrete 2D model is established that accurately describes the characteristics of RCS and allows us to adjust control and learning preferentially. Then, by employing a 2D Lyapunov functional and the singular-value decomposition (SVD) of the output matrix, a linear-matrix-inequality (LMI) based sufficient stability condition is derived. Two tuning parameters in the LMI are used to adjust control and learning. Finally, a numerical example demonstrates the effectiveness of the design method. Throughout this paper, $\mathbf{Z}^+$ is the set of non-negative integers, $\mathbf{C}^p$ is the p -dimensional vector space over complex numbers, $\aleph$ is the linear space of all the functions from $\{0, 1, 2, 3, \dots, N-1\}$ to $\mathbf{C}^p$, $l_2(\mathbf{Z}^+, \mathbf{C}^p)$ is the linear space of square integrable functions from $\mathbf{Z}^+$ to $\mathbf{C}^p$, and $l_2(\mathbf{Z}^+, \aleph)$ is the linear space of all the functions from $\mathbf{Z}^+$ to $\aleph$.

2 Problem description

This paper considers the discrete-time RCS in Fig. 2. It is assumed that the process dynamics have been sampled by the zeroth-order hold method at a uniform rate T_s . The dynamics model of the repetitive controller $C_R(z)$ is

$$v(k) = \begin{cases} e(k), & 0 \leq k < N \\ e(k) + v(k-N), & k \geq N \end{cases} \quad (1)$$

where $e(k) = r(k) - y(k)$ is the tracking error.

In Fig. 2, the single-input single-output (SISO) compensated plant with a relative degree of zero and time-varying structured uncertainties is

$$\begin{cases} x(k+1) = (A + \delta A(k))x(k) + (B + \delta B(k))u(k) \\ y(k) = Cx(k) + Du(k) \end{cases} \quad (2)$$

where $x(k) \in \mathbf{R}^n$ is the state of the plant, and $u(k) \in \mathbf{R}$ and $y(k) \in \mathbf{R}$ are the control input and output, respectively. A , B , C and D are real constant matrices. Assume that the structured uncertainties of the plant are

$$\begin{bmatrix} \delta A(k) & \delta B(k) \end{bmatrix} = MF(k) \begin{bmatrix} N_0 & N_1 \end{bmatrix} \quad (3)$$

where M , N_0 and N_1 are known constant matrices, and $F(k)$ is a real, unknown, and time-varying matrix with Lebesgue measurable elements satisfying

$$F^T(k)F(k) \leq I, \quad \forall k \in \mathbf{Z}^+. \quad (4)$$

We make the following assumption.

Assumption 1. The uncertainties $\delta A(k)$ and $\delta B(k)$ vary periodically with the same period as that of the reference input $r(k)$, i.e.,

$$\begin{cases} \delta A(k+N) = \delta A(k) \\ \delta B(k+N) = \delta B(k) \end{cases}, \quad \forall k \in \mathbf{Z}^+. \quad (5)$$

From (3) and (5), we have

$$F(k+N) = F(k). \quad (6)$$

The linear control law based on output feedback is

$$u(k) = K_e v(k) + K_y y(k), \quad K_e, K_y \in \mathbf{R} \quad (7)$$

where K_e is the feedback gain of the repetitive controller and K_y is the output feedback gain.

This paper considers the following design problem: Design suitable control gains K_e and K_y in (7) such that the RCS in Fig. 2 is robustly stable for the admissible uncertainties (3).

Equations (1), (2) and (7) describe the dynamics of the RCS in Fig. 2 in the time domain. Note that (7) can be rewritten as

$$u(k) = K_e[e(k) + v(k-N)] + K_y y(k) = K_e v(k-N) + \{K_e r(k) + (K_y - K_e)y(k)\}. \quad (8)$$

Equation (8) shows that the control law $u(k)$ contains two kinds of information: Information on the previous period (first term on the right-hand side) and information on the present period (second term).

Since we can view the past state as a kind of experience, RC actually involves two different types of actions: Control and learning. In other words, we use the information on the present and previous periods respectively to produce the present control input. A design method that enables independent adjustment of control and learning can potentially provide better transient performance. But from (8), we cannot preferentially adjust control and learning actions by directly changing the control gains K_e and K_y in (7). To solve this problem, we present a 2D description for the RCS in Fig. 2 in the following.

First, focusing on the periodicity of RC, we employ the lifting technique^[13] (Fig. 3) to slice the time axis into intervals of length N . That converts the vector-valued discrete-time signal sequence $\{\xi(k)\}$ into the function-valued signal sequence $\{\xi_i(j)\}$. Its element is denoted as $\xi(i, j)$ in this paper. That is,

Fig. 2 Configuration of the discrete output-feedback-based repetitive-control system

Fig. 3 Mapping $\mathcal{L}_D$

$$\mathcal{L}_D : l_2(\mathbf{Z}^+, \mathbf{C}^p) \rightarrow l_2(\mathbf{Z}^+, \mathbb{N})$$

$$\xi(i, j) = \xi_i(j) = \mathcal{L}_D(\xi(k))$$

$$k = iN + j, \quad i \in \mathbf{Z}^+, \quad j \in \{0, 1, 2, 3, \dots, N-1\}$$

$\mathcal{L}_D$ is an isometric, isomorphic transformation between $l_2(\mathbf{Z}^+, \mathbf{C}^p)$ and $l_2(\mathbf{Z}^+, \mathbb{N})$.

Since the stability of the system does not depend on an exogenous signal, we set $r(k) = 0$. Then, from (1), (2) and (8), we get the following 2D representation for the RCS in Fig. 2.

$$\begin{cases} x(i, j+1) = (A + \delta A(i, j))x(i, j) + (B + \delta B(i, j))u(i, j) \\ v(i, j) = -Cx(i, j) - Du(i, j) + v(i-1, j) \end{cases} \quad (9)$$

with the 2D feedback control law of the form

$$u(i, j) = F_e v(i-1, j) + F_p Cx(i, j) \quad (10)$$

where

$$\begin{aligned} F_e &= \frac{K_e}{1 + (K_e - K_y)D} \\ F_p &= \frac{K_y - K_e}{1 + (K_e - K_y)D}. \end{aligned} \quad (11)$$

Its dynamic boundary conditions are

$$x(i+1, 0) = x(i, N), \quad i \in \mathbf{Z}^+. \quad (12)$$

In (9), the first equation describes the control action within one repetition period, and the second equation describes the learning behavior between two adjacent periods. We can adjust them preferentially by changing F_p and F_e in (10) and thereby accelerate the convergence of the tracking error. This is a big advantage over the 1D methods. Note that there is a coupling relationship between control and learning, and we cannot adjust them completely and independently. This can be observed from the second equation of (9). Meanwhile, from (11), the control gains in Fig. 2 can be rewritten as

$$\begin{aligned} K_e &= \frac{F_e}{1 + DF_p} \\ K_y &= F_e + \frac{F_p(1 - DF_e)}{1 + DF_p}. \end{aligned} \quad (13)$$

Based on the 2D model (9), the design of the robust RCS in Fig. 2 is reformulated as the following problem: Find an admissible control law (10) for discrete 2D system (9) under boundary conditions of the form (12) such that the system is robustly stable and achieves fast convergence.

3 Design of discrete output-feedback based RCS

Substituting the control input (10) into (9) yields a representation of the 2D closed-loop RCS

$$\delta\eta(i, j) = A_l \eta(i, j) + B_l \Gamma(i, j) \quad (14)$$

where

$$\begin{aligned} \delta\eta(i, j) &= \begin{bmatrix} x^T(i, j+1) & v^T(i, j) \end{bmatrix}^T \\ \eta(i, j) &= \begin{bmatrix} x^T(i, j) & v^T(i-1, j) \end{bmatrix}^T \\ A_l &= \begin{bmatrix} A + BF_p C & BF_e \\ -C - DF_p C & 1 - DF_e \end{bmatrix} \\ B_l &= \begin{bmatrix} M \\ 0 \end{bmatrix} \\ \Psi &= \begin{bmatrix} N_0 + N_1 F_p C & N_1 F_e \end{bmatrix} \\ \Gamma(i, j) &= F(i, j) \Psi \eta(i, j). \end{aligned} \quad (15)$$

From (4),

$$\Gamma^T(i, j) \Gamma(i, j) \leq \eta^T(i, j) \Psi^T \Psi \eta(i, j). \quad (16)$$

Below, we derive a sufficient robust stability condition for the closed-loop 2D system (14) by constructing a 2D Lyapunov functional and combining the SVD of the output matrix.

Definition 1^[14]. Assume that the output matrix C in (2) has full row rank and $\text{rank}(C) = m$. The SVD of C is

$$C = U \begin{bmatrix} S & 0 \end{bmatrix} V^T \quad (17)$$

where $S \in \mathbf{R}^{m \times m}$ is a diagonal matrix with positive, diagonal elements in decreasing order, $0 \in \mathbf{R}^{m \times (n-m)}$ is a zero matrix, $U \in \mathbf{R}^{m \times m}$ and $V \in \mathbf{R}^{n \times n}$ are unitary matrices.

For the SVD (17), the following lemma presents an equivalent condition for matrix equation $CX = \bar{X}C$.

Lemma 1^[15]. For a given $C \in \mathbf{R}^{m \times n}$ with $\text{rank}(C) = m$, if $X \in \mathbf{R}^{n \times n}$ is a symmetric matrix, then there exists a matrix $\bar{X} \in \mathbf{R}^{m \times m}$ such that $CX = \bar{X}C$ holds if and only if

$$X = V \text{diag}\{X_{11}, X_{22}\} V^T$$

where $X_{11} \in \mathbf{R}^{m \times m}$ and $X_{22} \in \mathbf{R}^{(n-m) \times (n-m)}$.

The following two lemmas are also employed in the derivation of the stability condition for the 2D system (14).

Lemma 2 (Schur complement^[16]). For any real matrix $\Sigma = \Sigma^T$, the following assertions are equivalent:

- 1) $\Sigma = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} < 0$
- 2) $S_{11} < 0$ and $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$
- 3) $S_{22} < 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$

Lemma 3 (S-procedure^[17]). Let $\Omega_0(x)$ and $\Omega_1(x)$ be two quadratic matrix functions over $\mathbf{R}^n$, and let $\Omega_1(x) \leq 0$ for all $x \in \mathbf{R}^n - \{0\}$. Then, $\Omega_0(x) < 0$ holds for all $x \in \mathbf{R}^n - \{0\}$ if and only if there exists an $\varepsilon \geq 0$ such that

$$\Omega_0(x) - \varepsilon \Omega_1(x) < 0, \quad \forall x \in \mathbf{R}^n - \{0\}.$$

The above lemmas lead to Theorem 1.

Theorem 1. For given positive scalars α and β , if there exist symmetric, positive-definite matrices X_{11} , X_{22} , X_2 , and arbitrary matrices W_1 and W_2 such that the LMI

$$\begin{bmatrix} -\alpha X_1 & 0 & 0 & \Theta_{14} & \Theta_{15} & \Theta_{16} \\ * & -\beta X_2 & 0 & \Theta_{24} & \Theta_{25} & \Theta_{26} \\ * & * & -I & M^T & 0 & 0 \\ * & * & * & -\alpha X_1 & 0 & 0 \\ * & * & * & * & -\beta X_2 & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (18)$$

holds, where

$$\begin{aligned} X_1 &= V \text{diag}\{X_{11}, X_{22}\} V^T \\ \Theta_{14} &= \alpha X_1 A^T + \alpha C^T W_1^T B^T \\ \Theta_{15} &= -\alpha X_1 C^T - \alpha C^T W_1^T D^T \\ \Theta_{16} &= \alpha X_1 N_0^T + \alpha C^T W_1^T N_1^T \\ \Theta_{24} &= \beta W_2^T B^T \\ \Theta_{25} &= \beta X_2 - \beta W_2^T D^T \\ \Theta_{26} &= \beta W_2^T N_1^T \end{aligned} \quad (19)$$

then the discrete RCS (14) in Fig. 2 is asymptotically stable. And the 2D feedback gains in (10) are

$$\begin{aligned} F_e &= W_2 X_2^{-1} \\ F_p &= W_1 U S X_{11}^{-1} S^{-1} U^T \end{aligned} \quad (20)$$

where U and S are defined in SVD (17).

Proof. Let

$$P_1 = X_1^{-1}, \quad P_2 = X_2^{-1}.$$

Choose a Lyapunov functional candidate to be

$$V(i, j) = V_1(i, j) + V_2(i, j) \quad (21)$$

where

$$V_1(i, j) = x^T(i, j) \frac{1}{\alpha} P_1 x(i, j) \quad (22)$$

$$V_2(i, j) = v^T(i-1, j) \frac{1}{\beta} P_2 v(i-1, j). \quad (23)$$

Consider the associated increment $\Delta V(i, j)$ as

$$\Delta V(i, j) = V_1(i, j+1) - V_1(i, j) + V_2(i+1, j) - V_2(i, j). \quad (24)$$

Along the time trajectory of (14), we have

$$\Delta V(i, j) = \tilde{\eta}^T(i, j)(-P + \hat{A}_l^T P \hat{A}_l) \tilde{\eta}(i, j) \quad (25)$$

where

$$\begin{aligned} \tilde{\eta}^T(i, j) &= \begin{bmatrix} x^T(i, j) & v^T(i-1, j) & \Gamma^T(i, j) \end{bmatrix} \\ P &= \text{diag}\left\{\frac{1}{\alpha} P_1, \frac{1}{\beta} P_2, 0\right\} \\ \hat{A}_l^T &= \begin{bmatrix} A^T + C^T F_p^T B^T & -C^T - C^T F_p^T D^T & 0 \\ F_e^T B^T & 1 - F_e^T D^T & 0 \\ M^T & 0 & 0 \end{bmatrix} \end{aligned}$$

Furthermore, (15) and (25) yield

$$\Delta V(i, j) - [\Gamma^T(i, j) \Gamma(i, j) - \eta^T(i, j) \Psi^T \Psi \eta(i, j)] = \tilde{\eta}^T(i, j) \Lambda \tilde{\eta}(i, j) \quad (26)$$

where

$$\begin{aligned} \Lambda &= -\tilde{P} + \tilde{A}_l^T \tilde{P} \tilde{A}_l \\ \tilde{P} &= \text{diag}\left\{\frac{1}{\alpha} P_1, \frac{1}{\beta} P_2, I\right\} \\ \tilde{A}_l^T &= \begin{bmatrix} \tilde{A}_{l11}^T & \tilde{A}_{l12}^T & \tilde{A}_{l13}^T \\ F_e^T B^T & 1 - F_e^T D^T & F_e^T N_1^T \\ M^T & 0 & 0 \end{bmatrix} \\ \tilde{A}_{l11}^T &= A^T + C^T F_p^T B^T \\ \tilde{A}_{l12}^T &= -C^T - C^T F_p^T D^T \\ \tilde{A}_{l13}^T &= N_0^T + C^T F_p^T N_1^T. \end{aligned}$$

Combining (16) and (26), and applying S-procedure (Lemma 3) to (26), we find that if $\Lambda < 0$, then for any $\tilde{\eta}(i, j) \neq 0$, $\Delta V(i, j) < 0$; thus the closed-loop system (14) is asymptotically stable.

Also, from Schur complement Lemma 2, $\Lambda < 0$ is equivalent to the following matrix inequality

$$\Xi = \begin{bmatrix} -\frac{1}{\alpha} P_1 & 0 & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & -\frac{1}{\beta} P_2 & 0 & \Xi_{24} & \Xi_{25} & \Xi_{26} \\ * & * & -I & \Xi_{34} & 0 & 0 \\ * & * & * & -\frac{1}{\alpha} P_1 & 0 & 0 \\ * & * & * & * & -\frac{1}{\beta} P_2 & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (27)$$

where

$$\begin{aligned} \Xi_{14} &= \frac{1}{\alpha} (A^T + C^T F_p^T B^T) P_1 \\ \Xi_{15} &= \frac{1}{\beta} (-C^T - C^T F_p^T D^T) P_2 \\ \Xi_{16} &= N_0^T + C^T F_p^T N_1^T \\ \Xi_{24} &= \frac{1}{\alpha} F_e^T B^T \\ \Xi_{25} &= \frac{1}{\beta} (1 - F_e^T D^T) P_2 \\ \Xi_{26} &= F_e^T N_1^T \\ \Xi_{34} &= \frac{1}{\alpha} M^T P_1. \end{aligned}$$

Since $X_1 = V \text{diag}\{X_{11}, X_{22}\} V^T$, from Lemma 1, there exists $\bar{X}_1$ such that

$$CX_1 = \bar{X}_1 C \quad (28)$$

and

$$\bar{X}_1^{-1} = USX_{11}^{-1}S^{-1}U^T. \quad (29)$$

Define

$$W_1 = F_p \bar{X}_1, \quad W_2 = F_e X_2. \quad (30)$$

Pre-multiplying and post-multiplying the matrix on the left-hand side of (27) by

$$\text{diag}\{\alpha X_1, \beta X_2, I, \alpha X_1, \beta X_2, I\}$$

and substituting (28) and (30) into the corresponding matrix, we have LMI (18).

So, if LMI (18) holds, then the discrete closed-loop RCS (14) is robustly stable for the admissible uncertainties (3).

Finally, we obtain (20) from (29) and (30). $\square$

Remark 1. Theorem 1 provides an LMI-based sufficient robust stability condition for discrete 2D system (9) under control law (10). The condition can be easily used to directly design the controller parameters of the RCS in Fig. 2 using the *feasp* solver in robust control toolbox^[18]. The two tuning parameters α and β in LMI (18) manipulate the preferential adjustment of control and learning. More specifically, α adjusts the weighting matrix P_1 in $V_1(i, j)$, and β adjusts the weighting matrix P_2 in $V_2(i, j)$. Note that $V_1(i, j)$ and $V_2(i, j)$ are quadratic terms related directly to control and learning. Accordingly, α and β regulate the feasible solutions F_p and F_e in (20), respectively.

In addition, from Theorem 1, we can derive a sufficient stability condition for the nominal discrete 2D closed-loop system

$$\delta\eta(i, j) = A_l \eta(i, j) \quad (31)$$

where $\delta\eta(i, j)$, $\eta(i, j)$ and A_l are defined in (14).

Corollary 1. For given positive scalars α and β , if there exist symmetric, positive-definite matrices X_{11} , X_{22} , X_2 , and arbitrary matrices W_1 and W_2 such that the following LMI holds

$$\begin{bmatrix} -\alpha X_1 & 0 & \tilde{\Theta}_{13} & \tilde{\Theta}_{14} \\ * & -\beta X_2 & \beta W_2^T B^T & \tilde{\Theta}_{24} \\ * & * & -\alpha X_1 & 0 \\ * & * & * & -\beta X_2 \end{bmatrix} < 0 \quad (32)$$

where X_1 is defined in (19), and

$$\begin{aligned} \tilde{\Theta}_{13} &= \alpha X_1 A^T + \alpha C^T W_1^T B^T \\ \tilde{\Theta}_{14} &= -\alpha X_1 C^T - \alpha C^T W_1^T D^T \\ \tilde{\Theta}_{24} &= \beta X_2 - \beta W_2^T D^T \end{aligned}$$

then the 2D system (31) is asymptotically stable. And the 2D feedback gains in (10) are

$$\begin{aligned} F_e &= W_2 X_2^{-1} \\ F_p &= W_1 USX_{11}^{-1}S^{-1}U^T. \end{aligned} \quad (33)$$

4 Numerical example

Consider the problem of designing a controller for a direct current motor driven manipulator with a proportional integral regulator. The control input is the voltage applied to the armature, and the output is the rotational torque of the manipulator. The state-space model of the motor can be described using (2). Assume that the parameters of the uncertain plant (2) are

$$\begin{cases} A = \begin{bmatrix} -0.93 & 0 \\ 0.3 & -0.85 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.6 \end{bmatrix} \\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 2 \\ M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad N_0 = \begin{bmatrix} 0 & 0.01 \\ 0 & 0.01 \end{bmatrix} \\ N_1 = \begin{bmatrix} 0 \\ 0.05 \end{bmatrix}, \quad F(k) = \begin{bmatrix} \sin \pi k & 0 \\ 0 & \sin \pi k \end{bmatrix} \end{cases} \quad (34)$$

and the sampling frequency is $f_s = \frac{1}{T_s} = 10$ Hz.

We consider the problem of tracking the reference input

$$r(k) = \sin \pi k + 0.5 \sin 2\pi k + 0.5 \sin 3\pi k \quad (35)$$

and then the repetition period of the RCS in Fig. 2 is

$$N = \frac{2}{T_s} = 20. \quad (36)$$

Choose the performance index

$$J_5 = \frac{1}{2f_s} \sum_{i=0}^4 \sum_{j=1}^{20} e^2(iN + j) \quad (37)$$

as a criterion for the selection of the tuning parameters α and β . Fig. 4 shows the effect of adjusting the tuning parameters through simulation on three parameter sets:

$$\begin{cases} \text{(a):} & \alpha = 0.6, \quad \beta = 0.05 \\ \text{(b):} & \alpha = 0.6, \quad \beta = 0.13 \\ \text{(c):} & \alpha = 0.2, \quad \beta = 0.13. \end{cases} \quad (38)$$

The performance indices are

$$J_5^a = 3.3208, \quad J_5^b = 0.8068, \quad J_5^c = 0.1841. \quad (39)$$

Note that, in Fig. 4, the tracking error in the first period shows only the control performance because there is no learning behavior within this period, and the convergence speeds of the tracking error in different periods show different learning efficiencies. Comparing the simulation results of sets (a) and (b), it is clear that the tracking speed is accelerated by tuning β from 0.05 to 0.13. And comparing the simulation results of sets (b) and (c), we know that the control performance is greatly improved by tuning α from 0.6 to 0.2. These simulation results show that adjusting α can mainly affect control, and adjusting β can mainly affect learning. Note that, due to the coupling relationship between control and learning, tuning α and β also strongly influences learning and control, respectively. This can be observed from Fig. 4. Among the three parameter sets, (c)

provides the best overall control and learning performance. In this way, we use the optimization algorithm

$$\min J_5 \quad \text{s.t. LMI (18)} \quad (40)$$

to calculate the best tuning parameters.

Fig. 4 Tracking errors for parameter sets (38)

A combination of (40) and the fixed-step method in the ranges

$$\alpha \in (0, 1], \quad \beta \in (0, 1] \quad (41)$$

yields the best parameters set

$$\alpha = 0.2, \quad \beta = 0.99. \quad (42)$$

From Theorem 1, we obtain the corresponding control gains

$$K_e = 68.9590, \quad K_y = -2.9357. \quad (43)$$

Simulation results in Fig. 5 show that the system is asymptotically stable for the admissible uncertainties and it enters into the steady state in the third period. Moreover, $J_5 = 0.0034$.

Based on the above analysis, we present a design algorithm for the output-feedback-based RCS in Fig. 2.

Algorithm 1.

- Step 1. Use (40) to find α and β that minimize J_5 .
- Step 2. Use Theorem 1 to calculate F_e and F_p .
- Step 3. Use (13) to calculate K_e and K_y .

In addition, using the guaranteed cost algorithm in [11], we carried out simulation (in Fig. 6) for the uncertain plant (34). We found that $J_5^{[11]} = 0.4248$. The comparison of the simulation results (Figs. 5 and 6) shows that the preferential adjustment of control and learning greatly improves the transient performance.

Fig. 5 Simulation results for output-feedback-based RCS for $\alpha = 0.2$ and $\beta = 0.99$

Fig. 6 Tracking error using the method in [11]

5 Conclusions

This paper described an LMI-based design method for a discrete-time robust RCS based on static output-feedback for a class of linear plants with periodic uncertainties. Exploiting the inherent 2D structural characteristics of RC, we

established a discrete 2D model, making it possible to preferentially adjust control and learning by means of the gains in the 2D control law. The stability theory of 2D systems and the SVD of the output matrix were applied to derive an LMI-based sufficient stability condition for the closed-loop system. The two tuning parameters in the condition manipulated the preferential adjustment of control and learning. Finally, a numerical example illustrated the design and tuning procedures, and simulation results demonstrated the effectiveness of the method.

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