Recent advances of computing coseismic deformations in theory and applications

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Abstract This paper reviews the recent advances in computing coseismic deformations, and their contributions to seismology and geodesy. At first, an overview on the history of the dislocation theory development is given in the introduction section. Then, emphasis are given on some new developments through few examples in the following sections, such as the new dislocation theory for a 3D Earth model, a new computing scheme on coseismic deflection change of vertical, the relation of dislocation Love number and the conventional Love numbers, the application of dislocation theory applied in satellite gravity observations, the coseismic deformations observed by GRACE, and a new method to determine dislocation Love numbers by GRACE. Furthermore, some advanced theoretical and cases studies are introduced to illustrate how dislocation theory is important in interpret geodetic data, or invert seismic slip for co- and post-seismic processes, using seismic and geodetic data. Final remarks are given in the last section, with discussions, conclusions, comments on existing problems, and expected methods to solve them.

Keywords Dislocation theory · Coseismic deformations · Earthquake · Displacement · Gravity · Modern geodesy

1 Introduction

Modern geodetic techniques (GPS, InSAR, GRACE, altimetry, etc.) allow better detection of coseismic deformations such as displacement, strain, geoid, and gravity

changes. Such geophysical–geodetic information is useful for studying seismic mechanisms, seismic fault inversion, Earth structure, and so on. A quasi-static dislocation theory is necessary to properly apply the observed geophysical phenomena to invert the seismic parameters or interpret the geodetic observation data.

One of the most advanced developments in seismology in the last decade was that an earthquake was found to occur along a fault. Then seismology describing source mechanics and static/dynamic movement of the fault has been quickly established and well developed. At the same time, the quasi-static dislocation theory (simply called dislocation theory) has been presented. Due to its analytical and static property, it is somehow different with the wave propagation theory in seismology. In 60s of last century, numerous studies have been undertaken by many scientists to study coseismic deformation in a half-space Earth model, among them are Steketee (1958), Maruyama (1964), Press (1965), Jovanovich et al. (1974a, b), and Okada (1985).

Those studies presented analytical expressions for calculating surface displacement, tilt, and strain resulting from various dislocations buried in a semi-infinite (half-space) medium. Especially, Okada (1985) summarized previous studies and presented a complete set of analytical formulae for calculating these geodetic deformations. Okubo (1991, 1992) proposed closed-form expressions to describe potential and gravity changes resulting from dislocations. Because of their mathematical simplicity, these dislocation theories (e.g., Okada 1985; Okubo 1991, 1992) have been applied widely to study or invert seismic faults. However, the validity of these theories is strictly limited to a near field because Earth's curvature and radial heterogeneity are ignored. Wang et al. (2006) presented a new theory to compute co- and post-seismic deformations for a layered

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half-space Earth model, which is now applied in practical computation of coseismic deformation, or fault slip model inversions. Modern geodesy can detect and observe far-field crustal deformation, even a global coseismic deformation. Consequently, a dislocation theory for a more realistic Earth model is demanded to interpret the far-field deformation.

Efforts to develop formations for a spherical Earth model have been advanced through numerous studies (e.g., Ben-Menahem and Singh 1968; Ben-Menahem and Israel 1970; Smylie and Mansinha 1971). Such studies have revealed that Earth's curvature effects are negligible for shallow events, whereas vertical layering might have considerable effects on deformation fields.

For a more realistic Earth model, e.g., the PREM model (Dziewonski and Anderson 1981), Saito (1967) presented a theory to calculate the amplitudes of free oscillations caused by a point source in a spherically symmetric sphere. He expressed his results in terms of normal mode solutions and source functions. Rundle (1982) studied viscoelastic gravitational deformation by a rectangular thrust fault in a layered Earth. Pollitz (1992) solved the problem of regional displacement and strain fields induced by a dislocation in a viscoelastic, non-self-gravitating model. Pollitz et al. (2004) explained strain accumulation in San Francisco Bay. Ma and Kuszniir (1994) modified elastic dislocation theory to derive subsurface displacements for faults in a three-layer elastic-gravitational medium and their application to examine coseismic and post-seismic surface and subsurface displacement during continental extensional faulting. Piersanti et al. (1995), Sabadini et al. (1995), and Soldati et al. (1998) studied the gravity, displacement, and rates induced by a dislocation in viscoelastic, stratified Earth models, accounting for sphericity and self-gravitation using a self-consistent approach. They produced results of surface displacement and velocities in the near field and far field for various viscosity profiles in the mantle. Antonioli et al. (1998) discussed stress diffusion following large strike-slip earthquakes to observe the spherical and layered effects by comparing results of spherical and flat-Earth models. Nostro et al. (1999, 2001) studied the effects of Earth's sphericity and layering through coseismic and post-seismic deformations. Banerjee et al. (2005) and Hearn and Burgmann (2005) dealt with the same issue for the 2004 Sumatra earthquake and a strike-slip source, respectively. All of those studies used the normal mode approach; some investigations were performed for either specified modes (Antonioli et al. 1998; Nostro et al. 1999) or specified seismic source, e.g., strike-slip fault (Antonioli et al. 1998). Wang (1999) investigated surface vertical displacements, potential perturbations, and gravity changes of a viscoelastic Earth model induced by internal point dislocations. On the other hand, the computing accuracy of the normal-

mode method is a suspect because it has intrinsic numerical difficulties and because it requires artificial assumptions of compressibility and layer numbers (Tanaka et al. 2006). Tanaka et al. (2006) studied the same problem using a new method that overcomes some previous numerical difficulties and guarantees accuracy. Fu and Sun (2008) presented a new theory for computing coseismic gravity changes in a three-dimensional inhomogeneous Earth model. Melini et al. (2008) presented a new method for computing post-seismic observables in spherical geometry.

Pollitz (1996) investigated coseismic displacement and strain from earthquake faulting on a layered spherical Earth using normal-mode technique, ignoring Earth's self-gravitation. He asserted that the effect of sphericity is generally less than about 2 % of maximum deformation within 100 km of an earthquake source at crustal depths, and that up to 20 % error would be introduced if the Earth's layered structure was ignored. Sun and Okubo (2002) study comparing discrepancies between a half-space and a homogeneous sphere (accounting for self-gravitation) and between a homogeneous sphere and a stratified sphere indicates that both curvature and vertical layering exert marked effects on coseismic deformation, reaching 25 % in maximum. Amelung and Wolf (1994) studied the spherical effect problem for surface loading. They compared spherical Earth models with incremental gravitational force (IGF) and plate Earth models without IGF and found that errors resulting from neglect of sphericity and the IGF partially compensate each other. Sabadini and Vermeersen (1997) investigated the influence of lithospheric and mantle stratification on global coseismic and post-seismic deformation based on the normal-mode technique. They found that the mantle viscosity structure has a major influence on post-seismic deformation in a far field.

On the other hand, before and after great earthquakes, remarkable coseismic displacements have been observed frequently in the last several decades, such as the Taiwan Chi-Chi earthquake ($M7.6$) in 1999 (Yang et al. 2000), the Kunlun earthquake ($M7.8$) in 2001 (Lin et al. 2002), and the Tokachi-Oki earthquake ($M8.0$) in 2003 (Imanishi et al. 2004). These observations indicate that the dominating deformations appear in the near field and attenuate rapidly with increased epicentral distance. In a far field, e.g., beyond 1,000 km distant from the epicenter, coseismic displacements are usually difficult to detect. However, because of well-developed geodetic observation techniques and the extremely large seismic event, evident geodetic deformations caused by the Sumatra–Andaman Earthquake were detected in the far field. For example, coseismic horizontal displacements were observed using GPS at distances of up to 4,500 km from the epicenter (Banerjee et al. 2005; Khan and Gudmundsson 2005; Ammon et al. 2005; Vigny et al. 2005; Boschi et al. 2006; Fu and Sun 2006). Even the satellite

gravity mission GRACE detected gravity changes from space (Sun and Okubo 2004; Han et al. 2006). All these observations imply that coseismic deformation accompanying the Sumatra–Andaman Earthquake occurs not only near the epicenter, but also over the entire Earth. For that reason, we are compelled to investigate global coseismic deformations using both a theoretical model and practical observation. Especially, to interpret the observed geodetic and geophysical changes, a spherical dislocation theory is considered to be necessary to model/invert the fracture fault and to calculate the coseismic deformations over the Earth surface. Furthermore, the earthquake provides an opportunity to investigate global coseismic deformations and observe effects of spherical curvature and a layered structure of the Earth.

An important advance was made in computing coseismic gravity change. Hagiwara (1977) was the first one to study the coseismic gravity change, by considering a Mogi model buried in a half-space medium, and he gave a set of analytical expressions. Okubo (1991, 1992) proposed closed-form expressions to describe potential and gravity changes resulting from dislocations. Like Okada (1985), his expressions for a point source or a finite fault model are analytical, concise, and convenient in use. The theory becomes classic and it is widely applied even today.

To take the effects of curvature of the Earth and stratified structure of the Earth on gravity change, stratified sphere models, such as the 1066A model (Gilbert and Dziewonski 1975) or the PREM model (Dziewonski and Anderson 1981), should be considered. For such an Earth model, Sun and Okubo (1993, 1998) presented methods to calculate coseismic changes in gravity in spherically symmetric Earth models. They defined dislocation Love numbers and Green's functions for four independent seismic sources. They applied the Green's functions in numerical integration over fault plane, and interpreted the gravity changes caused by the 1964 Alaska earthquake ($M_w 9.2$). The theoretical modeled gravity changes agree well with the observed ones. Their results showed that the Earth's sphericity can cause a 10 % difference when the epicentral distance is greater than 10° ; radial heterogeneity should be considered when the epicentral distance is greater than 0.5° . In order to reduce the numerical computation of the coseismic deformations, Okubo (1993) proposed a reciprocity theorem for connecting solutions of dislocation and tidal, shear, and load deformations. That study found that deformation on the Earth's surface caused by dislocations at source radius $r = r_s$ is expressible simply using a linear combination of the tide, shear, and load deformations at $r = r_s$.

Sun et al. (2006a, b) extended their theory (Sun and Okubo 1993; Sun et al. 1996) to calculating coseismic strain caused by four independent seismic sources in a spherically symmetric, non-rotating, perfectly elastic, and

isotropic (SNREI) Earth model. The corresponding expressions are derived assuming that the seismic sources are located at the polar axis. The proper combination of these expressions allows calculation of the coseismic strain components resulting from an arbitrary seismic source at any Earth position. A case study is performed, and effects of Earth model, spherical curvature, and the stratified structure are investigated. To simplify the computation, Sun (2003, 2004a, b) presented a set of asymptotic expressions of the coseismic deformations. Due to their analytic form, the expressions can be easily applied in practical computation, with the Earth's curvature and layered structure being considered at the same time.

All of the above theories were developed for a deformed Earth surface because most traditional measurements are performed on the Earth surface. However, the advances in satellite gravity mission GRACE enable better detection of coseismic gravity change from space, e.g., the co- and post-seismic gravity change caused by the 2004 Sumatra earthquake was detected by GRACE (Gross and Chao 2001; Sun and Okubo 2004; Han et al. 2006; Chen et al. 2007; Panet et al. 2007). Han et al. (2006) calculated the gravity changes caused by the earthquake, and interpreted the gravity changes using a very simple method based on a half-space Earth model. In this case, a more reasonable dislocation theory must be used instead. For this purpose, Sun et al. (2009) presented formulas applicable to the space observation. In addition, a smoothing technique, e.g., a Gaussian filter, is necessary to damp the high-frequency contribution, so that the theory can be applied reasonably.

Recently, the dislocation theory has been developed through a number of theoretical and application studies. For example, Fu and Sun (2008) proposed a new dislocation theory for a 3D Earth model as mentioned above; Sun and Zhou (2012) presented a theoretical computing scheme on deflection change of vertical, which can be applied in interpretation of satellite gravity mission GRACE data; Sun and Dong (2013) gave the relation of dislocation Love number and the conventional Love numbers. Most of these new progresses have not been included in the book by Sun (2012). This paper mainly introduces these recent advances of computing coseismic deformations in theory and applications.

2 Theoretical study on coseismic deformations in a 3D Earth model

The most advanced theoretical study on dislocation theory is that for a 3-dimensional heterogeneous spherical Earth model. Fu and Sun (2008) proposed a computing scheme on surface gravity changes caused by dislocations within a 3D heterogeneous Earth. This theory is described using six independent dislocations: a vertical strike-slip, two vertical

dip-slips perpendicular to each other, and three tensile openings on three perpendicular planes. A combination of the six independent dislocations can be used to compute coseismic deformations resulting from an arbitrary seismic source at an arbitrary position. Based on the 3D lateral inhomogeneous P wave velocity model, they deduced the 3D density and S wave velocity models using the relation of Karato (1993). Finally, numerical computations are performed for a location south of Japan (30°N, 135°E). They calculated the coseismic gravity changes resulting from the six independent dislocations for source depths of 100, 300, and 637 km, respectively. Numerical results show that the maximum 3D effect varies concomitantly with the dislocation type and the source depth. Their results indicate that the perturbed gravity change by the 3D structure of the Earth, in percent comparing to the unperturbed results, reaches 1.6 %.

Note that the 3D Earth model used in this study is limited to $5^\circ \times 5^\circ$ in spatial resolution, meaning that the maximum harmonic degree is about 36. The corresponding coseismic gravity changes and conclusions are obtained for such an Earth model. In order to predict the effect of higher degree Earth model on gravity change, some computations are performed for different harmonic degrees to investigate the effects on the Earth model. They considered the 3D Earth model up to degree 6, 12, 18, 24, 30, and 36. Using these Earth models, they calculated the coseismic gravity changes caused by different dislocations. The maximum 3D effects on gravity changes for each harmonic degree show that the results converge as the harmonic degree increases. The 3D effects increase concomitantly with the increase in degrees of the 3D Earth models. The result implies that the conclusion obtained based on degree 36 in this study is not the ultimate one. The 3D effect should be even greater when a more detailed 3D Earth model is available, especially for a 3D earth model with detailed local structure. However, because of the large structural difference in the local area, a real 3D Earth model is expected to be even larger than the present one in the lateral increments. For example, the lateral increment in the Japanese ocean area reaches about 20 percent from PREM.

Fu et al. (2010) further developed their study to the coseismic displacements caused by point dislocations in a three-dimensional heterogeneous spherical Earth model, based on their previous study scheme. They expressed the coseismic deformation by the sum of unperturbed displacements on a spherically symmetric Earth model and the effects of the 3D effect. Then they decompose the 3D effect into two contributions: the effect of seismic sources and the effect of the Earth's lateral increments. The formulations were given for six point seismic sources; a combination of the six dislocations can be used to compute the 3D effect resulting from an arbitrary seismic source at

an arbitrary position. Results show that the maximum 3D effect on coseismic displacements is about 1 %–2 % of the corresponding unperturbed solutions, almost the same order as the coseismic gravity change.

3 Relation of dislocation Love numbers and conventional Love numbers

Recently, Sun and Dong (2013) studied the relation between the dislocation Love numbers and the conventional Love numbers, i.e., the tidal, press, and shear Love numbers, based on the Okubo (1993) reciprocity theorem. According to Sun and Okubo (1993) and Sun et al. (2009), if a dislocation occurs in a spherical Earth, such as in a homogeneous sphere or SNREI Earth (Dahlen 1968), the excited displacement $\mathbf{u}(a, \theta, \phi)$ (radius, colatitude, and longitude), and geopotential change $\psi(a, \theta, \phi)$ on the Earth surface are describable in the form of dislocation Love numbers $h_{nm}^{ij}(a)$, $l_{nm}^{ij}(a)$, $k_{nm}^{ij}(a)$, and $l_{nm}^{t,ij}(a)$, defined as

$$h_{nm}^{ij}(a) = y_1^{S,ij}(a; n, m) a^2 \quad (1)$$

$$l_{nm}^{ij}(a) = y_3^{S,ij}(a; n, m) a^2 \quad (2)$$

$$k_{nm}^{ij}(a) = y_5^{S,ij}(a; n, m) a^2 / g_0 \quad (3)$$

$$l_{nm}^{t,ij}(a) = y_1^{t,ij}(a; n, m) a^2 \quad (4)$$

where a denotes the Earth radius, g_0 signifies the gravity on Earth surface, superscript S represents spheroidal deformation, and t stands for toroidal deformation. Superscripts and subscripts i and j , respectively, denote the three components of slip vector $\mathbf{v}$ and normal $\mathbf{n}$. The y variables $y_{k,m}^{n,ij}(a)$ and $y_{k,m}^{t,n,ij}(a)$ are obtainable by solving the linearized first-order equations of equilibrium, stress–strain relation, and Poisson's equation for excited deformation (Takeuchi and Saito 1972), with the discontinuity condition at the source and the following free boundary conditions:

$$\forall n, m, i, j : y_{2,m}^{n,ij}(a) = y_{4,m}^{n,ij}(a) = y_{6,m}^{n,ij}(a) = y_{2,m}^{t,n,ij}(a) = 0. \quad (5)$$

Using the dislocation Love numbers defined in (1)–(4), one might compute Green functions that are useful to calculate coseismic deformations (displacement, potential, geoid and gravity changes, and strain vector) excited by independent sources of four types buried in a spherically symmetric Earth model. In combination, these components allow calculation of a displacement field that is excited by an arbitrary seismic source (Sun and Okubo 1993; Sun et al. 2009).

On the other hand, scientists have fully made studies on conventional deformation problems excited by external forces, such as tidal, press (load), and shear. For convenience, the boundary conditions and the definition of the tide, press, and shear Love numbers are introduced, i.e.,

(h_n, l_n, k_n) represent the conventional tidal Love numbers, (h'_n, l'_n, k'_n) are press Love numbers, and (h''_n, l''_n, k''_n) are shear Love numbers.

According to the reciprocity theorem (Okubo 1993), one can express y-solutions of the vertical strike-slip source in terms of the conventional forced deformations on the Earth surface. Then, according to the definition of dislocation Love numbers in (1)–(4) and the conventional Love numbers, one can obtain the relation of the dislocation Love numbers and the conventional Love numbers (Sun and Dong 2013) for a vertical strike-slip as shown below (results for other seismic sources refer Sun and Dong 2013):

$$h_{n\pm 2}^{12}(a) = \mp i \frac{G\mu U dS}{2g_0^2} l'_n \quad (6)$$

$$l_{n\pm 2}^{12}(a) = \pm i \frac{G\mu U dS}{2g_0^2} l''_n \quad (7)$$

$$k_{n\pm 2}^{12}(a) = \pm i \frac{G\mu U dS}{2g_0^2} l_n \quad (8)$$

$$l_{n\pm 2}^{\mu, 12}(a) = \frac{G\mu U dS}{2g_0^2} l'_n \quad (9)$$

The above relation shows that dislocation Love numbers for the vertical strike-slip source can be obtained simply from the conventional Love numbers by multiplying a scale factor $G\mu U dS/2g_0^2$, but different Love numbers correspond to different components of the dislocation Love numbers: e.g., the press Love number l'_n is related to dislocation Love number $h_{n\pm 2}^{12}$, and so on.

In summary, Sun and Dong (2013) obtained all the dislocation Love numbers for the four independent seismic sources on the Earth surface, expressed either by the conventional (tide, press, or shear) Love numbers, or known boundary conditions on the Earth surface, using the Okubo (1993) reciprocity theorem. They are useful to compute the Green functions on the surface. These relations mean that the coseismic deformation of the Earth is not independent, but has intrinsic relations with other external-force deformation problems. This fact implies that one can compute coseismic deformations by a combination of the conventional deformations caused by tidal, press, and shear forces.

The results of this study completed the dislocation theory of Sun et al. (2009) with the special case of a surface rupture considered. The previous computing codes (Sun et al. 2009) are valid only for inner sources because of a lack of the Green functions for a surface rupture. In this case, they assumed that the fault slip occurred inside the Earth with a small surface rupture ignored. Although the treatment does not affect the numerical computation too much, it is theoretically incomplete and causes numerical difficulty for computing coseismic deformations for a shallow source. Therefore, numerical results of the Green functions for the surface rupture are useful

to improve earlier computing codes. They are better applied in a practical numerical integration over a fault plan when it ruptures at the Earth surface. It can serve as a good constraint in the numerical integration even for inner rupture of the fault.

4 Study on coseismic deflection change of the vertical

Since GRACE was launched in 2002 by GFZ and NASA, it provides global gravity model to study geophysical and geodynamical changes. For example, it was successfully used to detect coseismic gravity changes resulting from the 2004 Sumatra earthquake ($M_w 9.3$) and the 2010 Chile earthquake ($M_w 8.8$) (Han et al. 2006; Panet et al. 2007; Ogawa and Heki 2007; Chen et al. 2007; De Linage et al. 2009; Han et al. 2010; Heki and Matsuo 2010; Zhou et al. 2011), and the 2011 Tohoku earthquake ($M_w 9.0$) (Zhou et al. 2012).

GRACE is not only to be used to compute gravity and geoid changes, but also possibly to compute the vertical deflection change. The vertical deflection is defined as the horizontal derivative of the geoid; it is expected to be more sensitive to coseismic deformation than geoid. It can open a new approach to the study of coseismic deformation with GRACE data, because the differentiation of the GRACE geoids in the North–South direction that corresponds to the NS vertical deflection change takes advantage of the higher precision along-track rather than across-track. This nicely appears when one compares the performance of earthquake signal recovery in terms of geoid or vertical deflection, and allows discriminating between different rupture models, showing the interest of GRACE for earthquake studies.

For this purpose, Sun and Zhou (2012) presented a new scheme to compute the deflection change of the vertical, with extending the current dislocation theory for a spherical Earth model (Sun and Okubo 1993; Sun et al. 2009). Once the excited geopotential change $\psi(r, \theta, \phi)$ is obtained, the corresponding coseismic geoid and deflection vertical changes can be derived as presented below:

$$N(a, \theta, \phi) = \frac{\psi(a, \theta, \phi)}{g_0} \quad (10)$$

$$\xi(a, \theta, \phi) = \frac{1}{a} \frac{\partial N(a, \theta, \phi)}{\partial \theta} \quad (11)$$

$$\eta(a, \theta, \phi) = -\frac{1}{a \sin \theta} \frac{\partial N(a, \theta, \phi)}{\partial \phi} \quad (12)$$

Using the same scheme of Sun et al. (2009), the north–south and east–west components of the deflection of the vertical changes can be further expressed using Green's functions for the four independent components. The theoretical expressions are useful to interpret the GRACE-observed data. The dislocation theory above is valid for a solid elastic Earth, and the corresponding computing

program assumed dry Earth, so that the surface subsidence on the Earth surface is replaced with air.

However, for a large earthquake occurring in or near an oceanic area, the deformation that occurred in the ocean bottom is replaced by seawater, and the sea bottom displacement causes additional potential and gravity changes. This seawater effect must be addressed specially so that the modeled coseismic deformation can be compared reasonably with GRACE data. Sun and Zhou (2012) discussed how to make seawater corrections to the potential/geoid and deflection changes because the treatment differs from that of gravity. Furthermore, they conclude that the ocean contribution can change the modeled coseismic gravity pattern on the solid Earth surface and is a major signal observed by GRACE for the Tohoku-Oki earthquake. It should be pointed out that since GRACE gives each single harmonic component of the potential, it is possible to compare separately the contribution from each harmonic component before summing. In practical computation, they applied an approximated approach to make seawater corrections.

According to the above computing scheme, Sun and Zhou (2012) first process the GRACE-observed data to obtain coseismic geoid and deflection vertical changes caused by the 2011 Tohoku-Oki earthquake ($M_w 9.0$). They use the GRACE Level-2 datasets composed of Stokes' coefficients for $n = 60$, released by Center for Space Research at the University of Texas. Since geophysical effects such as ocean tide, solid Earth and ocean pole tides, and atmosphere effect, have already been removed by modeling, the GRACE datasets are presumed to contain the coseismic deformation reflecting the mass redistribution.

After the seawater correction, the modeled coseismic geoid and deflection changes indicate that both the coseismic geoid and deflection changes can be detected clearly by GRACE observation. Results show that the coseismic geoid change is not sensitive to the fault slip models: the three slip models yield identical coseismic geoid changes, although the coseismic deflection changes are very sensitive to the fault slip models because the modeled deflection changes indicate a large difference, especially for the E–W component. These behaviors provide a new and useful approach to invert the seismic fault using GRACE-observed deflection changes as a constraint.

5 Dislocation theory applied in satellite gravity observations

5.1 Gravity changes on deformed Earth surface and at space-fixed point

Usually, the coseismic gravity change $\delta g(a, \theta, \phi)$ on the deformed Earth surface ($r = a + u$) (note that $r = a + u$

is denoted as a for convenience) is expressed as (Sun and Okubo 1993)

$$\delta g(a, \theta, \phi) = \Delta g(a, \theta, \phi) - \beta u_r(a, \theta, \phi) \quad (13)$$

where the first term $\Delta g(a, \theta, \phi)$ of the right-hand side of (13) is the gravity change at a fixed-space point ($r = a$) and $\beta = 2g(a)/a$, where $g(a)$ is the mean gravity on the Earth surface. The last term $-\beta u_r(a, \theta, \phi)$ gives the free-air correction caused by the vertical displacement on the Earth surface. This free-air correction is considered to convert the gravity change from the fixed-space point to the deformed Earth surface. This correction is necessary if a theoretical coseismic gravity change is computed to compare with the observed gravity change on the deformed Earth surface. However, if one wants to study the coseismic gravity change observed in space, such as by GRACE, Eq. (13) is not applicable, and the free-air correction term must be eliminated because the satellite gravity mission does not “see” this part.

We need only consider the first term on the right-hand side in (13) to interpret the gravity change observed from space. This term $\Delta g(a, \theta, \phi)$ is actually the gravity change at a fixed-space point on the undeformed Earth (see level) surface $r = a$. Finally, the gravity change $\Delta g(a, \theta, \phi)$ is obtainable as

$$\Delta g(a, \theta, \phi) = \frac{g}{a^3} \sum_{n,m}^{\infty} (n+1) k_{nm}^{ij} Y_n^m(\theta, \phi) v_i n_j U dS \quad (14)$$

where U denotes the dislocation slip; v_i and n_j , respectively, indicate the slip and normal components; and Y_n^m is the spherical function. The variable k_{nm}^{ij} is the dislocation Love number of the potential change, and $k_{nm}^{ij} = y_{5,m}^{n,ij}$. Its numerical computation is similar to that in Sun and Okubo (1993).

To compare the gravity changes for the two cases, Sun et al. (2009) computed the coseismic gravity changes $\Delta g(a, \theta, \phi)$ at the space-fixed point $r = a$ in the same computing scheme as $\delta g(a, \theta, \phi)$, caused by the 2004 Sumatra earthquake ($M_w 9.3$). Comparing $\Delta g(a, \theta, \phi)$ and $\delta g(a, \theta, \phi)$ reveals a great difference in amplitude and sign. The coseismic gravity changes $\Delta g(a, \theta, \phi)$ appear negative on the land side, and positive in ocean side, opposite with $\delta g(a, \theta, \phi)$.

5.2 Coseismic deformations detectable by satellite gravity missions

Before the 2004 Sumatra earthquake ($M_w 9.3$), Sun and Okubo (2004) investigated whether the satellite gravity mission GRACE could detect coseismic deformations. According to Sun and Okubo (1993), for an arbitrary shear fault on the polar axis, the coseismic geoid changes can be written as the following:

$$\zeta^{\text{Shear}}(a, \theta, \phi) = \sum_{n=2}^{\infty} \left\{ \cos \lambda [-k_{n2}^{12} \sin \delta Y_n^2(\theta, \phi) + k_{n1}^{13} \cos \delta Y_n^1(\theta, \phi)] + \sin \lambda \left[\frac{1}{2} (k_{n0}^{33} - k_{n0}^{22}) \sin 2\delta Y_n^0(\theta, \phi) + k_{n1}^{32} \cos 2\delta Y_n^1(\theta, \phi) \right] \right\} \times \frac{UdS}{a^2} \quad (15)$$

where k_{nm}^{ij} (related to the gravitational potential change) are the dislocation Love numbers defined by Sun and Okubo (1993), functions of the spherical harmonic degree, order, source depth, and source type. δ and λ are dip-angle and slip-angle of the fault, respectively. U is dislocation on the infinitesimal fault area dS , a is the radius of the Earth, and $Y_n^m(\theta, \phi)$ is the spherical harmonic function of degree n and order m . The dislocation factors, UdS/a^2 , define the earthquake magnitude and give the unit of geoid change.

Eq. (15) is a function of spherical harmonic degree n ; one may compute that the degree variance c_n^2 (i.e., the power spectral density) of the gravitational potential anomaly, which gives the contribution of the degree n terms to the total variance, is defined as (ESA 1999)

$$c_n^2 = \sum_{m=0}^n (C_{nm}^2 + S_{nm}^2) = \sum_{m=-n}^n |K_{nm}|^2 \quad (16)$$

which is also known as the root-mean-square value per degree. Actually, the coefficients (the dislocation Love numbers and the geographical parameters of the fault) of $Y_n^m(\theta, \phi)$ in (15) are nothing but the Stoke's coefficients. Correspondingly, the degree variances for several earthquakes are calculated using the dislocation numbers. Results show that the 1964 Alaska earthquake causes the coseismic geoid in low degree part with near two orders in magnitude larger than the GRACE error. This fact indicates that the global geoid (or gravity) change is sufficiently large to be detected by GRACE.

Comparison of the degree variances of the 2002 Alaska and the 2003 Hokkaido earthquakes showed that geoid and gravity effects for the 2003 Hokkaido earthquakes are much larger than those of the 2002 Alaska earthquake at almost all the harmonic degrees, but that their fault sizes are almost identical. This fact further confirms that the geographical position, especially the dip-angle, plays an important role in these geophysical effects. It is found that the degree variances for a shear fault movement include both the shear and tensile components; whereas the degree variances for a tensile fault also include the two components, but are unconnected with the dip-angle. The above results and discussions imply that coseismic geoid and gravity change for an earthquake with a magnitude greater than $m > 7.5$ for a tensile fault, or $m > 9.0$ for a strike fault, coseismic geoid and gravity changes are anticipated to be detectable by GRACE. This conclusion has been

confirmed later by the 2004 Sumatra earthquake ($M_w 9.3$) (Han et al. 2006; Ogawa and Heki 2007).

5.3 Dislocation Love numbers determined by satellite gravity

Sun et al. (2006a, b) proposed a method to determine dislocation Love numbers using satellite gravity observations. According to Sun and Okubo (1993), coseismic gravitational potential change caused by a tensile source could be expressed as

$$\psi^{\text{Tensile}}(a, \theta, \phi) = \sum_{n=2}^{\infty} \sum_{m=-n}^n [(k_{nm}^{33} \cos^2 \delta + k_{nm}^{22} \sin^2 \delta) Y_n^m(\theta, \phi) - k_{nm}^{32} \sin 2\delta Y_n^m(\theta, \phi)] \times \frac{g_0 UdS}{a^2} \quad (17)$$

Theoretically, it should equal to the potential $T(a, \theta, \phi)$ observed by satellite gravity mission (such as GRACE), i.e., $\psi(a, \theta, \phi) \equiv T(a, \theta, \phi)$. If the satellite observed data are used as conditions, and assuming the dislocation Love numbers as unknowns, we obtain the following equation:

$$f_1(\theta, \phi) k_{n2}^{12} + f_2(\theta, \phi) k_{n1}^{32} + f_3(\theta, \phi) k_{n0}^{22} + f_4(\theta, \phi) k_{n0}^{33} = g(\theta, \phi) \quad (18)$$

where $f_1(\theta, \phi) = (\sin \lambda \sin 2\delta \cos 2\phi - 2 \cos \lambda \sin \delta \sin 2\phi) P_n^2(\cos \theta) \frac{g_0 UdS}{a^2}$ is known once the parameters of an earthquake are provided (f_2, f_3 , and f_4 are similar). The only unknowns are the dislocation Love numbers k_{n2}^{12}, k_{n1}^{32} , and $k_{n0}^{33} - k_{n0}^{22}$. While the function $g(\theta, \phi)$ is a function related to observation data, $g(\theta, \phi) = a \sum_{k=-n}^n (\Delta c_{nk} \cos k\phi + \Delta s_{nk} \sin k\phi) P_n^k(\cos \theta)$, where Δc_{nk} and Δs_{nk} are differences of two sets of spherical harmonic coefficients (C_{nm}^1, S_{nm}^1) and (C_{nm}^2, S_{nm}^2) of the geopotential model observed by the GRACE mission, containing the information of coseismic gravity changes. Eq. (18) contains only 4 unknown variables. Since the observation is made globally, it is easy to discretize the Eq. (18), and form a linear equation $FK=G$. Then the unknown K can be easily resolved (refer Sun et al. (2006a, b), for detail).

6 Dislocation theory applied in studying co- and post-seismic deformations

Since dislocation theories have been developed, they are often applied in studying co- and post-seismic deformations. In this section, among many advanced theoretical and case studies, only few of them are introduced, to illustrate how dislocation theory is important in interpret geodetic data, or invert seismic slip for co- and post-seismic processes.

Tanaka et al. (2006, 2007) presented a new method to compute global viscoelastic post-seismic deformation in a realistic Earth model for displacement and gravity variation. Previous methods are based on simplified Earth models that neglect compressibility and/or the continuous variation of the radial structure of Earth. This is because the previous mode summation technique cannot avoid intrinsic numerical difficulties caused by the *innumerable* poles that appear in a realistic Earth model that considers such effects. In contrast, the proposed method enables both of these effects to be taken into account simultaneously. They carried out numerical inverse Laplace integration, which allows evaluation of the contribution from all of the innumerable modes of the realistic Earth model. Using this method, a complete set of Green's functions is obtained, including functions of the time variation of the displacement, gravity change, and the geoid height change at the surface for different point dislocations. They employed the PREM model and a convex viscosity profile, and investigated the effects of fine layering of the viscoelastic structure and compressibility on a time series of the post-seismic deformation. They computed the post-seismic deformation due to the Sumatra–Andaman Islands earthquake ($M_w 9.3$). They claimed that the rate of post-seismic vertical displacement and gravity change possibly detected in the far field even exceeds 400 km.

Tanaka et al. (2008) applied spectral finite element approach to study post-seismic deformation in a viscoelastic self-gravitating spherical Earth. They derived interface conditions for an arbitrary shear fault in the form of double-couple forces that are equivalent to a prescribed dislocation and simulate a relaxation process for an incompressible Maxwell Earth with a 3D viscoelastic structure. Computational results are validated for a spherically symmetric model by an independent method based on the inverse Laplace integration, and good agreement is obtained. As an example, we apply this approach to the 2004 Sumatra–Andaman earthquake and simulate a large-scale post-seismic gravity potential variation by a forward calculation.

Melini et al. (2008) studied post-seismic rebound of a spherical viscoelastic Earth, using analytical scheme within the framework of normal modes, based on the application of propagator methods. The authors pointed out that the main drawback related to the numerical solution of the secular equation is that the harmonic degree increases linearly with the number of viscoelastic layers so that only coarse-layered models are practically solvable. However, a viable alternative to the standard normal-mode approach, based on the Post–Widder Laplace inversion formula, has been proposed in the realm of postglacial rebound models. The main advantage of this method is to bypass the explicit solution of the secular equation, while retaining the

analytical structure of the propagator formalism. At the same time, the numerical computation is much simplified so that additional features such as linear non-Maxwell rheologies can be simply implemented. Therefore, the authors applied the Post–Widder Laplace inversion formula to a post-seismic rebound model. They tested the method against the standard normal-mode solution and performed various benchmarks aimed to tune the algorithm and to optimize computation performance while ensuring the stability of the solution. As an application, they address the issue of finding the minimum number of layers with distinct elastic properties needed to accurately describe the post-seismic relaxation of a realistic Earth model. Finally, they demonstrated the potentialities of their code by modeling the post-seismic relaxation after the 2004 Sumatra–Andaman earthquake.

Pollitz et al. (2011) applied the dislocation theory to investigate the slip model of the 2011 $M 9.0$ Tohoku earthquake using geodetic data. They sampled the three-dimensional GPS crustal displacement field, and employ a spherically layered Earth structure and use a combination of onland GPS, out to $\sim 4,000$ km from the rupture, and offshore GPS. Then they inverted the displacement field for dip slip, and obtained a compact slip maximum of about 33 m located 200 km east of Sendai. The geodetic moment is $4.06 \times 1,022$ N m, corresponding to $M_w = 9.0$. The area of maximum slip is concentrated at a depth of about 10 km; is up dip of the rupture areas of the earthquakes of 1933, 1936, 1937, and 1978; and roughly coincides with the rupture area of the $M 7.1$ Miyagi-oki earthquake in 1981.

Cambiotti and Sabadini (2012) studied the seismic source for the great 2011 Tohoku earthquake ($M_w 9.1$) from inversion of GRACE Level 2 data time series and self-gravitating, compressible 1D Earth model. After spatial localization of space gravity data in the surrounding of the USGS epicenter by means of orthogonal Slepian functions, they estimated the long-wavelength coseismic gravity signature. They found that the pattern is bipolar: the positive pole offshore in the Pacific Ocean and the negative pole in the northern Japan and Japan sea. They inverted the GRACE data to resolve average features of finite fault models, and found that the total seismic moment agrees with the centroid-moment-tensor solution. The mean depth of the rupture they obtained is just below the Moho discontinuity.

Wang et al. (2012) investigated the same problem for the 2010 $M_w 8.8$ Maule, Chile, earthquake. They conducted a spatio-spectral localization analysis, based on Slepian basis functions, of data from GRACE to extract coseismic gravity change signals of the Maule earthquake with improved spatial resolution (350 km half-wavelength). Their results reveal discernible differences in the average slip between the GRACE observation and predictions from

various coseismic models. The sensitivity analysis reveals that GRACE observation is sensitive to the size of the fault, but unable to separate depth and slip. They assume the depth of the fault is known, and simultaneously invert for the fault-plane area and the average slip using the simulated annealing algorithm. The GRACE-inverted fault-plane length and width and the estimated slip indicate that most of the strain accumulated since 1835 was released by the 2010 Maule earthquake.

Diao et al. (2014) investigated the post-seismic deformation processes of afterslip and viscoelastic relaxation following the 2011 M_w 9.0 Tohoku earthquake, using surface displacement data as observed by the onshore global positioning system network. They explored two different inversion modeling strategies: pure afterslip and combined effect of afterslip and viscoelastic relaxation. Temporal analysis of our model inversions suggests that the rate of afterslip rapidly decreases with time, consistent with the state- and rate-strengthening frictional law. The spatial pattern of afterslip coincides with the locations of aftershocks, and also with the area of coseismically increased Coulomb failure stress (CFS). Only a small part of the coseismically increased CFS was released by the afterslip in 564 days after the event. The effect of the viscoelastic relaxation within this initial stage only plays a secondary role, but it shows an increasing tendency.

7 Final remarks

Dislocation theories have been developed for a long time since 1960s by numerous scientists. They are developed for different media models, from homogeneous half-space to 3D structured Earth model and from static deformation to dynamic changes. A large number of scientific papers have been published; however, due to the volume limit the overview in this paper contains only limited achievements. This paper only introduces some recent advances in theory and applications in computing coseismic deformation. It should also be pointed out that the dislocation theory is still under development, and some more complicated physical models need to be further considered.

Although Fu and Sun (2008) studied the 3D effect on coseismic deformation, the 3D Earth model is still too rough to reflect the truth. However, the problem is that even though a detailed 3D Earth model is available, the computation by the current scheme will meet a theoretical difficulty, because the real 3D perturbation of the Earth could be beyond 100 % (e.g., the local P-wave structure in the crust). On the other hand, the current theory is based on the perturbation theory, and it is no more valid for such an Earth model. To overcome such a difficulty, the finite element method (FEM) might be an efficient choice.

Another physical amount, i.e., the topography, has not been taken into account so far. How large is the terrain effect on seismic deformation is still an open question. Actually, the terrain effect is really difficult to be considered in a theoretical description like the above theories, due to its irregularity. Same as the 3D effect, the terrain effect is probably solved only by the FEM method.

As the Earth model has been advanced from half-space to 3D viscoelastic sphere, the corresponding dislocation theory becomes more complicated, in both expression and numerical computation. To reduce the complicity of the theories, one may try to search a new approach to simplify them, like the asymptotic expressions of the coseismic deformations.

On the other hand, modern geodetic techniques develop quickly; especially the satellite gravity makes more advanced progress in observations. Those advances in geodetic observation and the dislocation theory will certainly enhance the studies in seismology and geodesy, particularly in inverse of inner structure, fault slip distribution, and interpretation of geodetic observation data.

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