The spatiotemporal variation of the b -value and its tectonic implications in North China

Yujie Zheng · Shiyong Zhou

Received: 25 February 2014 / Accepted: 25 April 2014 / Published online: 29 May 2014
© The Seismological Society of China, Institute of Geophysics, China Earthquake Administration and Springer-Verlag Berlin Heidelberg 2014

Abstract In this study, we adopt an improved Bayesian approach based on free-knot B -spline bases to study the spatial and temporal distribution of the b -value. Synthetic tests show that the improved Bayesian approach has a superior performance compared to the Bayesian approach as well as the widely used maximum likelihood estimation (MLE) method in fitting the real variation of b -values. We then apply the improved Bayesian approach to North China and find that the b -value has a clear relevance to seismicity. Temporal changes of b -values are also investigated in two specific areas of North China. We interpret sharp decreases in the b -values as useful messages in earthquake hazard analysis.

Keywords Improved Bayesian approach · B -splines · b -value · Frequency–magnitude distribution · North China · Earthquake hazard analysis

1 Introduction

Ever since Gutenberg and Richter (1944) observed the empirical law of the magnitude–frequency distribution:

$$\log_{10}N(M) = a - bM, \quad (1)$$

where N is the cumulative number of earthquakes with magnitude $\geq M$, the coefficient b has been playing a central role in earthquake hazard models. The coefficient

b corresponds to the slope of the power law and describes the size distribution of events, i.e., a high b -value indicates a larger proportion of small events and vice versa. Results from rock fracture experiments have indicated that the b -value is inversely dependent on the applied shear stress (Amitrano 2003; Scholz 1968; Schorlemmer et al. 2005); therefore, regions with high stress level often have relatively low b -values. Temporal change of the b -value has also been attracting attention since differences in the b -value were found between fore- and aftershock sequences (Suyehiro 1966). Evidences from observational data as well as laboratory and numerical models have all suggested a clear relevance between the b -value and seismicity. Therefore, it is reasonable to interpret the b -value as an indirect “stressmeter” in the Earth’s crust, based on which an earthquake hazard model can be built (Wiemer and Schorlemmer 2007).

Numerous studies have been performed in the past 40 years to investigate the spatiotemporal variation of the b -value and its probable physical properties. Most of these studies adopt the maximum likelihood estimation (MLE) approach to calculate the b -value. However, the MLE approach deals poorly with the heterogeneity of earthquake hypocenter distributions. Besides, the choice of the size of the space–time window is quite subjective and can bring difficulties in evaluating the error of estimates. A Bayesian approach with smoothness prior was suggested by Ogata et al. (1991) as an alternative approach to obtain the b -value, and it performs significantly better than the MLE approach in overcoming the above-mentioned drawbacks. However, since the Bayesian approach is built upon B -spline bases with equally spaced knots (Splines are piecewise-defined smooth curves. The joined points where polynomial segments meet are known as knots), its ability to deal with the clustered nature of earthquake occurrence

Y. Zheng · S. Zhou (✉)
School of Earth and Space Sciences, Peking University,
Beijing 100871, China
e-mail: zsy@pku.edu.cn

Y. Zheng
e-mail: zyjatpku@163.com

is relatively limited. Hence, our goal is to improve the original Bayesian approach and make it more adaptive to real earthquake catalogs.

A typical earthquake catalog has several areas of high activity and areas consisting of medium or low seismicity. Including only a few widely spaced knots generally leads to very smooth curves, which can result in the blurring of local variation of the b -value. On the other hand, for multiple knots in close proximity, it often occurs that too few earthquakes are utilized to calculate b -values in the low seismicity areas and, therefore, leads to considerable errors. Hence, a local rather than a global distribution of knots may give a better representation of the b -value. In this study, we improve the Bayesian approach by alternating splines with uniform knot distribution to free-knot splines. This improved approach is tested with a synthetic catalog and applied to North China to obtain the spatio-temporal variation of the b -value.

2 Method

In this study, we use free-knot spline functions to estimate and interpolate b -values in space. Since many parameters are required for the spline function, we maximize the log-likelihood function which measures data fitting to obtain parameter estimations. On the other hand, we adopt the objective Bayesian method to avoid unnecessary fluctuation of the estimated spline surface.

2.1 The penalized log-likelihood function

The Gutenberg–Richter's law (1) indicates the probability density function of the earthquakes with magnitude M as below:

$$f(M) = \beta e^{-\beta(M-M_c)}, M > M_c, \quad (2)$$

where $\beta = b \ln 10$ and M_c is the minimum magnitude of a complete catalog, above which almost all the earthquakes are detected.

Since we assume that b is dependent on the location of the space, the current likelihood function can be rewritten as:

$$L(\theta) = \prod_{i=1}^n \beta_{\theta}(x_i, y_i) \exp \{-\beta_{\theta}(x_i, y_i)(M_i - M_c)\}. \quad (3)$$

The parameterization of the function $\beta(x, y)$ is carried out by

$$\beta_{\theta}(x, y) = e^{\phi_{\theta}(x, y)}, \quad (4)$$

where ϕ is the 2-D B -spline function:

$$\phi_{\theta}(x, y) = \sum_{l=0}^{L+3} \sum_{m=0}^{M+3} c_{l,m} F_l(x) G_m(y), \quad (5)$$

where the parameter $\theta = \{c_{l,m}\}$ is a set of coefficients and the functions F_l and G_m are cubic B -spline bases which can be deduced from the following recurrence scheme for the $(k-1)$ th degree (normalized) B -spline basis $N_j^k(x)$ (De Boor 1978):

$$N_j^1 = \begin{cases} 1 & (\xi_j \leq x < \xi_{j+1}) \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

and

$$N_j^k(x) = \frac{\xi_{j+k} - x}{\xi_{j+k} - \xi_{j+1}} N_{j+1}^{k-1}(x) + \frac{x - \xi_j}{\xi_{j+k-1} - \xi_j} N_j^{k-1}(x), \quad (7)$$

where ξ_j represent knot sequences. Instead of using equally spaced knots for the sake of simplicity, we adopt variable knot intervals which can better adapt to the heterogeneity of earthquake hypocenter distributions. The free-knot spline functions are extremely useful when dealing with irregularly spaced data. However, as can be seen in Eq. (5), a large number of parameters are required for the estimation, resulting in a rapidly fluctuated spline surface. Thus, to avoid too much rapid local variation, we adopt the roughness penalties or smoothness constraints of the spline function (5), which measure the local variation of a surface

$$\varphi_1(\theta|w_1) = \int_v w_1 \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right] dx dy \quad (8)$$

and

$$\varphi_2(\theta|w_2) = \int_v w_2 \left[\left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2} \right)^2 \right] dx dy, \quad (9)$$

where the non-negative constants w_1 and w_2 are weights controlling the strength of the penalties. Here, we consider an isotropic plane so that x and y directions are not distinguished. Thus, given suitable weights w_1 and w_2 , the estimates of the parameters $\theta = \{c_{lm}\}$ can be obtained by maximizing the penalized log-likelihood function (Ogata et al. 1991)

$$\begin{aligned} Q(\theta|w_1, w_2) &= \log L(\theta) - \varphi_1(\theta|w_1) - \varphi_2(\theta|w_2) \\ &= \sum_{i=1}^n [\phi_{\theta}(x_i, y_i) - e^{\phi_{\theta}(x_i, y_i)}(M_i - M_c)] \\ &\quad - \varphi_1(\theta|w_1) - \varphi_2(\theta|w_2) \end{aligned} \quad (10)$$

for the data set $\{(x_i, y_i, M_i), i = 1, 2, \dots, n\}$.

2.2 The objective Bayesian method

The smoothness constraints are set in the form of (8) and (9) for specific reasons: the first derivative norm represents the total fluctuation of the function, thus by minimizing (8), unnatural oscillations can be efficiently suppressed; the second derivative norm is a measure of curvature, and it is minimized as in ordinary methods of smoothing and interpolation (Inoue 1986). Therefore, the determination of weights w_1 and w_2 are crucial in our method.

In order to obtain the optimal weights, we introduce the Bayesian Likelihood (Akaike 1980; Good 1965) into our method. The exponential of the negative sum $-(\varphi_1 + \varphi_2)$ is considered to be proportional to a prior distribution $\pi(\theta|w_1, w_2)$ which is characterized by the hyperparameters w_1 and w_2 . The penalties are quadratic with respect to coefficients $\theta = \{c_{l,m}\} = (c_p)_{p=1,2,\dots,P}$:

$$\frac{1}{2}\theta\Sigma\theta^T = \varphi_1(\theta|w_1) + \varphi_2(\theta|w_2), \quad (11)$$

where Σ is a non-negative definite matrix. Since the matrix Σ turns out to be degenerate in the present case, we make some adjustments to the parameter θ and the prior distribution π can be written as:

$$\pi(c^r|w_1, w_2, c_p) = \frac{(\det \Sigma_r)^{1/2}}{\sqrt{2\pi}^{P-1}} e^{-\theta\Sigma\theta^T/2}, \quad (12)$$

where c_p is the last component of θ , c^r is the rest, and Σ_r is the cofactor of the last diagonal element of Σ .

Then, we can consider the Bayesian Likelihood

$$L(w_1, w_2, c_p) = \int L(\theta)\pi(c^r, c_p|w_1, w_2)dc^r. \quad (13)$$

To obtain w_1 , w_2 , and c_p , we need to maximize the Bayesian Likelihood L . Nevertheless, the integral in (13) is not analytical, and hence we have to apply the Gaussian approximation method (Ishiguro and Sakamoto 1983) to get an approximated log Bayesian Likelihood (see Ogata et al. (1991) for details)

$$\log L(w_1, w_2, \hat{c}_p) = Q(\hat{\theta}|w_1, w_2) + \frac{1}{2}\log\{\det \Sigma_r\} - \frac{1}{2}\log\{\det \mathbf{H}_r\}, \quad (14)$$

where $\hat{\theta}$ is the vector that maximizes Q for fixed w_1 and w_2 , and $\mathbf{H}$ is the Hessian matrix of the penalized log-likelihood at $\hat{\theta}$.

An iteration algorithm is used to calculate the b -value. As shown in Fig. 1, we first maximize Q in (10) with respect to θ for fixed weights w_1 and w_2 . Then, we maximize (14) with respect to w_1 and w_2 for fixed θ obtained from the previous step. Further, we substitute the newly

Fig. 1 A flow chart of the steps used to calculate the b -value

calculated weights into (10) and get a better estimation of θ . The loop is terminated when the b -value converges. Since both of the penalized log-likelihood and the log Bayesian Likelihood are nonlinear, we use Davidon–Fletcher–Powell method (Fletcher and Powell 1963) for maximization. The convergence is quite rapid in spite of the large dimension of θ which shows the stability of our calculations.

For the calculation of temporal b -value changes, we use the same method described above, straightforwardly replacing epicentral coordinate (x, y) by time t .

3 Synthetic test

To validate our approach of using free-knot B -spline functions to calculate the variations of b -values, we simulate a synthetic catalog of events with a minimum magnitude of completeness $M_c = 2.0$. Magnitudes above M_c follow a Gutenberg–Richter law with the b -value varying within the range of 0.6–1.4. For simplicity, we adopt a 1-D temporal model with piecewise rate of earthquake occurrence:

$$\lambda = \begin{cases} 2500 \text{ per year,} & t \in (2.5, 7.5) \\ 1000 \text{ per year,} & t \in [0, 2.5] \cap [7.5, 12.5] \cap [17.5, 20]. \\ 200 \text{ per year,} & t \in (12.5, 17.5) \end{cases} \quad (15)$$

Figure 2a shows the magnitude against occurrence time of events in the simulation. The three rates are designed to represent high, medium, and low level of seismicity, respectively. The real rate of earthquake occurrence in North China is used as a reference in designing the three rates. The dashed line in Fig. 2b depicts the variation of the b -value in the studied time range. The temporal change of the b -value is also characterized by a piecewise function:

Fig. 2 **a** Earthquake magnitudes in the synthetic catalog against occurrence time. The 2.5th to the 7.5th year and the 12.5th to the 17.5th year are designed to represent high and low seismicity time zone, respectively. **b** Comparison between the true variations of the b -value, MLE, the Bayesian approach with global spacing 0.4 and 0.2, respectively, and the improved Bayesian approach. **c** Comparison of estimation errors of MLE, the Bayesian approach with a global spacing 0.4, the Bayesian approach with a global spacing 0.2 and the improved Bayesian approach

$$b(t) = \begin{cases} 1 + 0.2 \sin(2\pi t) + 0.2 \sin(3\pi t), & t \in [4, 6] \\ 1 + 0.2 \sin(\pi t) + 0.2 \sin(1.5\pi t), & t \notin [4, 6] \end{cases} \quad (16)$$

Here, the short-period variation zone is designed to reveal the probable relationship between grid spacing and a fine local variation of the b -value.

The widely used MLE method with a constant sample volume, the Bayesian approach with fixed grid spacing of 0.4 and 0.2, and our modified Bayesian approach are all applied to this synthetic catalog, and the results are shown in Fig. 2b. The estimation errors are presented in Fig. 2c. As can be seen in Fig. 2c, the Bayesian approach performs significantly better than the MLE method in low seismicity areas and generally outperforms the MLE method in the

medium and high seismicity zones. However, it should be noted that due to the wide grid spacing, the Bayesian approach with knot interval of 0.4 fails to give a precise estimation of the b -value where there is a fine local variation of the b -value (between the 4th and the 6th year, high seismicity period), though it gives the best estimation in all the rest time period. Apparently, the improved Bayesian approach has a superior performance. Its flexible grid spacing which can be adapted to the local density of events allows it to combine the advantages of both two grid spacings. Furthermore, the estimation error of the improved Bayesian approach is generally less than 0.05 in both high and medium seismicity zones and less than 0.1 in the low seismicity zone, which shows the reliability of our method.

4 Application to North China

4.1 Data

In this study, we focus on the region bounded by 108°E and 124°E meridians, and 32°N and 42°N parallels, which

Fig. 3 Epicenter distribution of earthquakes in North China. Epicenters of $M \geq 2.3$ events from 1985.1 to 2012.12 are shown as dots, scaled proportional to magnitude. Stars indicate the locations of major cities in North China. Major tectonic units in North China are labeled, including the Tanlu Fault Zone, Luxi Uplift (LU), Taihang Mountains (TM), Yan Mountains (YM), Ordos Plateau, Yinchuan–Hetao (Y–H), and Shaanxi–Shanxi (S–S) rift systems

Fig. 4 **a** Magnitude against occurrence time of events in the subsets of the CENC catalog. M_c is indicated with a thick black line. **b** Frequency–magnitude distribution of the CENC catalog in the investigated spatiotemporal range. M_c is indicated with an arrow

includes most of North China. The earthquake catalog we use is compiled by China Earthquake Networks Center (CENC) between the year 1985 and the year 2012. Figure 3 shows an epicenter map of North China. The minimum magnitude of completeness is estimated using the Maximum curvature-method (MAXC) in combination with the bootstrap and add a correction value of 0.2 (Wiemer and Wyss 2000; Woessner and Wiemer 2005). Figure 4a shows the plot of magnitude against occurrence time of events, and Fig. 4b depicts the cumulative frequency–magnitude distribution of events of the catalog investigated. Taking spatial variation of the minimum magnitude of completeness into consideration, we adopt an overall $M_c = 2.3$, which gives us 12,468 effective events in the considered space–time range.

4.2 The spatiotemporal variation of the b -value

In the present paper, we study the spatiotemporal variation of the b -value in North China. To obtain the spatial variation, we divide the area of North China into $L \times M = 16 \times 10$ rectangular subareas with sides of 1° in

both meridian and parallel. Due to the limitation of the number of events that we can use and the need to guarantee enough spatial resolution, we adopt uniform grid instead of non-uniform grid here, and the result is shown in Fig. 5a. The b -value varies significantly from $b \approx 0.60$ – 1.70 in North China. There are mainly three regions that have relatively low b -values: Baotou Region (on the North-West corner), regions A and B (marked by polygons shown in Fig. 5a). According to the inverse relationship between the b -value and the shear stress accumulation, these three regions, therefore, may have relatively higher stress accumulation than the other areas. In regions with low seismicity, the b -value appears to be generally above 1.0.

Further, we investigate the temporal changes of the b -value in regions A and B. The knot sequences are calculated by an adaptive algorithm which guarantees at least 30 events in each grid with a minimum grid spacing of 0.1 . Figure 6 shows the distribution of the knot sequences and the number of events located within each grid. Therefore, the error of the estimated b -value should be less than 0.1 according to the results shown by the synthetic test. Figure 5b, c illustrates the temporal changes of the b -value in

Fig. 5 **a** Spatial variation of the b -value in North China. Filled yellow circles represent earthquakes with magnitudes over 5.5 from 1985 to 2012. **b** Temporal changes of the b -value in region A. The red dashed line depicts the result calculated using earthquake recordings before the year 1998. **c** Temporal changes of the b -value in Region B. Regions A and B are marked by the polygons shown in Fig. 5a

Fig. 6 Number of events evaluated in each grid against the distribution of the knot sequences when calculating the temporal change of b -values in **a** region A and **b** region B

the investigated regions. Region A is located in the Zhangjiakou-Bohai Tectonic Zone which is one of the areas that have the highest seismicity in North China. As shown in Fig. 5b, the b -value in region A is quite low from 1985 to 2000 when seven $M \geq 5.5$ events took place. After 2000, the b -value starts to rise and remains above 1.0 for about 10 years. No moderate or strong earthquakes happened in this time period. Region B corresponds to the North China Plain. This region, unlike region A, does not possess any $M \geq 5.0$ events in the investigated time range. The temporal change of the b -value in this region is much milder compared to that of region A. As can be seen in Fig. 5c, the b -value remains within the range 0.85–1.15. Though a drop of 0.3 in the b -value took place from 1985 to 2005, the b -value is still above 0.85 at its lowest point,

whereas the b -value is generally below 0.8 when $M \geq 5.5$ events took place in region A.

5 Discussion and conclusions

We present an improved Bayesian approach based on free-knot B -spline bases to calculate the spatiotemporal variation of the b -value. The result of the synthetic test shows that our approach has the best performance in dealing with the heterogeneity of earthquake hypocenter distributions. The application to North China shows significant variations in frequency–magnitude distribution as a function of both space and time.

North China, though one of the oldest cratons in the world (Liu et al. 1992; Zheng et al. 2004), is a tectonically complex and important region. It consists of three major parts: the eastern North China Craton (NCC), the western NCC, and the central NCC in between. The eastern NCC is characterized by widespread thinned lithosphere (60–100 km) in contrast to the central and western parts where both preserved thick and dramatically thinned lithosphere coexist (Chen et al. 2009). The thick lithosphere (>200 km) underlies the stable Ordos Plateau, where there is little seismicity, magmatic activity, and relative low heat flow (Hu et al. 2000; Wang et al. 1996). The thinned lithosphere (up to 80 km) is found around the late Cenozoic Yinchuan–Hetao and Shaanxi–Shanxi rift areas (see Fig. 3 for locations), with sharp changes occurring over a lateral distance of 200–400 km (Chen et al. 2008). As can be seen in Fig. 5a, the two low b -value anomalies (Baotou region and Yuncheng region) in the western and central NCC locate on the two above-mentioned rift areas where significant higher seismicity can be found. Given that historically only two M_6 earthquakes took place in Yuncheng Region, the low b -value may indicate high stress level in this region. The highest b -value occurs around the Yan Mountains in the eastern NCC where seismicity is much lower than the surrounding areas. Central Shandong area in the eastern NCC, in contrast, has relatively low b -values. Since Shandong area has not suffered from any $M > 5.0$ events since 1970s and locates on the active Tanlu Fault Zone, the low b -value is likely to suggest that the stress level in this area is relatively high. The major two low b -value areas (Region A and B) locate on the boundary of the western and central NCC and the central and the eastern NCC, respectively. Both regions enjoy high seismicity, and most of the $M > 5.5$ events took place in region A, including the largest event, the 1998 $M_{6.3}$ Zhangbei earthquake, in the investigated earthquake catalog.

While spatially highly variable, b -values in most regions are quite stationary with time (Schorlemmer et al. 2004; Wiemer and Wyss 2002). As can be seen in Fig. 5c, the b -value remains in close proximity to 1.0 in Region B from 1985 to 2012. In some regions, however, the temporal variation of the b -value can be significant, providing us abundant information about recent tectonic events that took place in the vicinity (Nanjo et al. 2012; Wiemer et al. 1998). As can be seen in Fig. 5b, the b -value in region A shows considerable fluctuation with time. Besides, all $M > 5.5$ events took place when the b -value is smaller than 0.8. Before the 1998 $M_{6.3}$ Zhangbei earthquake, the b -value experienced a significant drop of 0.5 within 2 years time. However, it is not yet safe to say that the drop in the b -value is an indication of the upcoming $M_{6.3}$ earthquake for the increase of the mean magnitude will lead to decrease of the b -value as well. Therefore, we further

investigate the temporal variation of the b -value before the 1998 $M_{6.3}$ event, and the result shows that the b -value still undergoes a drop of 0.3 within 1 year (Fig. 5b, shown as the red dashed line). Hence, a low b -value accompanied by a sharp decrease may reflect that the shear stress accumulation is close to the failure strength of the local faults and can be used in the earthquake hazard analysis.

The above analysis shows that spatiotemporal variation of the b -value provides valuable insights into the study of earthquake hazard models. It is possible to establish a 3-D space–time model of b -values by carrying out the similar procedure used in the present paper. However, due to the lack of events, 3-D model is not feasible in this study. Besides, 3-D models are much more exhaustive than 1- or 2-D models. Therefore, in this paper, we first calculate the spatial distribution of the b -value. Then, we focus specifically on the low b -value regions and calculate the temporal change of b -values in these regions. By combining spatial and temporal variations of the b -value in this way, we have effectively used events in the investigated earthquake catalog and obtained equally abundant useful information regarding earthquake hazards.

Acknowledgments We are grateful to Prof. Jiancang Zhuang for his constructive comments and suggestions and China Earthquake Networks Center for providing the earthquake catalog investigated. This research is jointly funded by the National Natural Science Foundation of China (Grant No.41274052) and the Seismological Research Project of China (Grant No. 201208009). The first author is financially supported by Peking University President's Research Funding for undergraduate students (2012–2013).

References

- Akaike H (1980) Likelihood and the Bayes procedure. *Trabajos de estadística y de investigación operativa* 31(1):143–166
- Amitrano D (2003) Brittle-ductile transition and associated seismicity: experimental and numerical studies and relationship with the b value. *J Geophys Res Solid Earth* 108(B1):2044. doi:[10.1029/2001JB000680](https://doi.org/10.1029/2001JB000680)
- Chen L, Tao W, Zhao L, Zheng T (2008) Distinct lateral variation of lithospheric thickness in the Northeastern North China Craton. *Earth Planet Sci Lett* 267(1):56–68
- Chen L, Cheng C, Wei Z (2009) Seismic evidence for significant lateral variations in lithospheric thickness beneath the central and western North China Craton. *Earth Planet Sci Lett* 286(1):171–183
- De Boor C (1978) A practical guide to splines. Springer-Verlag, New York
- Fletcher R, Powell MJ (1963) A rapidly convergent descent method for minimization. *Comput J* 6(2):163–168
- Good IJ (1965) The estimation of probabilities: an essay on modern Bayesian methods. MIT Press, Cambridge, MA
- Gutenberg B, Richter CF (1944) Frequency of earthquakes in California. *Bull Seismol Soc Am* 34(4):185–188
- Hu S, He L, Wang J (2000) Heat flow in the continental area of China: a new data set. *Earth Planet Sci Lett* 179(2):407–419

- Inoue H (1986) A least-squares smooth fitting for irregularly spaced data: finite-element approach using the cubic -spline basis. *Geophysics* 51(11):2051–2066. doi:[10.1190/1.1442060](https://doi.org/10.1190/1.1442060)
- Ishiguro M, Sakamoto Y (1983) A Bayesian approach to binary response curve estimation. *Ann Inst Stat Math* 35(1):115–137
- Liu D, Nutman A, Compston W, Wu J, Shen QH (1992) Remnants of ≥ 3800 Ma crust in the Chinese part of the Sino-Korean craton. *Geology* 20(4):339–342
- Nanjo K, Hirata N, Obara K, Kasahara K (2012) Decade-scale decrease in b value prior to the *M*₉-class 2011 Tohoku and 2004 Sumatra quakes. *Geophys Res Lett* 39(20):L20304
- Ogata Y, Imoto M, Katsura K (1991) 3-D spatial variation of *b*-values of magnitude–frequency distribution beneath the Kanto District, Japan. *Geophys J Int* 104(1):135–146. doi:[10.1111/j.1365-246X.1991.tb02499.x](https://doi.org/10.1111/j.1365-246X.1991.tb02499.x)
- Scholz CH (1968) The frequency–magnitude relation of microfracturing in rock and its relation to earthquakes. *Bull Seismol Soc Am* 58(1):399–415
- Schorlemmer D, Wiemer S, Wyss M (2004) Earthquake statistics at Parkfield: 1. Stationarity of b values. *J Geophys Res Solid Earth* (1978–2012) 109(B12):B12307
- Schorlemmer D, Wiemer S, Wyss M (2005) Variations in earthquake-size distribution across different stress regimes. *Nature* 437(7058):539–542
- Suyehiro S (1966) Difference between aftershocks and foreshocks in the relationship of magnitude to frequency of occurrence for the great Chilean earthquake of 1960. *Bull Seismol Soc Am* 56(1):185–200
- Wang J, Huang S, Chen M (1996) Terrestrial heat flux map, Geophysical Atlas in China. Geophysical Publishing House, Beijing, p 102
- Wiemer S, Schorlemmer D (2007) ALM: an asperity-based likelihood model for California. *Seismol Res Lett* 78(1):134–140. doi:[10.1785/gssrl.78.1.134](https://doi.org/10.1785/gssrl.78.1.134)
- Wiemer S, Wyss M (2000) Minimum magnitude of completeness in earthquake catalogs: examples from Alaska, the Western United States, and Japan. *Bull Seismol Soc Am* 90(4):859–869. doi:[10.1785/0119990114](https://doi.org/10.1785/0119990114)
- Wiemer S, Wyss M (2002) Mapping spatial variability of the frequency–magnitude distribution of earthquakes. *Adv Geophys* 45:259–302
- Wiemer S, McNutt SR, Wyss M (1998) Temporal and three-dimensional spatial analyses of the frequency–magnitude distribution near Long Valley Caldera, California. *Geophys J Int* 134(2):409–421
- Woessner J, Wiemer S (2005) Assessing the quality of earthquake catalogues: estimating the magnitude of completeness and its uncertainty. *Bull Seismol Soc Am* 95(2):684–698
- Zheng J, Griffin W, O'Reilly SY, Lu F, Wang C, Zhang M, Wang F, Li H (2004) 3.6 Ga lower crust in central China: new evidence on the assembly of the North China Craton. *Geology* 32(3):229–232