Amplification of in-plane seismic ground motion by group cavities in layered half-space (I)*

Jianwen Liang^{1,2,†} Ji Zhang^{1,2} and Zhenning Ba^{1,2}

¹ Department of Civil Engineering, Tianjin University, Tianjin 300072, China

² Tianjin Key Laboratory of Civil Engineering Structures and New Materials, Tianjin 300072, China

Abstract Amplification of in-plane seismic ground motion by underground group cavities in layered half-space is studied both in frequency domain and time domain by using indirect boundary element method (IBEM), and the effect of cavity interval and spectrum of incident waves on the amplification are studied by numerical examples. It is shown that there may be large interaction between cavities, and group cavities with certain intervals may have significant amplification to seismic ground motion. The amplification of PGA (peak ground acceleration) and its PRS (peak response spectrum) can be increased up to 45.2% and 84.4%, for an example site in Tianjin, under the excitation of Taft wave and El Centro wave; and group cavities may also affect the spectra of the seismic ground motion. It is suggested that the effect of underground group cavities on design seismic ground motion should be considered.

Key words: underground group cavity; amplification; in-plane; seismic ground motion; time domain; frequency domain; indirect boundary element method (IBEM)

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1 Introduction

Amplification of seismic ground motion by underground spaces or structures has been paid attention in recent years (Chen et al., 2004; He et al., 2009; Zhu et al., 2011; Liang et al., 2011).

All these studies calculated the amplification using FEM (finite element method) combined with artificial boundaries, since FEM has advantages of dealing with nonlinearity and spaces or structures of arbitrary shapes, however, most of these studies only considered single space or structure (Chen et al., 2004; He et al., 2009; Zhu et al., 2011), which did not deal with the interaction between two spaces or two structures. Liang et al. (2011) studied group tunnels composed of two tunnels, but they used the simple model of rigid bedrock, which unavoidably results in some approximation.

Liang and Ba (2012) studied amplification of plane SH waves by a cavity in layered half-space by using

indirect boundary element method (IBEM), and IBEM does not need artificial boundaries because it uses Green's functions that automatically satisfy the radiation conditions at infinity, and the numerical results are so accurate that they can be comparable to the analytical solutions. Liang et al. (2012) further presented time-domain amplification of out-of-plane seismic ground motion by group cavities in layered elastic half-space, and it was shown that, group cavities have significant amplification to seismic ground motion, and cavity interval has great effect on the amplification of the seismic ground motion.

This paper, as a continuation study of Liang and Ba (2012) and Liang et al. (2012), further studies amplification of in-plane seismic ground motion by group cavities in layered half-space both in frequency domain and in time domain by using IBEM, and it is shown by numerical examples that group cavities have significant amplification to seismic ground motion.

2 Method

The two-dimensional plane strain model is shown in Figure 1. Two circular cavities with radius a and depth d are located in a layered half-space. The half-

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† Corresponding author. e-mail: liang@tju.edu.cn

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space is composed of multiple soil layers with total thickness H and bedrock (homogeneous half-space). The layered half-space is excited by seismic waves incident from the bedrock with angle θ . It should be noted that, for the brevity only two circular cavities are considered in this paper, however, multiple cavities with arbitrary shape can be solved in the same way.

Figure 1 The model of two cavities in layered half-space for incident plane SV wave. The cavities are featured with radius a , depth d and interval B . The layered half-space is composed of n soil layers and bedrock. The plane SV wave is incident from the bedrock with angle θ .

We use the indirect boundary element method (IBEM) to determine the amplification of seismic ground motion by group cavities in layered half-space. First, we calculate the free-field response, so that we can determine the horizontal and vertical displacements on the surface of the half-space, and the horizontal and vertical stresses on the surface of the cavities. Then, horizontal and vertical distributed fictitious source loads are applied on the surface of the cavities in the free field. The corresponding Green's functions for the displacements and stresses are then calculated. Next, the amplitudes of the fictitious source loads are determined by boundary conditions that the stresses resulted from the free field and from the fictitious loads on the surface of the cavities vanish. These boundary conditions can be satisfied in an averaged sense by using weighted residual method. Finally, the time-domain response can be obtained from the frequency-domain response.

The exact dynamic-stiffness matrix (Wolf, 1985) of a layered half-space and the Green's function (Wolf, 1985; Liang and Ba, 2007) for the distributed loads acting on an inclined line in a layered half-space are used in this study. Compared with Wolf (1985), the Green's functions with non-singularity (Liang and Ba, 2007) can significantly raise the accuracy and computation efficiency.

2.1 Free-field response

By assembling the dynamic-stiffness matrices (Wolf, 1985) of the soil layers and the bedrock half-space, the equation of motion of a layered half-space can be expressed as

$$\mathbf{S}_{P-SV}\mathbf{U} = \mathbf{Q}, \quad (1)$$

where $\mathbf{S}_{P-SV}$ is the stiffness matrix of the layered half-space, $\mathbf{U}$ is the vector of the displacement, and $\mathbf{Q}$ is the vector of the external loads. Solving this equation gives the free-field responses, which consists of the in-plane free-field displacements on the surface of the half-space, and the stresses on the surface of the cavities.

2.2 Green's functions

Green's function denotes the dynamic response at any point in the free field (layered half-space) when a unit load is applied somewhere in the free field.

At first, the two interfaces in each layer are assumed to be fixed, and the corresponding reaction forces are calculated. The directions of these forces are then reversed, and they are applied as loads to determine the response. The total response can then be obtained by adding the response of fixed layers portion. More details can be found in Wolf (1985) and Liang and Ba (2007).

The displacements $\{u_g, w_g\}^T$ and stresses $\{t_{xg}(s), t_{zg}(s)\}^T$ of the scattered field can be expressed as

$$\begin{Bmatrix} u_g \\ w_g \end{Bmatrix} = \mathbf{g}_d(s) \begin{Bmatrix} p(s) \\ r(s) \end{Bmatrix} \quad (2)$$

and

$$\begin{Bmatrix} t_{xg}(s) \\ t_{zg}(s) \end{Bmatrix} = \mathbf{g}_t(s) \begin{Bmatrix} p(s) \\ r(s) \end{Bmatrix}, \quad (3)$$

where $\mathbf{g}_d(s)$ and $\mathbf{g}_t(s)$ are the Green's functions for the displacement and stress, respectively; and the elements in the vector $\{p(s), r(s)\}^T$ are the fictitious source loads in the x - and z -directions on the surface of the cavities.

2.3 Boundary conditions

The boundary conditions on the surface of the cavities can be expressed as

$$\int_s \mathbf{W}^T(s) \left(\begin{Bmatrix} t_{xg}(s) \\ t_{zg}(s) \end{Bmatrix} + \begin{Bmatrix} t_{xf}(s) \\ t_{zf}(s) \end{Bmatrix} \right) ds = 0 \quad (4)$$

with the weight function $\mathbf{W}(s)$. If $\mathbf{W}(s)$ takes unit matrix, the integral can be evaluated over each element separately. Substituting equation (3) into equation (4) gives

$$\mathbf{T}_p \begin{Bmatrix} p(s) \\ r(s) \end{Bmatrix} = \mathbf{T}_f \quad (5)$$

with

$$\mathbf{T}_p = \int_s \mathbf{W}^T(s) \mathbf{g}_t(s) ds$$

and

$$\mathbf{T}_f = - \int_s \mathbf{W}^T(s) \{t_{xf}(s), t_{zf}(s)\}^T ds.$$

Combining the solutions of equations (5) and (2), the surface displacements become

$$\begin{Bmatrix} u \\ w \end{Bmatrix} = \begin{Bmatrix} u_f \\ w_f \end{Bmatrix} + \mathbf{g}_d(s) \mathbf{T}_p^{-1} \mathbf{T}_f. \quad (6)$$

It should be noted that, the boundary conditions for zero stress on the surface of the half-space are satisfied automatically, since the free-field response and layered half-space Green's functions are obtained under the corresponding boundary conditions.

3 Verification of the accuracy

To verify the accuracy of the method of IBEM of this paper, we compare our results with those computed using FLUSH (Lysmer et al., 1975). Figure 2 shows verification of the accuracy, with the model (Figure 2a) and the comparison (Figure 2b) for the case of two cavities in a layered half-space with parameters in Table 1, subjected to the excitation of vertically incident El Centro wave at the bedrock. Both two cavities are featured with radius $a=5$ m and depth $d=10$ m, and with an interval $B=3a$. The horizontal acceleration time histories at $x/a=0$ and $x/a=4$ on the surface of the half-space are illustrated, with the upper ones in Figure 2b being the results by FLUSH and with the lower ones in Figure 2b the results by IBEM of this paper. Because the computer program FLUSH has the limitation in computation capacity, the total thickness of three 10 m soil layers

Figure 2 Verification of the accuracy. (a) Verification model. (b) Comparison of surface accelerations obtained by FLUSH and IBEM.

is used in the verification. Also, the computer program FLUSH can only handle rigid bedrock, so we take the bedrock of infinite stiffness in our model. It can be seen that the results by IBEM of this paper are in very good agreement with those by FLUSH.

Table 1 Soil parameters for verification of the accuracy

Soil layer No.	Thickness /m	Shear velocity /(m·s ⁻¹)	Density /(10 ³ kg·m ⁻³)	Damping ratio
1	10	150	1.75	0.05
2	10	175	1.775	0.05
3	10	200	1.80	0.05
Bedrock	∞	∞	2.10	0.02

4 Numerical results and analyses

We study the amplification of seismic ground motion due to two cavities in layered half-space, taking a typical site in Tianjin as an example. The parameters for the soil layers and bedrock are shown in Table 2. Both two cavities are featured with radius $a=5$ m and depth $d=10$ m.

4.1 Frequency-domain amplification

Figures 3–6 show the spectral amplification of motions at the surface of the half-space due to two cavities in layered half-space for $B/a=2.5, 5, 10$ and 20 , respectively, with x -axis expressing dimensionless frequency and corresponding true frequency, and with y -axis the ratio of surface accelerations to input accelerations at bedrock. The spectral amplification of surface motion at

$x/a=0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ and 15 are illustrated. For the purpose of comparison, Figure 7 shows the spectral amplification of motions at the surface of the half-space with only one cavity. A dimensionless wave frequency is defined as $\eta=\omega a/\pi\bar{c}_s$, where, ω is the circular frequency of incident wave, $\bar{c}_s$ is the averaged shear velocity of the soil layers where the cavities are located, and $\bar{c}_s=(150+175)/2=162.5$ m/s for the soil parameters in Table 2. For an acceleration time history of interval $\Delta t=0.02$ s, the highest frequency (Nyquist frequency) is $1/(2\Delta t)=25$ Hz, and the corresponding dimensionless frequency is 1.538 .

Table 2 Soil parameters of layered half-space for numerical analysis

Soil layer No.	Thickness /m	Shear velocity /(m·s ⁻¹)	Density /(10 ³ kg·m ⁻³)	Damping ratio
1	10	150	1.75	0.05
2	10	175	1.775	0.05
3	10	200	1.80	0.05
4	10	250	1.85	0.05
5	10	300	1.90	0.05
6	10	350	1.95	0.05
7	10	400	2.00	0.05
8	10	450	2.05	0.05
Bedrock	∞	500	2.10	0.02

It can be seen from Figure 3 (the case of $B/a=2.5$) that, when the incident frequency is low, the incident waves cannot feel the cavities, and spectral

Figure 3 Spectral amplification in the case of two cavities for $B/a=2.5$.

Figure 4 Spectral amplification in the case of two cavities for $B/a=5$.

Figure 5 Spectral amplification in the case of two cavities for $B/a=10$.

amplification is almost same as that of the free field; the maximal horizontal peak is 2.89 for $\eta=0.128$ at $x/a=3$ in the case of two cavities, which is amplified by 25.1% compared with 2.31 (for $\eta=0.069$), the maximal peak of the free field. The maximal vertical peak is up to 3.11 for $\eta=0.146$ at $x/a=2$, which is even larger than both the corresponding maximal horizontal peak in the case of two cavities and the maximal peak of the free field. It should be noted that the vertical free-field re-

sponses are resulted from the scattering and diffraction of waves around cavities, therefore, the vertical response at $x/a=0$ (middle point of the two cavities) is zero. It can also be seen that, the spectral amplification becomes larger and the band with the large spectral amplification becomes wider due to overlap of the free-field response and of the scattering and diffraction of waves around cavities.

From Figure 4 (the case of $B/a=5$) we can see that,

Figure 6 Spectral amplification in the case of two cavities for $B/a=20$.

Figure 7 Spectral amplification in the case of single cavity.

the maximal horizontal peak of the spectral amplification for the case of two cavities is 3.27 for $\eta=0.157$ at $x/a=0$, which is enlarged by 41.6% compared with that of the free field.

Now, let's see Figure 6 (the case of $B/a=20$), it can be observed that, the maximal horizontal peak of the spectral amplification for the case of two cavities is 2.90 for $\eta=0.166$ at $x/a=8$, while the maximal ver-

tical peak is 2.08 for $\eta=0.176$ at $x/a=9$, which shows that even when the two cavities are far from each other ($B=20a=100$ m), there is still large interaction between them, compared with the case of single cavity (Figure 7) that the maximal horizontal and vertical peaks of the spectral amplification are 2.48 and 1.53, respectively.

It can be seen from Figures 3–6 that, the spectral amplification depends strongly on observation po-

sition (x/a), cavity interval (B), and incident frequency (η), which shows that there is large interaction between cavities. Also, it can be observed that within frequency 1–5 Hz in each figure there are some bands, in which the horizontal responses in the case of two cavities are always larger than the free-field response, and most of energies of the vertical response are also focused within 1–5 Hz, which can be attributed to the amplification in time domain that will be discussed in the following section.

Therefore, it can be concluded that, there may be large interaction between cavities, and group cavities with certain interval may have significant effect on the spectral amplification of ground motion.

4.2 Time-domain amplification

For time-domain amplification of seismic ground motion by group cavities in layered half-space, Taft wave and El Centro wave with peak acceleration of 0.1 g are used as the excitation at the bedrock with the incident angle $\theta=90^\circ$. Figure 8 is the acceleration time histories of Taft wave and El Centro wave.

Figure 9 shows the envelopes of the horizontal and vertical PGAs (peak ground accelerations) on the sur-

face of the half-space under the excitation of Taft wave and El Centro wave, for $B/a=2.5, 3, 4, 5, 6, 7, 8, 9, 10$ and 20. Figures 10–11 show the acceleration time histories with the maximal values of the first two extreme PGAs and their response spectra. For the purpose of comparison, Figure 12 shows the envelopes of the horizontal and vertical PGAs on the surface of the half-space for the case of single cavity, and Figures 13–14 show the acceleration time histories with the maximal PGAs and their response spectra.

It can be seen from Figures 9 and 12 that, in the case of two cavities for Taft wave, the maximal horizontal PGA is 0.225 g, appearing at $x/a=0$ for $B/a=4$, and the amplification is up to 45.2% compared with 0.155 g, the PGA of the free-field response; and the amplification is 27.8% compared with 0.176 g, the PGA in the case of single cavity. While in the case of two cavities for El Centro wave, the maximum horizontal PGA is 0.208 g, emerging at $x/a=3$ for $B/a=2.5$, and the amplification can be 30.8%, compared with 0.159 g, the PGA of the free-field response; and the amplification can be 19.5%, compared with the 0.174 g, the PGA in the case of single cavity. For vertical PGA, the maximum value is 0.150 g

Figure 8 Incident Taft wave and El Centro wave.

Figure 9 Envelope of PGA in the case of two cavities.

Figure 10 Acceleration time histories in the case of two cavities.

at $x/a=1.5$ for $B/a=2.5$ in the case of two cavities for Taft wave, and 0.171 g at $x/a=1.5$ for $B/a=2.5$ in the case of two cavities for El Centro wave; while for the case of single cavity, the maximal value is 0.079 g for Taft wave, and 0.088 g for El Centro wave.

Therefore, it is easily concluded that group cavities may amplify the seismic ground motion significantly.

It is observed from Figures 10–11 and 13–14 that, in the case of two cavities for Taft wave, the horizontal PRS (peak response spectrum) is 0.983 g at $x/a=0$ for $B/a=4$, and 0.879 g at $x/a=2.5$ for $B/a=2.5$, and the amplifications are up to 84.4% and 64.9%, compared with 0.533 g, the PRS of the free field; and the amplifications are up to 31.4% and 17.5%, compared with 0.748 g, the PRS in the case of single cavity. Both the predominant periods of the two response spectra are 0.44 s, identical to that of the response spectrum of

the free field. While in the case of two cavities for El Centro wave, the horizontal PRS is 0.662 g at $x/a=3$ for $B/a=2.5$, and 0.641 g at $x/a=0$ for $B/a=3$, and the amplifications are 50.8% and 46.0%, compared with 0.439 g, the PRS of the free field; and the amplifications are 25.1% and 21.2%, compared with 0.529 g, the PRS in the case of single cavity. The predominant periods of the two response spectra are 0.54 s and 0.52 s, both close to 0.56 s, the predominant period of the free-field response spectrum.

For vertical response spectra in the case of two cavities for Taft wave, the peak values are 0.922 g at $x/a=1.5$ for $B/a=2.5$ and 0.656 g at $x/a=2$ for $B/a=3$, and both the predominant periods of the response spectra are 0.44 s; while in the case of two cavities for El Centro wave, the peak values are 0.522 g at $x/a=1.5$ for $B/a=2.5$ and 0.410 g at $x/a=2$ for $B/a=6$, and the

Figure 11 Acceleration response spectra (RS) in the case of two cavities.

Figure 12 Envelope of PGA in the case of single cavity.

Figure 13 Acceleration time histories in the case of single cavity.

Figure 14 Acceleration response spectra (RS) in the case of single cavity.

predominant periods of the response spectra are 0.32 s and 0.26 s, significantly different from 0.56 s, the predominant period of the free-field response spectrum.

So it is concluded that group cavities may also affect the spectra of the seismic ground motion.

5 Conclusions

The amplification of in-plane seismic ground motion by underground group cavities in layered half-space is studied using indirect boundary element method (IBEM). It is shown that group cavities may induce significant amplification to seismic ground motion, and

the amplification of PGA (peak ground acceleration) and its PRS (peak response spectrum) can be up to 45.2% and 84.4%, respectively, and group cavities may also affect the spectrum of seismic ground motion. It is suggested that the effect of underground group cavities on seismic ground motion should be considered in seismic microzonation.

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References

- Chen G, Zhuang H and Xu Y (2004). A study on influence of excavated shallow cavities on design parameters of ground motion in the soft site. *Chinese Journal of Geotechnical Engineering* **26**(6): 739–744 (in Chinese with English abstract).
- He W, Chen J and Yu P (2009). A study on influence of underground structure on ground surface response spectra. *Chinese Journal of Underground Space and Engineering* **5**(6): 1 098–1 102 (in Chinese with English abstract).
- Liang J W and Ba Z N (2007). Surface motion of an alluvial valley in layered half-space for incident plane SH waves. *Journal of Earthquake Engineering and Engineering Vibration* **27**(3): 1–9 (in Chinese with English abstract).
- Liang J W and Ba Z N (2012). Amplification of plane SH waves by a cavity in layered half-space. *Journal of Earthquake Engineering and Engineering Vibration* **32**(2): 14–24 (in Chinese with English abstract).
- Liang J W, Li F and Liu Z X (2011). Amplification of ground motion by metro cavities group. *Journal of Earthquake Engineering and Engineering Vibration* **31**(2): 31–39 (in Chinese with English abstract).
- Liang J W, Zhang J and Ba Z N (2012). Time-domain amplification of seismic ground motion by group cavities in layered half-space. *China Civil Engineering Journal* **45**(S1): 152–157 (in Chinese with English abstract).
- Lysmer J, Udaka T, Tsai C-F and Seed H B (1975). *FLUSH: A Computer Program for Approximate 3-D Analysis of Soil-Structure Interaction Problems*. EERC 75-30, Earthquake Engineering Research Center, University of California, Berkeley, 1–139.
- Wolf J P (1985). *Dynamic Soil-Structure Interaction*. Prentice-Hall, Englewood Cliffs, 179–368.
- Zhu X, Lou M and Kong X (2011). Influence of underground structures on ground motion field of engineering site. *Technology for Earthquake Disaster Prevention* **6**(1): 49–58 (in Chinese with English abstract).