Teleseismic receiver function using stacking and smoothing of multi seismic-records at a single station*

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Abstract Receiver functions (RFs) obtained using teleseismic wave records at a seismic station and synthetic seismograms indicate that RF with a single teleseismic wave record is related to the selection of record section and to the calculating parameters of the RF. The scatter noise contained in the seismogram also affects the quality of RF. A new method for calculating receiver function, stacking and smoothing multi-seismic records in a single station, is presented in this paper. The RF results using some records and some synthetic seismograms with different noises indicate prominent mantle discontinuity and thus prove that the method is effective and satisfied.

Key words: receiver function; mantle discontinuity; deconvolution; stability

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1 Introduction

The teleseismic receiver function (RF) is a common method to study the buried depths of discontinuities in crust and upper mantle (Ammon, 1991; Langston, 1979; Liu et al., 1996; Lawrence and Shearer, 2006; Yang and Zhou, 2001). By using a station's three-component teleseismic records, RF is an effective method to obtain the information about crust and upper mantle. Radial receiver function were used in popular. Radial receiver function is deconvolution of radial record of a teleseismic seismogram to vertical. If the noises in the teleseismic seismogram can be ignored, the RF would retain more information of the Earth media, and the instrument response and earthquake source influence would be gotten rid of effectively. Whereas if the noises contained in the teleseismic seismogram can not be ignored, the deconvolution of the radial record to vertical would be instable and variant (Liu et al., 1996).

The frequency domain deconvolution of the radial

record to vertical for calculating RF is used universal. To lower the influence of noise on RF and then get stable RF, some methods were used in many studies for investigating the discontinuities of crust and upper mantle, one of which is appending “water level” and Gauss filter (for example, Owens et al., 1984). The other methods can refer to Gurrola et al. (1995), Ligorria and Ammon (1999), Li et al. (2000) and Liu et al. (1996). The above-mentioned methods are effective for focal depth no more than 100 km (in crust or upper mantle) to study the discontinuities and crustal and mantle velocity structure, while for focal depth more than 100 km (especially discontinuities in mantle, like 410- or 660-discontinuities), the stability of RF needs further discussion.

It is well known that the phases related to 410- or 660-discontinuities are very weak, and these phases are mixed with noise. Accidental, these phases can be found in difficulty from the original seismogram. If noise contained in the teleseismic wave can be omitted, the phases of 410- or 660-discontinuities may be more clearly in RF than those in the original seismogram. If the noise contained in the teleseismic wave can not be omitted, it is suspectable that the RF is response of Earth media when incident seismic body waves pass through from bottom.

By using Owens' formula to calculate RF (Owens

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et al., 1984), we discuss the stability of the RFs and the influence of the record data section, “water level” and noise contained in record on RFs respectively. And we propose a new calculating method of RF, which stack and smooth multi-seismic records at a single station to obtain teleseismic receiver function (RFSSMS), and get clear discontinuities, proving the new method is effective.

2 Instability of RF

2.1 Influence of c on RF

To study the velocity structure of crust and upper mantle, Owens et al. (1984) defined the RF as

$$r_R(t) = \int \frac{R(\omega)Z^*(\omega)}{\varphi(\omega)} G(\omega) e^{i\omega t} d\omega \quad (1)$$

where $Z^*(\omega)$ is the complex conjugate of $Z(\omega)$, $\phi(\omega) = \max\{|Z(\omega)|^2, \omega \cdot \max[|Z(\omega)|^2]\}$, α is the width factor of Gauss filter $G(\omega) = e^{-\omega^2/4\alpha^2}$, c is “water level” for avoiding computing instability when $Z(\omega)$ near zero. α and c were gotten by trials and errors. Equation (1) is used extensively in studying the discontinuities in crust and mantle. Through RF computation with observed records and synthetic seismograms, we consider that $\alpha=1.5$ is suitable. The Gauss filter with $\alpha=1.5$ could suppress high-frequency noise and retain the information about discontinuities in crust and mantle. c is a subjective parameter, and different values of c can lead to different RFs. For discontinuities in crust and mantle (depth no more than 100 km) and with seismogram of satisfied signal to noise ratio, a suitable c can be found by trials and errors. While for discontinuities in mantle (410- or 660-discontinuities, for example), especially using the record section of 30–80 s after P phase, it is difficult to find a suitable c and thus obtain a stable RF. Therefore it is unsuitable using c parameter to increase the stability of RF in the case of studying mantle discontinuities.

2.2 Influence of data section selection of seismogram on RF

The seismogram data used for RF must contain the information about mantle discontinuities and not include the information about direct S wave. For this reason data section selection of seismogram must be proceeded before calculating RF. Because the limitation of Fourier transformation in formula (1), 2 048 or 4 096 data points may be used (corresponding to 102.4 s or 204.8 s respectively when sampling rate is 50 points per second). n_b express the data number before direct

P wave. The RFs are calculated with $n_b=100, 110, 120, \dots, 1\,900, 2\,000$ separately, we find different values of n_b always result in different RFs, and the difference in RFs is more obviously especially in the section of 30–90 s after direct P wave (Figure 1, $n_b=100$ to 300), which may contain the information about velocity structure and discontinuities of mantle. Because the stability of RFs are dependent on the value of n_b , even almost any difference between two value of n_b the RFs are different obviously, the RFs using formula (1) are unstable.

3 Influence of noises on RF

The observed seismogram can be expressed as

$$\begin{aligned} R(t) &= I(t) * S(t) * E_R(t) + N_R(t) + N_{mR}(t) \\ Z(t) &= I(t) * S(t) * E_Z(t) + N_Z(t) + N_{mZ}(t) \end{aligned}$$

Here, $I(t)$, $S(t)$ and $E_R(t)$ are responses of the instrument, earthquake source and seismic wave propagating media separately, $*$ stands for deconvolution, $N_{mR}(t)$ and $N_{mZ}(t)$ are station background noises in R and Z components, $N_R(t)$ and $N_Z(t)$ are scatter noises with seismic wave in R and Z components. And we can write seismogram in frequency domain,

$$\begin{cases} R(\omega) = D(\omega)E_R(\omega) + D(\omega)n_R(\omega) + N_{mR}(\omega) \\ Z(\omega) = D(\omega)E_Z(\omega) + D(\omega)n_Z(\omega) + N_{mZ}(\omega) \end{cases} \quad (2)$$

We discuss the stability of RF under different noises with synthetic seismograms. The synthetic seismograms are made by the matrix method for seismic wave propagation in stratified media (Kennett, 1983).

3.1 Synthetic RF with no noise

If noise do not exist in seismic records, we compute the synthetic RFs with formula (1) using Iasp’91 model. When $c=0-0.018$, the RFs are the same, n_b have no effect on the RFs. Other models are used randomly based on Iasp’91 model, the RFs are stable and unique also.

3.2 Influence of white background noise in station on RF

If the seismograms contain white background noise in station and amplitude of direct P is larger than that of the noise, the synthetic RFs are stable only when $c=0-0.018$ and with different values of n_b .

3.3 Influence of periodic low frequent Earth pulsation on RF

We add harmonic waves with period of 3–10 s on synthetic seismogram using Iasp’91 model, and calculate the RFs. We find periodic low frequent Earth pulsation do not affect RFs if amplitude of direct P is larger

Figure 1 Influence of data section selection of seismogram on RFs. (a) The R and Z components in a seismogram in MDJ station. Earthquake occurrence date: 1994-05-02; epicenter location: 1.09°S , 97.34°E ; magnitude: $M_S 6.4$; epicentral distance: 53.8° ; epicentral azimuth: 221.29° . (b) Different RFs with different values of n_b . For comparison the difference conveniently, these RFs are stacked together (on bottom of the figure).

than that of the noise.

3.4 Influence of scatter noise on RF

The observed seismogram contains scatter noise produced by Earth unwell-distributed medium under seismic station. The scatter noise is companied with seismic signal, and it is difficult to be removed from the seismic signal using filters. If we consider the single scatter of direct P wave, A_R and A_Z are the amplitudes of P coda wave in R and Z directions separately (Zhou, 1990),

$$\begin{aligned} A_R &= A_{PR} C_R e^{-\frac{\omega t}{2Q}}, \\ A_Z &= A_{PZ} C_Z e^{-\frac{\omega t}{2Q}}. \end{aligned}$$

Here ω is angular frequency, Q is medium quality factor, A_{PR} and A_{PZ} are the amplitudes of direct P wave in R and Z directions respectively, C_R and C_Z are proportional factors. We let $\omega=0.6$ Hz, and $Q=100$. Then $n_R(t)$ and $n_Z(t)$ could be

$$\begin{aligned} n_R(t) &= A_R f_R(t) \\ n_Z(t) &= A_Z f_Z(t) \end{aligned}$$

$f_R(t)$ and $f_Z(t)$ are white noise time sequences. The synthetic seismogram superpose on these scatter noises, we get hundreds of synthetic seismograms with coda noise, and the RFs are calculated. When both C_R and C_Z are greater than 0.4, the RFs are much different from the RFs of no noise. It is obviously that $N_R(t)$ can affect the result of RF when the amplitude of $N_R(t)$ reaches a level. We can say in some way that the receiver function can not be used in studying the mantle discontinuities if the scatter effect is strong enough under the seismic station.

4 Stacking and weighted smoothing multi seismic-records at a single station

The noise in seismic waves, especially the scatter noise, can lead to different RFs although relevant earthquake events occurred in the same region. We calculate RFs using equation (1) for each of events, and we find that the times of amplitude peaks in every RF are different from each other obviously. Therefore, it is not suitable to stack these RFs in time domain to get relevant phases of discontinuities. But in frequency domain, the stacking technique can achieve good result. In formula (1), the medium response in Z direction is

regarded as a δ function in frequency domain (Ammon, 1991; Langston, 1979), but the observed medium response in Z direction is not a δ function because of existing jump phenomenon in amplitude-frequency curve of Z direction. According to frequency spectrum theory, smoothing these jumps appropriately can remove noise effectively.

We consider N teleseismic records at a single station, and these N telequakes are in the same region. If we restrict the range of the region, the seismic paths of N quakes are the same under the seismic station, especially after P transform to S at a discontinuities in mantle. We stack the N records in frequency domain at R and Z directions separately, and smooth the stacked result with five-point weighted smoothing technique, the smoothing factor $h(\omega)$ is

$$h(\omega) = \frac{1}{9}(1, 2, 3, 2, 1)$$

After the stacking and weighted smoothing, $R(\omega)$ and $Z(\omega)$ would be replaced by $\tilde{R}(t)$ and $\tilde{Z}(t)$. The RF is written as

$$r_R(t) = \int \frac{\tilde{R}(\omega)}{\tilde{Z}(\omega)} G(\omega) e^{i\omega t} d\omega$$

This method for RF is named stacking and smoothing multi-seismic records (RFSSMS). Here, no “water lever” c used.

5 Case study on MDJ and HIA stations

When using RFSSMS to compute RF at a single seismic station for exploring the velocity structure under the station, the selection of teleseismic events is important. The number and the location of the events can affect the quality of RFs. Before calculating RFs with actual seismic wave data, we selected and preprocessed the seismic records. The principle of selection is that the data length is long enough, the ratio of signal to noise is high enough and P phase is clear in original seismogram. We choose some records at MDJ station and HIA station to test the effectiveness of RFSSMS. Here, we choose more than five teleseismic records for each station (Figures 2 and 3).

From Figures 2 and 3, the RFs obtained using RFSSMS have stable and obvious $P_{410}S$ and $P_{660}S$ phases in RF curves.

Figure 2 Using RFSSMS method to process the records of M_1 group at MDJ station. (a) The selected teleseismic records at MDJ station (M_1 group); (b) ①–⑤ are RFs using the single record with different values of n_b and stacked together, ⑥ is the RF using RFSSMS with different values of n_b and stacked together.

Figure 3 Using RFSSMS method to process the records of H_1 group at HIA station. (a) The selected teleseismic records at HIA station (H_1 group); (b) ①–⑤ are RFs using the single record with different values of n_b and stacked together, ⑥ is the RF using RFSSMS with different values of n_b and stacked together.

6 Conclusions

1) Through theoretical and actual RF calculation, we get a stable RF in a single station after stacking several records in R and Z directions separately in frequency domain, and smoothing the stacked result with five-point weighted smoothing technique. The RFSSMS can effectively suppress the influence of noise on RF. The RF using RFSSMS do not use "water level" c and thus avoid the subjectivity for RF, and relevant phases of major discontinuities in mantle are prominent and clear in the RFs. But the forms and arrival times of these phases in RFs using formula (1) are not the same with different calculating parameters.

2) For the number limitation of teleseismic records, RFSSMS can not get rid of all noise effect, but if the number of data is larger, the quality of RF would be better. Whether the data is enough in a region plays a important role in effectivity of RFSSMS.

3) The related phases for mantle discontinuities are weak, and often mixed with other strong phases. We can still not improve RFSSMS to distinguish all the phases from seismic records.

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