Meshing effects of the 3-D FEM numerical modeling in seismo-electromagnetics: An application in selectivity of seismic electric signal (SES)*

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Abstract We investigated how density and quality of mesh around interest domain affect electromagnetic (EM) responses of 3D Earth layered media using finite element method (FEM). Effect of different mesh shapes was also investigated using a method of mixing structured and unstructured mesh. As a case study, we estimated the effects of meshing on selectivity phenomenon of seismic electric signal (SES). Our results suggest that the relative errors resulting from mesh effects may not be negligible, which may lead to some unconvincing explanation of the SES selectivity based on the numerical modeling results.

Key words: 3D electromagnetic numerical modeling; FEM; meshing; SES

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1 Introduction

As a new branch of geoelectromagnetism, seismo-electromagnetics focuses on the electromagnetic phenomena associated with earthquake, and aims at understanding the nature of seismogenic process from the viewpoint of electromagnetism (Huang, 2004). There are numerous independent reports on the electromagnetic (EM) phenomena associated with earthquakes (Central Earthquake Working Committee, 1969; Sobolev, 1975; Zhao and Qian, 1978; Varotsos and Alexopoulos, 1984a, b; Qian et al., 1985; Fraser-Smith et al., 1990; Johnston, 1997; Hayakawa et al., 2000; Nagao et al., 2002; Hattori et al., 2004; Huang and Liu, 2006). Such EM phenomena, especially ultra-low-frequency (ULF) EM signals, have ever been attempted to being applied to short-term prediction of earthquakes (Varotsos, 2005; Hayakawa et al., 2008). Although the seismo-EM phenomena have not been fully understood till now, there are some continuous attempts to the study on possible mechanisms of seismo-EM phenomena (Bernard,

1992; Johnston, 1997; Vallianatos and Tzanis, 1998; Guo et al., 1999; Huang, 2002). Much attention has been paid to some debated phenomena such as the selectivity phenomenon of seismic electric signal (SES) (Varotsos and Lazaridou, 1991). Besides theoretical model analysis (Varotsos et al., 1998; Varotsos, 2005), some laboratory analogue experiments (Meng and Dosso, 1990; Huang and Ikeya, 1998, 1999; Huang, 2005; Lin, 2010) and numerical modeling (Sarlis et al., 1999; Kulchitsky et al., 2004; Huang and Lin, 2010a, b) have been adopted in the explanation of the SES selectivity phenomenon.

Due to low cost and convenience for field tests and indoor experiments, numerical modeling becomes a useful tool in solving geo-EM problems, especially EM sounding. Recently, there are also some attempts to applying numerical technique to seismo-EM problems, e.g., Sarlis et al. (1999) made a numerical modeling with a conductive channel to explain the SES selectivity using the EM1DSH software, which was developed for calculating either the layered Earth model including displacement currents or the two-layer model with embedded thin sheets but without displacement currents (Hoversten and Becker, 1995). As a further study, Huang and Lin (2010a, b) made a numerical simulation

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on SES selectivity based on 3D finite element method (FEM, Xu, 1994). Their results indicated that either the heterogeneous ground surface or the underground conductive channel could lead to the selectivity phenomenon of SES.

In 3D EM numerical modeling, there are mainly three methods: finite-difference method (FDM), integral equation method (IEM) and FEM. In comparison with FDM and IEM, FEM is a flexible tool for complex models, because either triangular or tetrahedral elements it uses can deal with arbitrary geometry shapes (ore-bodies, cylindrical wells, topography, etc.) (Nam et al., 2007). Although many improvements have been achieved in FEM, such as use of nodal and edge elements (Ndlec, 1980, 1986; Sugeng, 1998; Jin et al., 1999; Yoshimura and Oshiman, 2002; Ma and Liu, 2006; Nam et al., 2007) for addressing the discontinuities of EM fields along material discontinuities, high accuracy solutions for modeling with complex geometry still requires much computational cost. Some researchers have investigated structured (rectangular, rectangle-triangle and hexahedral) and unstructured (triangular and tetrahedral) meshes for decreasing required memory and time in computation while increasing convergence and accuracy (Ren et al., 2008; Tang and Wang, 2008; Key and Weiss, 2006; Li and Key, 2007). Generally, unstructured mesh can improve accuracy of complex boundaries with the same nodes comparing with structured mesh, especially near some singular points (Tang and Wang, 2008). However, unstructured mesh also has its own deficiencies, e.g., the prior information about the domains under interest should be known in order to interact with the computer and the nodes distribution resulting from it is still not optimal. Consequently, the adaptive finite element method (AFEM) is developed recently in simulating the EM response of a petroleum engineering resistivity logging instrument in a borehole environment (Demkowicz and Vardapetyan, 1998; Rachowicz and Demkowicz, 2000, 2002; Cecot et al., 2003), which can automatically control the optimal distribution of the nodes to minimize the error of a prescribed quantity of interest with respect to the problem.

Huang and Lin (2010b) investigated the possible influences on 3D FEM numerical simulation of SES using a commercial FEM software, COMSOL Multiphysics[®]. After unstructured meshing, they adopted different types of boundary condition, varied size of calculated region, various combinations of electrical parameters of the media, and different models containing heterogeneous ground surface in-homogeneity of

conductivity or underground conductive channel. Their results indicated that selection of model parameters should be paid attention to. SES selectivity may depend on the detailed electric structure and contrast of conductivity in the model.

In this study, we investigated the effects of meshing on the 3D numerical modeling of SES using the FEM software, COMSOL Multiphysics[®]. After the validation test of the software, we investigated the effects of mesh quality, density and type based on various models with different meshing methods. As an example of modeling SES selectivity, we investigated numerically the response of SES to the meshing effects for a revised model after Sarlis et al. (1999) and Lin (2010).

2 Methods

We used the Maxwell's equations in the frequency domain (Zhdanov, 2009) under quasi-static approximation and the Dirichlet boundary condition (Abubakar et al., 2008) to deal with the boundary value problem. However, COMSOL does not solve Maxwell's equations directly. Instead it uses the Helmholtz potential decomposition with the Coulomb gauge condition to ensure that there are no spurious solutions in the EM potentials (Badea et al., 2001; Mitsuhata and Uchida, 2004). Following this approach we can get the weak form (Xu, 1994) using Galerkin weighted residuals method. In this study, we only considered the case with low frequency in Earth media, so multi-grid solver (Mulder, 2006) was adopted in COMSOL, which has a high efficiency to solve the large system of linear-equations especially in 3D.

In this study, we conducted quality measurement for elements. The quality measure is related to the aspect ratio, which means that anisotropic elements can get a low quality measure even though the element shape is reasonable (COMSOL, 2008). For tetrahedral elements, COMSOL Multiphysics[®] computes the mesh quality as

$$q = \frac{72\sqrt{3} V}{\left(\sum_{i=1}^6 h_i^2\right)^{\frac{3}{2}}}, \quad (1)$$

where V is the volume, and h_i are the edge lengths of the tetrahedron. The values of q would affect the numerical solutions significantly if q is very small. The numerical results with a large q will be more credible than those with a small one (COMSOL, 2008).

We judged whether a mesh is coarse or fine by es-

timating the number of elements on unit length, λ , as

$$\lambda = \frac{n}{L}, \quad (2)$$

where n is the number of elements distributed on lines with a length of L . In this study, we specified some lines to increase or decrease n according to our models.

3 Validation test

As a validation test of the COMSOL FEM software, we calculated the electric field x -component at 1 Hz on the top surface of a three-layer model with a point electrical dipole source (with x -directed dipole moment) in the third layer (Figure 1a). The point electric dipole source can be given in the model conveniently by the COMSOL FEM software. In this example, we

adopted a point dipole source with a moment magnitude of 10^4 A·m along the x -direction. We can obtain the theoretical solutions (Wei et al., 2007) for the same model. The numerical results obtained from COMSOL are in good consistency with the theoretical ones (Figure 1b). This can be further confirmed by the relative error given in Figure 1c.

Note that we conduct the following two element numbers in the validation test of FEM. The first is the ground line on the top of the first layer (Figure 1a) with $n_1=200$ and $L_1=20$ km. The second is the cross-line on the horizontal plane of the point electric dipole source at a depth of 5 km (Figure 1a) with $n_2=500$ and $L_2=10$ km. Figure 2 shows the mesh quality on the horizontal plane of the point electric dipole source.

Figure 1 Validation test of the COMSOL FEM software. (a) An x -directed vertical cross-section passing through a point electric dipole source in a 3D three-layer model, which does not draw the air domain. The computation size is five times as much as the displaying model including air domains which has a conductivity of 10^{-14} S·m $^{-1}$. (b) The x -components of electric field at 1 Hz are consistent well with the theoretical ones (black line). Circles represent the results of point electric dipole source. (c) The relative error between the numerical and theoretical results.

Figure 2 The mesh quality on the horizontal plane of the point electric dipole source with $n_2=500$ and $L_2=10$ km (see text for details).

4 Results

4.1 Mesh density effect

Huang and Lin (2010b) studied the effects of the boundary condition and the size of calculated region comprehensively. In this study, we investigated the influence of the density of mesh around point source on numerical results.

As an example, Figure 3a shows the numerical results obtained from the mesh conditions of $n_1=50$,

$L_1=20$ km, and $n_2=1$, $L_2=10$ km for the model given in Figure 1a. It is coarser than that from the mesh conditions in Figure 1b. Figure 3a indicated that there are some clear deviations of the above numerical results from the theoretical results. Such phenomenon can be further confirmed by their relative errors (Figure 3b), which is at least one order larger than those obtained from the mesh conditions of $n_1=200$, $L_1=20$ km, and $n_2=500$, $L_2=10$ km (Figure 1c).

Figure 3 Results of electric field x -component at 1 Hz with different density of mesh. (a) The numerical results (circles) obtained from a relative coarse mesh around the point electric dipole source. The solid line represents the theoretical ones. (b) The relative error between the numerical and theoretical results.

4.2 Meshing way effect

In order to investigate the possible effects due to the ways of meshing, we adopted different ways of meshing for the model given in Figure 1a and made the numerical calculations. Figure 4a shows the mesh quality of the meshing way with all the domains of the model

using tetrahedral elements. The related numerical results are given in Figure 1b. As another meshing way, we first let the regions from top surface to the depth of point source, which are called “main domain”, be tetrahedral elements, and let remaining regions, which are called “remained domain”, be triangular prismatic

Figure 4 Mesh qualities for different ways of meshing. (a) The quality of mesh using tetrahedral elements for all domains. The average mesh quality is about 0.703, with a maximum and a minimum of 0.958 and 0.174, respectively. (b) The quality of mesh using tetrahedral elements and triangular prisms, “swept meshing”. The average q is about 0.498, with a maximum and a minimum of 0.996 and 0.006.

elements by sweeping.

4.2.1 Mixture meshing

The average mesh quality, q , for the meshing way in Figure 4a, is about 0.703, with a maximum and a minimum of 0.958 and 0.174, respectively. In the case of the mixture meshing given in Figure 4b, the average q is about 0.498, with a maximum and a minimum of 0.996 and 0.006.

Figure 5a indicated that the numerical results of mixing elements of tetrahedron and prism have large deviations from those of the theoretical ones and tetra-

hedral elements. The relative error between the numerical results of mixture meshing and the theoretical ones is more than 10% (Figure 5b). Furthermore, its calculating time is about three times as much as that of the tetrahedral elements (see Table 1). Table 1 also gives the comparisons of some other solution parameters for these two ways of meshing method. Actually, prismatic elements are comparatively, simpler to implement, but the tetrahedral approach is potentially much more flexible, allowing high resolution where warranted, and hence more efficient 3D modeling (Sarakorn, et al., 2008).

Figure 5 The x -component of electric field at 1 Hz for the mesh of Figure 4. (a) Comparison between the numerical results (circles and triangles) and the theoretical ones (black line). Circles are the same as results in the Figure 1b, and triangles represent the results from the mesh in Figure 4b. (b) Relative errors. The triangles (circles) represent the relative errors between numerical results and theoretical ones.

Table 1 Comparison of solution parameters between the different ways of meshing, tetrahedral and mixed elements

Meshing method	Number of elements	Number of freedom degrees	Elapsed solution time/s	Number of iteration for 1 Hz	Number of iteration for 10 Hz
Tetrahedral elements	294 869	1 873 976	419.3	3	7
Mixed elements	219 610	2 052 578	1 283.8	31	81

4.2.2 Model with semi-torus using tetrahedral elements

We added a semi-torus into the three-layer model. Figure 6a is its cross section, and Figure 6b is its mesh quality under the conditions with $n_1=200$ and $L_1=20$ km, $n_2=500$ and $L_2=10$ km, and $n_3=200$ and $L_3=3.77$ km. L_3 is an arc line along the top of the semi-torus. Its average q is about 0.794 with a maximum and a minimum of 0.977 and 0.385. In the case of a relative coarse meshing given in Figure 6c, we let $n_1=50$, $n_2=2$, $n_3=2$, and let L_1 , L_2 , and L_3 , respectively be the same as the above ones given in Figure 6b. It has a relative small-

er average q of 0.539 with a maximum and a minimum of 0.821 and 0.368. Figure 7a indicated that some significant deviations occur between the numerical results from the meshes of Figures 6b and 6c. The maximum relative error is more than 10% near the center of semi-torus (see Figure 7b).

5 Discussion and conclusions

As a case study, we adopted the model (see Figure 8) containing a relative conductive semi-torus channel and an ellipsoid underlying, which is revised from Sarlis

Figure 6 Cross sections of the semi-torus model with different mesh density. (a) Cross section for the resistivity structure of the semi-torus model. (b) A relative fine mesh. (c) A relative coarse mesh.

Figure 7 The x -component of electric field at 1 Hz with different mesh densities for the model in Figure 6a. (a) Dashed line represents the results of the relative fine mesh in Figure 6b. Triangles represent the results of relative coarse mesh in Figure 6c. (b) The relative error between the results of meshing conditions in Figure 6b and Figure 6c.

Figure 8 The 3D model with a conductive semi-torus channel and an ellipsoid modified from Sarlis et al. (1999) and Lin (2010). The color bar is logarithm scale for conductivity of the model. The air, the first layer (with 500 m thickness) and the second layer have the resistivity of $10^{14} \Omega\cdot\text{m}$, $200 \Omega\cdot\text{m}$ and $4\,000 \Omega\cdot\text{m}$, respectively. The conductive semi-torus channel and the ellipsoid have the same resistivity of $10 \Omega\cdot\text{m}$. The center of semi-torus is at the position of $(x=45.9 \text{ km}, y=0 \text{ km}, z=-9.9 \text{ km})$. The diameter of the channel is 1 km. The ellipsoid with a inclination of 45° is centralized at the position of $(x=47 \text{ km}, y=0 \text{ km}, z=-11 \text{ km})$. Its a -semiaxis, b -semiaxis and c -semiaxis are 1 km, 7 km and 15 km, respectively.

et al. (1999) and Lin (2010) to study the mesh quality effects on the numerical results of the response of SES. Sarlis et al. (1999) used the model which assumes that the dipole current source lies in the vicinity of a conductive path which terminates below the Earth's surface. Their results indicated that the electric field can be am-

plified in the region above the end of the conductive channel. Lin (2010) adopted a similar model to a conductive fault to study the effects of some parameters of the source and fault on the numerical results of electric field. His model results confirmed that the conductive fault or path could explain the selectivity phenomenon

of SES. His study on the frequency effect of the source indicated that the significant amplified phenomenon of SES would occur mainly at a frequency less than 2 Hz. Therefore, we focused our calculations of the electric field at 1 Hz in this study.

As the further investigation of meshing effects on the numerical simulation of SES selectivity phe-

nomenon, we arranged different elements (n_4) on the arc line ($L_4= 59.92$ km) along the top of the conductive path to get a fine or a coarse mesh. Figure 9a shows a relative fine meshing with $n_1=150$, $L_1=160$ km, $n_2=150$, $L_2=20$ km, $n_4=100$, and $L_4=59.92$ km. Its average q is about 0.701, with a maximum and a minimum of 0.998 and 0.059. For a relative coarse meshing in

Figure 9 Mesh qualities with different mesh density for the model in Figure 8. (a) A relative fine mesh. (b) A relative coarse mesh.

Figure 9b, we let $n_1=23$, $n_2=2$, and $n_4=35$, and keep L_1 , L_2 , and L_4 unchanged. Its average q is about 0.596, with a maximum and a minimum of 0.989 and 0.014, respectively.

In Figure 10a, the dotted line is the result from Figure 9a, the dashed line is from Figure 9b and the solid line is the theoretical result without conductive

channel and ellipsoid. We can find that on the outcrop (about $x=40\text{--}55\text{ km}$) of the channel, the results from a fine meshing condition (dotted line) are larger than that from a coarse meshing condition (dashed line). Their relative errors in the vicinity of the outcrop of the channel could reach 30% and more (see Figure 10b).

Figure 10 The numerical results of the electric field at 1 Hz for the model in Figure 8. (a) Dotted line represents the numerical result of a relative fine mesh in Figure 9a, dashed line represents the one of a relative coarse mesh in Figure 9b, and the solid line is the theoretical one without the conductive channel and ellipsoid. (b) The relative errors between the results of the fine mesh (Figure 9a) and coarse mesh (Figure 9b).

Figure 11 The relative errors at $x=50.8\text{ km}$ (see text for details) for different frequencies.

In order to investigate the possible frequency effects on the above relative errors, we performed the similar calculations at a frequency varied from 1 Hz to 10 Hz using the fine mesh in Figure 9a and the coarse mesh in Figure 9b. Because the point at $x=50.8\text{ km}$ is the location where a local maximum of the dotted line appears (Figure 10a), for simplicity, Figure 11 shows only the relative errors at $x=50.8\text{ km}$. It suggested that the relative errors varied non-linearly with frequency. Such effect may lead to a more complicated situation in the numerical simulation of SES selectivity.

Although FEM has been approved an efficient tool for 3D seismo-EM numerical modeling, our study indicated that great care should be taken in explaining SES selectivity based on the numerical results, because the explanation of SES selectivity phenomenon may not be convinced if there is a large relative error due to meshing effect (Figure 10) or other model parameters (Lin, 2010).

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