Nonlinear Control of Magnetically-gear Drive-trains

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Abstract: The paper considers certain impedimental issues related to the use of magnetic gearbox and magnetic coupling technologies in high performance servo control systems. A prototype magnetic coupling is used as a basis for demonstrating that the underlying torque transfer characteristic is significantly nonlinear when transmitted torque approaches the maximum designed pull-out torque of the device. It is shown that linear controllers for speed control proportional plus integral (PI) and position control proportional plus derivative (PD) result in acceptable performance provided the magnetic coupling operates below 80% of designed pull-out torque. To fully compensate for the inherent nonlinearity of the torque transfer characteristic, feedback linearizing control laws and state transformations are derived resulting in exactly linear input-output characteristic for position and speed control of magnetically-gear drive-trains. With the addition of state feedback, the closed-loop dynamics for both position and speed control of a magnetically-gear drive-train can be designed to satisfy the integral of time multiplied by absolute error (ITAE) optimized linear response for a step input. Outstanding results are demonstrated through simulation and experimental real-time implementation on a demonstrator magnetically-gear drive-train.

Keywords: Magnetic gears, magnetic couplings, linear control, nonlinear control, feedback linearization.

1 Introduction

In recent years, there has been growing interest in the use of magnetic gears as replacements for conventional mechanical gears^[1–5]. In a number of application sectors, magnetic gears offer advantages over their mechanical counterparts, e.g., reduced maintenance requirements, improved reliability, physical isolation between input and output shafts, elimination of gearbox jamming, and inherent overload protection^[6]. With the emergence of rare earth permanent magnet materials, the static torque transmission capability of magnetic gears can now compete with traditional mechanical gear counterparts^[7,8]. Moreover, previously reported applications with embedded mechanical planetary gears^[9,10] can now be replaced with integrated magnetic gear counterparts. For instance, [10] with an integrated magnetic gear resulted in the “pseudo” direct drive of [11]. Magnetic gears and magnetically-gear motors and generators are being considered for many applications, e.g., electric/hybrid vehicles^[12–14], marine^[15], wind^[16–18] and tidal-turbines^[19–21].

Some impediments to the adoption of magnetic gears are the high-compliance characteristics, which currently limit their use to systems with relatively low-bandwidth dynamic transients, the potential for overloaded gears to “pole-slip”, and the inherent nonlinearity of the torque transfer characteristic. Such problems become particularly acute for applications where it is prohibitive to use load-side feedback sensors, by virtue of their proximity, reliability, connection, specification, cost or load-side working environment.

These issues, in the context of a servo speed control system, have been investigated extensively^[22–24]. The problem of position servo control of magnetically-gear drive-trains incorporating only sensors on the prime mover (i.e., no load position sensing) has been explored^[25]. Furthermore, while [26] provides remedial strategies for recovery

of a magnetic gear that has already entered a pole-slipping regime, [27] details the development of more advanced control techniques to ensure that a pole-slip regime is avoided with the introduction of a model predictive control framework for pole-slip prevention using an explicit form of model predictive control (MPC).

This paper aims to address the issue of nonlinearity in the torque transfer characteristic of a magnetic coupling. As in the previous investigations^[22–27], pertinent issues are investigated using a demonstrator drive-train incorporating a specially constructed 1:1 magnetic coupling under the control of a dSPACE hardware development platform. The use of a 1:1 magnetic coupling, as opposed to the more general case of a 1:n magnetic gearbox, is justified more fully^[23], where it is shown that the latter can be modelled by the former through use of a simple transformation.

2 Experimental magnetic coupling

An experimental magnetic coupling with a peak torque transmission capability of 5.7 N·m has been designed and constructed. There are 3×pole-pairs on both the inner-rotors and outer-rotors. A schematic of the experimental magnetic coupling manufactured magnetic coupling are shown in Figs. 1 (a) and 1 (b), respectively.

Fig. 2 shows the torque transfer characteristic of the magnetic coupling and a linearized approximation over the principal displacement angle $-30^\circ \leq \theta_D \leq 30^\circ$ (It is defined so because the magnetic coupling begins to “pole-slip”^[27]). The transmitted torque is given by $T_C = T_G \sin(p\theta_D)$, where T_G is the maximum designed torque of the magnetic coupling and p is the number of pole-pairs.

The magnetic coupling can be modelled as a classical two-inertia servo drive, where the “magnetic spring” has dynamic representation as in Fig. 3.

For a conventional two-inertia servo-drive, the shaft has stiffness K N·m/rad and is linear within its operating range.

When considering the magnetic coupling, the linear torsion spring of traditional systems is replaced by the nonlinear torque transfer function given by $T_C = T_G \sin(p\theta_D)$.

Fig. 1 Experimental coupling

Fig. 2 Static holding torque versus mechanical displacement angle

Fig. 3 Equivalent model of magnetic gear

3 Speed and position control with integral of time multiplied by absolute error optimized linear controllers

Fig. 2 shows the torque transfer characteristic of the magnetic coupling and the linearized spring constant over dis-

placement angle $-30^\circ \leq \theta_D \leq 30^\circ$. In the region up to about 50 % of rated torque, the magnetic coupling's torque transfer is linear. Outside of this range, the transfer characteristic begins to deviate from an ideal linear torsion spring. Speed and position control of the load-side of the magnetic coupling can be achieved using linear proportional plus integral/proportional plus derivative (PI/PD) control respectively, as demonstrated in Fig. 4, and is investigated in detail^[23, 25].

Fig. 4 Linear control of magnetic gear

Closed-loop transfer functions from input reference to load-side speed and position are

$$\frac{\omega_L}{\omega_{ref}} = \frac{K_I \omega_a^2}{J_M s^4 + K_P s^3 + (J_M \omega_a^2 (1+R) + K_I) s^2 + K_P \omega_a^2 s + K_I \omega_a^2} \quad (1)$$

$$\frac{\theta_L(s)}{\theta_{ref}(s)} = \frac{K_P \omega_a^2}{J_M s^4 + K_D s^3 + (J_M \omega_a^2 (1+R) + K_P) s^2 + K_D \omega_a^2 s + K_P \omega_a^2} \quad (2)$$

where K_P is the proportional gain, K_I is the integral gain, K_D is the derivative gain, and for notational convenience, the anti-resonant frequency ω_a and inertia ratio R are defined as

$$\omega_a^2 = \frac{K_{lin}}{J_L} \quad (3)$$

$$R = \frac{J_L}{J_M}$$

K_{lin} is the linearized spring constant.

Controller parameters are selected on the basis of the integral of time multiplied by absolute error (ITAE) performance index, as this provides optimum step responses for speed or position tracking on the load-side of the magnetic coupling^[23, 25]. As shown in (1) and (2), the load-side closed-loop transfer functions are defined by 4th order

polynomials without closed-loop zeros. Consequently, it is possible to equate the denominators of (1) and (2) with the optimized 4th order ITAE polynomial for a step response given by

$$D(s) = s^4 + 2.1\omega_x s^3 + 3.4\omega_x^2 s^2 + 2.7\omega_x^3 s + \omega_x^4 \quad (4)$$

where $D(s)$ is the system denominator polynomial and $\omega_x = -3\text{dB}$ bandwidth. This results in identical closed-loop load-side dynamics for both speed and position.

Simulated step responses (using Matlab/Simulink) for ITAE optimized linear controllers for 50 % input and 100 % input are shown in Fig. 5. Also shown in Fig. 5, the response of the linearized model of the magnetic coupling with a linearized spring constant is given by $K_{lin} = pT_G = 17\text{ N}\cdot\text{m}/\text{rad}$. At 50 % input, the step response is almost identical to the theoretical linear response. However, at 100 % input, the step response is characterised by substantial increases in overshoot, settling and rise times. Experimental results for 50 % input and 100 % input can be seen in Fig. 6, showing excellent agreement with the simulated step responses.

Fig. 5 Simulated step responses for linear ITAE, 50 %, and 100 % inputs

Fig. 6 Experimentally measured responses

By simulating over the full input range and calculating the instantaneous percentage error between the linear and nonlinear step responses, the error surface is obtained as shown in Fig. 7.

The step error surface as shown in Fig. 7 demonstrates that optimized linear control of the magnetic-coupling enabled drive-train becomes rapidly under-damped as the in-

put exceeds 50 %. In this case, simple linear control can lead to significant increases in step response performance metrics, such as overshoot, rise and settling times.

Fig. 7 Simulated percentage of absolute load-side step error from linear control

4 Feedback linearization

Section 3 demonstrated how the step response of the magnetic coupling can rapidly deteriorate from the ideal optimized linear response as the input approaches 100 %. Utilizing a linearized model of the nonlinear characteristics is valid at, or close to, the chosen operating point. This section proposes a nonlinear control approach that will provide a uniform step response over the entire input range, i.e., 0 % to 100 %. This implies that the error surface of Fig. 7 must be a zero flat plane for any input within the specified range of the input. This can be achieved with feedback linearization, an entirely different technique to conventional linearization via system approximation^[28]. The objective of feedback linearization is to determine both a nonlinear control law and a nonlinear state transformation that produces an exact linearization, from input-output of the nonlinear system. However, the analysis is restricted by two important considerations: 1) No account is taken of external load-side torque disturbances, this essentially restricts the nonlinear model to single input single output (SISO). 2) The condition of pole-slipping^[27] is not considered, restricting the magnetic coupling to be within the principal mechanical displacement angle range $-30^\circ < \theta_D < 30^\circ$.

The derived nonlinear control law and state transformation results in a linear system when input-output feedback linearization is applied. Furthermore, the introduction of an outer loop state variable control structure allows an optimized ITAE step response to be obtained throughout the entire input range, resulting in a zero load-side step response error surface.

A nonlinear state space representation of the magnetic coupling is

$$\begin{bmatrix} \dot{\theta}_M \\ \dot{\omega}_M \\ \dot{\theta}_L \\ \dot{\omega}_L \end{bmatrix} = \begin{bmatrix} \omega_M \\ -\frac{1}{J_M} \sin(p(\theta_M - \theta_L)) \\ \omega_L \\ \frac{1}{J_L} \sin(p(\theta_M - \theta_L)) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_M} \\ 0 \\ 0 \end{bmatrix} T_{EM}. \quad (5)$$

For notational convenience, the following substitutions

are made as

$$\begin{bmatrix} \theta_M \\ \omega_M \\ \theta_L \\ \omega_L \end{bmatrix} \equiv \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (6)$$

$$\theta_D = (\theta_M - \theta_L) \equiv x_D = (x_1 - x_3).$$

To simplify the subsequent analysis, a finite polynomial approximation is determined for the torque transfer characteristic over the first principal mechanical displacement angle as shown in Fig. 8. The approximated torque transfer characteristic of Fig. 8 over the principal displacement angle is described by

$$T_C = \gamma \theta_D - \psi \theta_D^3 \equiv \gamma x_D - \psi x_D^3 \quad (7)$$

where $\gamma = 16.9$ and $\psi = 22.4$. Fig. 8 shows both the magnetic coupling's torque characteristic T_C and its cubic approximation over the entire 2π radians of mechanical displacement angle.

Fig. 8 Torque characteristic over 2π radians and cubic approximation

With the simplified torque transfer function, the nonlinear state space model of (5) is now described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{1}{J_M} (\gamma x_D - \psi x_D^3) \\ x_4 \\ \frac{1}{J_L} (\gamma x_D - \psi x_D^3) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_M} \\ 0 \\ 0 \end{bmatrix} T_{EM}. \quad (8)$$

While satisfactory results using linearization and linear PI/PD control have shown to be achievable^[23, 25], feedback linearization is adopted to fully account for the intrinsic nonlinearity of the magnetic coupling's torque transfer characteristic.

4.1 Input to output feedback linearization: Position control case

For position control of the load-side of the magnetic coupling, the SISO nonlinear state space model is

$$\dot{x} = \begin{bmatrix} x_2 \\ -\frac{1}{J_M} (\gamma x_D - \psi x_D^3) \\ x_4 \\ \frac{1}{J_L} (\gamma x_D - \psi x_D^3) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_M} \\ 0 \\ 0 \end{bmatrix} T_{EM} \quad (9)$$

$$y = h(x) = x_3 = \{\theta_L\}.$$

Differentiating (9), we can obtain that

$$\begin{aligned} \dot{y} &= \frac{\partial h}{\partial x} f(x) + \frac{\partial h}{\partial x} g(x) T_{EM} \\ \dot{y} &= L_f h(x) + L_g h(x) T_{EM} \\ \dot{y} &= x_4 + (0) T_{EM} = x_4 \end{aligned} \quad (10)$$

and the Lie derivative $L_g h(x) = 0$. Consequently, it is necessary to differentiate until $L_g h(x) \neq 0$. Repeated differentiation of the output results in

$$\begin{aligned} \ddot{y} &= \frac{-6\psi x_2 x_D}{J_L} (x_2 - x_4) - \\ &\frac{1}{J_M J_L} (\gamma x_D - \psi x_D^3) (\gamma - 3\psi x_D^2) + \frac{6\psi x_4 x_D}{J_L} (x_2 - x_4) - \\ &\frac{1}{J_L^2} (\gamma x_D - \psi x_D^3) (\gamma - 3\psi x_D^2) + \frac{1}{J_M J_L} (\gamma - 3\psi x_D^2) T_{EM}. \end{aligned} \quad (11)$$

As T_{EM} appears at the 4th derivative, the nonlinear system of (9) has relative degree $r = 4$ and the linearizing control law is given by

$$T_{EM} = \frac{1}{L_g L_f^3 h(x)} (v - L_f^4 h(x)) \quad (12)$$

where v is a "synthetic input", and the feedback linearized dynamics from input v to output y is

$$\ddot{y} = v. \quad (13)$$

For the output $y = x_3$, the relative degree $r = 4$, which is the same as the system order ($n = 4$), and input-output feedback linearization result in complete linearization of the original nonlinear SISO system. The linearization of (12) is not in any sense an approximation, but results in a totally linear system between the output y and the synthetic input v . In general, a feedback linearizing control law can be formed from

$$\begin{aligned} T_{EM} &= \frac{1}{L_g L_f^{r-1} h(x)} (v - L_f^r h(x)) \\ L_g L_f^{r-1} h(x) &\neq 0 \\ r &\leq n \end{aligned} \quad (14)$$

and the fully linearized ($r = n$) or partially linearized ($r < n$) nonlinear system is reduced to

$$\frac{d^r y}{dt^r} = v. \quad (15)$$

Fig. 9 illustrates how feedback linearization acts as an “inner loop” creating a linear relationship between the output y and the synthetic input v . It then becomes feasible to apply many of the well-known linear control techniques for the “outer loop” control to generate v . And in this case, simple state feedback has been adopted to satisfy the optimal ITAE criterion outlined in Section 3.

The feedback linearized system has a particularly simple canonical linear state space structure which can be obtained by adopting the Byrnes-Isidori normal form^[29,30]. As a consequence of the relative degree $r = n$, the z_i dynamics possess the simple form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v \quad (16)$$

$\dot{z} = Az + bv$

where A , b are defined obviously, and the system of (16) is linear and controllable. The z states can be viewed as (fictitious) internal states of an integrator chain, and these are obtained via the state transformation

$$z = \begin{bmatrix} L_f^0 h(x) \\ L_f^1 h(x) \\ L_f^2 h(x) \\ L_f^3 h(x) \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \\ \dddot{y} \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{1}{J_L} (\gamma x_D - \psi x_D^3) \\ \frac{1}{J_L} (x_2 - x_4) (\gamma - 3\psi x_D^2) \end{bmatrix}. \quad (17)$$

Fig. 9 Feedback linearization of a nonlinear system with state feedback

4.2 Input to output feedback linearization: Speed control case

The relative degree of a nonlinear system is defined to be the value of r , for which

$$\begin{aligned} L_g L_f^{r-1} h(x) &\neq 0 \\ L_g L_f^{r-2} h(x) &= 0. \end{aligned} \quad (18)$$

Consequently, the relative degree for the speed control case is $r = 3$ with the output derivative given by

$$\dddot{y} = L_f^3 h(x) + L_g L_f^2 h(x) T_{EM} \quad (19)$$

with

$$\begin{aligned} L_f^3 h(x) &= \frac{-6\psi x_2 x_D}{J_L} (x_2 - x_4) - \\ &\quad \frac{1}{J_M J_L} (\gamma x_D - \psi x_D^3) (\gamma - 3\psi x_D^2) + \\ &\quad \frac{6\psi x_4 x_D}{J_L} (x_2 - x_4) - \\ &\quad \frac{1}{J_L^2} (\gamma x_D - \psi x_D^3) (\gamma - 3\psi x_D^2) \\ L_g L_f^2 h(x) &= \frac{1}{J_M J_L} (\gamma - 3\psi x_D^2). \end{aligned} \quad (20)$$

The necessary state transformation is given by

$$z = \begin{bmatrix} x_4 \\ J_L^{-1} (\gamma x_D - \psi x_D^3) \\ J_L^{-1} (x_2 - x_4) (\gamma - 3\psi x_D^2) \end{bmatrix}. \quad (21)$$

The zero dynamics not “reached” by the feedback linearization must be asymptotically stable for the overall linearization to be effective. For the nonlinear system under present discussion, the zero dynamics are described by

$$\frac{\partial \eta}{\partial x_2} \frac{1}{J_M} = 0 \quad (22)$$

and consequently, the zero dynamic is stable.

5 Simulation of feedback linearization for position and speed control

The general form of the feedback linearizing control law is given in (14), where recursive solution of the Lie derivatives $L_g L_f h(x)$ and $L_f h(x)$ lead to the nonlinear control law for the magnetic coupling’s load-side position as

$$\begin{aligned} T_{EM} &= \frac{J_M J_L}{(\gamma - 3\psi x_D^2)} \left(\frac{6\psi x_2 x_D}{J_L} (x_2 - x_4) + \right. \\ &\quad \frac{1}{J_M J_L} (\gamma x_D - \psi x_D^3) (\gamma - 3\psi x_D^2) - \frac{6\psi x_4 x_D}{J_L} (x_2 - x_4) + \\ &\quad \left. \frac{1}{J_L^2} (\gamma x_D - \psi x_D^3) (\gamma - 3\psi x_D^2) + v \right) \\ x_D &= (x_1 - x_3). \end{aligned} \quad (23)$$

It results in the linear relationship $\dddot{x}_3 = \ddot{\theta}_L = v$, a 4th order integrator chain. The outer control loop is now selected to satisfy the ITAE optimized linear results of Section 3. Specifically, the entire closed-loop dynamics satisfies the optimal 4th order ITAE polynomial (4). With the state transformation of (17), the closed-loop poles of (16) can be relocated to (4) using state feedback.

Feedback linearization for the speed control case produces a 3rd order linear integrator chain, and the outer loop state feedback gains are optimized for the 3rd order ITAE polynomial as

$$D(s) = s^3 + 1.75\omega_x s^2 + 2.15\omega_x^2 s + \omega_x^3 \quad (24)$$

where $D(s)$ is the system denominator polynomial and $\omega_x = -3$ dB bandwidth.

As with the position control case, state feedback provides the ITAE optimized step response for a third order system.

For comparison purposes, Fig. 10 demonstrates the simulated step responses for both position and speed control. In the position control case, the step response is linearized to a 4th ITAE response. And for the speed control case, the step response is linearized to a 3rd order ITAE response. Clearly, in each case, the load-side output is identical to the required ITAE response, validating the feedback linearization with state feedback approach.

The principal aim of the feedback linearization and state feedback scheme so far derived is to ensure that the optimized step response remains linear throughout the entire input range, but prior to pole-slipping. A plot of the step response error surface over the input range of interest is indicated in Fig. 11. Perfect linearity is obtained for the entire input operating range, with load-side percentage step error between the linearized model and feedback linearized non-linear model being identically zero throughout the entire range space. (Note all simulation results are obtained using Matlab/Simulink.)

Fig. 10 Simulated step response feedback linearization with state feedback

Fig. 11 Simulated percentage absolute load-side step error with feedback linearization

6 Experimental results for feedback linearization

The feedback linearizing plus state feedback controllers simulated in Section 5 are realized on a demonstrator drive-train, under the control of a dSPACE hardware development platform as illustrated in Fig. 12.

The experimental test rig consists of two drives and two permanent magnetic synchronous machines (PMSMs). One

machine acts as the driving motor, the other machine acts as the dynamic load torque. Both machines are under the control of a dSPACE DS1104 platform.

Fig. 12 Experimental test rig incorporating magnetic coupling and dSPACE feedback linearizing controllers

Feedback linearizing control laws of (11) and (20), and state transformations of (17) and (21) are implemented in dSPACE for experimental testing of position and speed controllers. Fig. 13 demonstrates speed responses for 25 %, 75 % and 100 % of the maximum input.

From Fig. 13, the transient response is identical for all levels of command input. However, the required state transformations and linearizing control laws have a significant degree of complexity, particularly when compared with the classical PI case.

Experimental results for the feedback linearizing position controller are shown in Fig. 14 for transient position commands of $\theta_{ref} = 25\%$, 75% and 100% of maximum input respectively. Fig. 14 demonstrates excellent agreement with the theoretically expected results.

Fig. 13 Experimentally measured speed responses 25 %, 75 %, and 100 % inputs respectively

7 Conclusions

To compensate for the nonlinearity of the magnetic coupling's torque transfer characteristic, an approach based on the use of feedback linearization and state feedback has been developed. The derived control laws and state transformations result in exact linear behavior, for both speed

and position control, between input and output. Feedback linearization forms an inner control loop that renders the nonlinear input-output dynamics into a linear chain of integrators. With a feedback linearized “inner loop”, an “outer loop” is designed to produce the overall required dynamics. As previously discussed, the ITAE step response polynomials are used to provide an optimized linear step response over the entire input operating range, prior to the point at which pole-slipping occurs in the magnetic coupling.

Fig. 14 Experimentally measured transient position responses 25 %, 75 %, and 100 % inputs, respectively

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