The b -value and fractal dimension of local seismicity around Koyna Dam (India)

Arjun Kumar · S. S. Rai · Anand Joshi ·
Himanshu Mittal · Rajiv Sachdeva ·
Rohtash Kumar · Vandana Ghangas

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Abstract Earthquakes began to occur in Koyna region (India) soon after the filling of Koyna Dam in 1962. In the present study, three datasets 1964–1993, 1993–1995, and 1996–1997 are analyzed to study the b -value and fractal dimension. The b -value is calculated using the Gutenberg–Richter relationship and fractal dimension D_{corr} using correlation integral method. The estimated b -value and D_{corr} of this region before 1993 are found to be in good agreement with previously reported studies. In the subsequent years after 1995, the b -value shows an increase. The estimated b -values of this region are found within the limits of global average. Also, the pattern of spatial clustering of earthquakes show increase in clustering and migration along the three zones called North-East Zone, South-East Zone (SEZ), and Warna Seismic Zone. The earthquake events having depth ≤ 5 km are largely confined to SEZ. After 1993, the D_{corr} shows decrease, implying that earthquake activity gets clustered. This seismic clustering could be helpful for earthquake forecasting.

Keywords Koyna · Fractals · b -Value · Correlation dimension · Algorithm

1 Introduction

Earthquakes pose a major natural hazard that affects many parts of the world. One possible way to reduce this hazard is

earthquake prediction. Earthquakes are strongly confined to a few regions. Earth's surface is broken into a mosaic of relatively earthquake-free plates, whose margins release more than 99 % of the world's seismic energy (Scholz 1990). Since most earthquakes occur repeatedly along certain active segments of faults, it is important for long-term prediction to determine their locations and movement of tectonic plate in the past.

The spatial and temporal distribution of the past earthquakes of different sizes recorded historically and by seismic networks (in existence for only about a century), as well as many aspects of earthquake sources and near-source characteristics, has been studied in attempts to discover some patterns that might be useful for long- to short-term predictions of future earthquakes.

Earthquake epicenters can be considered as to be point events in space and time. If the occurrence of each earthquake is totally uncorrelated with other earthquakes then the distribution of events is called Poisson (random) distribution with well-understood mathematics. In regional seismicity studies, the distributions are not Poisson (e.g., Knopoff 1964; Singh and Sanford 1972; Smalley et al. 1987). Just as Poisson process is purely random, fractals geometry exhibiting scale-invariant properties such as scale-invariant clustering is applied to study earthquakes. Fractal geometry reveals that simple algorithms can generate complex forms. This describes temporal or spatial properties of geological patterns. It does not describe the mechanism that produces the fractal scaling, but it helps to sort out the possible mechanism or explanation.

2 Method of analysis

The seismicity has fractal structure with respect to space, time, and magnitude (Kagan and Knopoff 1978). The

A. Kumar (✉) · A. Joshi · H. Mittal ·
R. Sachdeva · R. Kumar · V. Ghangas
IIT Roorkee, Roorkee 247667, Uttarakhand, India
e-mail: arjundeq@gmail.com

S. S. Rai
National Geophysical Research Institute, Hyderabad, India

b -value describes the rate of increase of earthquakes as magnitude decreases. The fractal dimension is an efficient parameter to characterize the spatial distribution of the earthquake hypocenters (epicenters). Hirata et al. (1987) proposed the use of the decrease of fractal dimension of earthquake epicenters with the progress of creep, which may be useful for predicting large earthquakes. Ouchi and Ukawa (1986) reported that the degree of clustering of earthquakes increases before large earthquakes. Nanjo and Nagahama (2000) proposed that if distribution of earthquake epicenters is completely random and unpredictable, D_{corr} is 2. Thus, the use of decrease in the D_{corr} of earthquake hypocenters or epicenters may be valuable for predicting large earthquakes. Also, the b -value decreases before large earthquakes but D_{corr} appeared to be more sensitive than b -value (Hirata et al. 1987). In this study, we have estimated the b -value and correlation dimension (D_{corr}) of spatial clustering using the methods discussed below.

3 Seismic b -value

The b -value is usually computed using the “cumulative frequency–magnitude” distribution data and applying the Gutenberg–Richter relationship (Gutenberg and Richter 1954), which is widely used in seismology. The equation is as follows:

$$\log N = a - bM,$$

where N is the number of earthquakes with size greater than magnitude M ; and a and b are scaling constants depending on the location. The constant b also called the seismic b -value gives the information about the seismicity of the region, large b -value presents seismicity with small magnitudes and vice versa. Generally, b -value is in the interval $0.8 < b < 1.5$. This law shows the fractal behavior of earthquakes and the b -value is strongly related to the fractal dimension of earthquakes (Aki 1965; Hirata 1989a, b) in the form:

$$D = 2b$$

We use the data with lower limit of magnitude, threshold magnitude (Jia et al. 2012), above which the magnitude–frequency distribution follows the Gutenberg–Richter distribution law as shown in Fig. 1a, b.

3.1 Correlation dimension (D_{corr}) of spatial clustering

Spatial self-similarity can be demonstrated by examining the distribution of distances between pairs of points in a dataset over a range of distances. The angular distance r in degrees between two events is estimated using the following formula (Hirata 1989a, b):

Fig. 1 The figure shows **a** the estimation of threshold magnitude and **b** magnitude–frequency distribution (Gutenberg–Richter distribution) above the threshold magnitude

$$r = \cos^{-1}[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1 - \phi_2)],$$

where θ and ϕ are the latitudes and longitudes of the events, respectively. Then this angular distance (degrees) is converted to linear distance (kilometers) using:

$$1^\circ = 111 \text{ km.}$$

The fractal dimension based on the correlation integral (Grassberger 1983; Grassberger and Procaccia 1983) was used to estimate D_{corr} ,

$$C(r) \sim r^{D_{\text{corr}}}.$$

the correlation integral $C(r)$ is defined for the epicentral distribution as:

$$C(r) = \frac{2}{N(N-1)}N(R < r),$$

where N is the total number of events and $N(R < r)$ is the number of distances (R) between pairs of events with a distance smaller than r . For simplicity, $C(r)$ is the function of probability that two events will be separated by a

Fig. 2 The figure shows depopulation for distance less than r_n and saturation beyond distance r_s of correlation curve. The gradient of the least square fit between these points is called fractal dimension (D_{corr})

distance less than r . The resulting plot of $\log(r)$ versus $\log C(r)$ should be a straight line whose gradient gives the fractal dimension. But it is not so as shown in Fig. 2. For very large values of r , the gradient is artificially low, whereas for small values of r the gradient is high and these distances are called “saturation distance (r_s)” and “depopulation distance (r_n)” respectively. In order to estimate the fractal dimension, the straight line must be fitted in between these distances of the curve. Nerenberg and Essex (1990) suggested formulae to determine these distances of “depopulation (r_n)” and “saturation (r_s)” as:

$$r_n = 2R(1/N)^{1/d} \quad r_s = \frac{R}{d+1},$$

where d is the dimensionality of the data cluster and R is the maximum distance used in analysis. Also, Eneva (1996) pointed out that it is safe to start the scaling range as low as $r_n/3$.

4 Brief preview of study area

The study area of the present work is Koyna region of Maharashtra state, situated on Peninsular (Precambrian) Shield of India as shown in Fig. 3. Seismicity in this region started with impoundment of Koyna Dam in 1962. This region attracts the attention of Geophysicists and Seismologists throughout the world to study the mechanism of reservoir-induced earthquakes, which may be very fruitful to understand the phenomenon of earthquakes occurrence.

Our present knowledge about the seismicity of this region span over three decades. The record of seismicity prior to 1962 is limited due to the absence of seismic stations in this area. The seismic zoning map of India prepared by Indian Standards Institutions in 1962 and later revised in 1966 showed this region to be aseismic. No felt earthquake was reported in this region at least 100 years prior to impoundment of reservoir. Filling of the lake began in 1962, and by the end of that year the lake was partly filled. The first mild tremors were felt a few months later, in 1963. The maximum lake level was reached in 1965. Two severe shocks were felt in September 1967; and the largest shock (magnitude 6.4) occurred on December 10, 1967. This shock resulted in the death of 177 people and injuries to 2,300, and most of buildings in Koyna either collapsed or damaged. Some earthquakes have been listed in the nearby areas in the historical past, during 1594–1832 between Bombay (18.8°N, 74°E) and Goa (15.5°N, 74°E) there were at least 15 felt earthquakes (Gubin 1968; Kelkar 1968), also in 1952 and 1962 earthquakes were felt in the coastal town of Jaigarh (17.4°N, 72.3°E) and Ratnagiri (17°N, 72°E), suggesting that, although the Peninsular Shield has been geologically stable, it was wrong to rule out the occurrence of such an earthquake entirely. The impoundment of Warna dam,

Fig. 3 The map showing area of study—Koyna region, and seismological network deployed (after Gautam 1997)

about 35 km from Koyna dam, in 1985 may also have affected the earthquake activity in this region. Rastogi et al. (1990) observed a southward shift in the epicentral locations of earthquakes in the region following the impoundment of the Warna Dam.

5 Data description

The instrumental recording in Koyna region started in 1963 with four analog seismic stations using vertical-component Wood–Anderson seismographs. Four more seismic stations in 1973 and three more seismic stations in 1984 were installed consisting of Wood–Anderson seismographs and smoke drum recorders. All instruments had a manual option of impinging time using radio signals from BBC on the seismograms since 1985. Subsequently in 1993, eight more seismographs (five Portacorder and three Teledyne-Geotech's PDAS) were added to the exiting network of eleven seismic stations. Since these digital systems were not linked to any external timing devices like GPS, Omega clocks, etc., there was no mechanism to correct the drifts in the system's clock, and, therefore, only S–P data were used for the location. Time synchronization with BBC signal leads to an error 0.2–0.4 s, relative to GPS timings. Additionally, lower drum speed (60 mm/min prior to 1993 and 120 mm/min afterward) resulted in lower resolution as the reading accuracy of P-onset is restricted to ~ 0.2 s. Due to poor control on timing system, large inter-station separation with mostly vertical-component recording and the absence of stations in the near vicinity of the earthquake zone, the errors in the epicenter location vary between 2 and 3 km, as demonstrated in the recent study by Talwani et al. (1996). For the dataset 1993–1995, no detailed error statistics have been provided by Chadda et al. (1997) and Mandal et al. (1998). We used the relocated data with A, B, and C quality for 1964–1993 (listed in Talwani et al. 1996) and for 1993–1995 (Chadda et al. 1997 and Mandal et al. 1998).

Data for 1996–1997 was taken from the Koyna seismic tomography digital network comprising 24-bit REFTEK PASCAL (72A-07) recorder with GPS timing system. The digital seismic stations were equipped with three-component short-period sensors-Mark products L4-3D (three nos.) and SM3-KV seismometers (seven nos.) of the Institute of Physics of the Earth, Moscow. These were set on trigger mode to record local and distant earthquakes at 100 samples/s (Rai et al. 1999).

6 Results and discussion

In this study, three datasets; (1) 1964–1993 (Talwani et al. 1996), (2) 1993–1995 (Chadda et al. 1997; Mandal et al.

Fig. 4 Events occurred between Oct, 1964 and Aug, 1993

Fig. 5 Events occurred between Aug, 1993 and Dec, 1995

Fig. 6 Events occurred between April, 1996 and Nov, 1997

Fig. 7 Epicenters of an earthquake sequence which occurred between 22nd April, 1996 and 1st May, 1996

1998), and (3) 1996–1997 (Rai et al. 1999) and a particular sequence of events that occurred between 4th April, 1996 and 1st May, 1996 has been analyzed. The epicentral locations for these four sequences are shown in Figs. 4, 5, 6, and 7, respectively.

The b -value and correlation dimension ($D_{\text{corr.}}$) of spatial clustering of earthquake events have been estimated for all these three datasets and while the dataset (1996–1997) has been studied in detail considering hypocentral pattern laterally, depth wise, segment wise, and for a particular sequence of events. The results for these are listed in Table 1.

For datasets 1964–1993, 1993–1995, 1996–1997, and a particular sequence of events April, 1996; the b -value is 0.82, 0.845, 1.286, and 0.895, respectively.

The b -values as referred by Lee and Stewart (1981) for many areas of the world are within the range of 0.6–1.2. Also referred by Mogi (1987), Miyamura (1962) found that the b -value is moderate (0.7–1.0) in peninsulas, and large (1.0–1.8) in the oceanic regions including mid oceanic ridges and island arcs of small islets. The $b = 1.07$ as

Fig. 8 Events occurred between 1996 and 1997 having depth ≤ 5 km

computed by Dimri (1997) is much greater than our findings for 1964–1993 and 1993–1995, 0.82 and 0.845, respectively, but these values are much higher than the value 0.47 observed for Peninsular India (Gupta et al. 1969). Except for the b -value 1.286 of 1996–1997 data, our findings are in agreement with the previous studies of b -value in this area as suggested by various authors; Ram and Singh (1971), Ram and Rathor (1970), Gupta et al. (1969), and Gupta and Arora (2000). For 1996–1997 data, the b -value 1.286 shows an increase.

The $D_{\text{corr.}} = 1.407$ for data 1964–1993 is in agreement with 1.52 (Dimri 1997), and shows little clustering. The $D_{\text{corr.}}$ shows decrease of 1.12141 and 1.0534 for datasets 1993–1995 and 1996–1997, respectively, showing clustering of events.

For events that occurred in 1996–1997, the $D_{\text{corr.}}$ for depth ≤ 5 km (Fig. 8) and >5 km (Fig. 9) are 0.91135 and 0.9997, respectively. Which are in agreement with Kagan and Knopoff (1980): for shallow earthquakes, $D_{\text{corr.}} \sim 1.0$ and for deeper earthquakes $D_{\text{corr.}} \sim 1.5$ or more, as all the

Table 1 The b -value and $D_{\text{corr.}}$ with corresponding depopulation r_n and saturation r_s distances are shown for various datasets taken for analysis

Dataset	b -value	r_n (km)	r_s (km)	$D_{\text{corr.}}$
Oct, 1964–Aug, 1993	0.820 ± 0.0105	1.9560	14.993	1.4070 ± 0.00087
Aug, 1993–Dec, 1995	0.845 ± 0.00479	1.9797	16.563	1.1214 ± 0.00250
April, 1996–Nov, 1997	1.286 ± 0.012	1.1659	9.4359	1.0534 ± 0.024
1996–1997 depth ≤ 5 km	–	1.176149	8.662877	0.91135 ± 0.00085
1996–1997 depth >5 km	–	1.328873	9.944373	0.9997 ± 0.000214
1996–1997 (segment NEZ)	–	0.899	6.729	1.023 ± 0.00087
1996–1997 (segment SEZ)	–	0.519	4.011	1.17 ± 0.00169
1996–1997 (segment WSZ)	–	0.0692	4.1207	1.037 ± 0.000006
4th April–1st May 1996 (sequence)	0.895 ± 0.026	1.1659	9.4359	1.26 ± 0.00032

Fig. 9 Events occurred between 1996 and 1997 having depth >5 km

Fig. 10 D_{corr} analysis of events occurred in three fault segments of dataset 1996–1997

events of dataset 1996–1997 are confined within depth <15 km.

For segment-wise analysis of events (1996–1997), shown in Fig. 10, the respective D_{corr} are 1.023, 1.17 and 1.037, which shows better clustering and seems the presence of three faults. The D_{corr} for a particular sequence which occurred on April, 1996 is 1.26 showing moderate clustering. For depth ≤ 5 km earthquake events (1996–1997) show more clustering along South-East Zone (SEZ) segment while for depth >5 km clustering is uniform along all three segments.

The standard error σ has been estimated by linear regression of $\log(r)$ against $\log C(r)$ (Oncel et al. 1996). Its percentage comes out to be smaller than that found in case of b -value because of regression involving $N(N-1)/2$ individual two-point distances r rather than N magnitudes m .

7 Conclusions

The b -value and fractal dimension (D_{corr}) has been studied for earthquakes that occurred in Koyna region from 1964 to 1997. The b -value has been estimated for magnitudes above the threshold magnitude (Jia et al. 2012) that follows Gutenberg–Richter relationship and the D_{corr} using the correlation integral method of Grassberger and Procaccia (1983) by fitting straight line within the limits suggested by Nerenberg and Essex (1990). The b -values for 1964–1993 data and 1993–1995 data are in good agreement with the previous studies carried out in this area and by comparing the results with worldwide regional b -value, they are in the limits of regional b -values. It is suggested in the previous studies that this region has large b -value due to reservoir-induced seismicity but it seems nothing like this on comparing with worldwide regional b -values. The 1996–1997 data show a large increase in b -value. The D_{corr} of 1964–1993 data is also in agreement with the previous studies carried out by Dimri (1997). In subsequent years after 1993, the D_{corr} values go on decreasing and show increase in clustering, showing clear patterns along three zones namely North-East Zone (NEZ), SEZ, and Warna Seismic Zone (WSZ). The earthquake events having depth ≤ 5 km are found to be more confined to SEZ.

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