Theoretical and experimental study of 3-D initial fracture and its significance to faulting*

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Abstract The experimental results of 3-D fracture under compression are introduced in brief and the theory of stress criterion of 3-D fracture is studied. Methods to imitate initial fractures are developed. It is pointed that there are important defects in the extreme value (EV) method ever proposed by Palaniswamy and Knauss. The major defect lies in that only two Euler angles (2EA) are considered, but another one is neglected. If the variation of all the three Euler angles (3EA) are considered, one can get better result which is consistent with the observation of faulting that extends on curved surfaces but not on planes. The method of evaluating maximal normal stress direction vector (NSDV) is proposed and further proved to be equivalent to the 3EA method. It is proved that the NSDV method can be further optimized to the method of composition of the first principal differential plane (CFPDP). The results from CFPDP method can fit the curved surfaces of initial growth observed in the experiments of 3-D fracture. The CFPDP method can also be used to interpret the 3-D fractures of the slipping section between the asperities in the buried fault plane that is modeled as ellipse crack. The results of 3-D fracture can be applied to interpreting the related problems of faulting including the mechanism of a lot of shatter rocks with different dimensions, the cause of earthquakes occurred at the edge of plate under low shear stress, and the mechanism of anisotropy caused by the extensive dilatancy anisotropy (EDA) cracks.

Key words: fracture mechanics; 3-D fracture; fault; fracture criterion

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1 Introduction

During the conduction of the Wenchuan Earthquake Fault Scientific Drilling (WFSD) project, Fan et al. (2009) found a great deal of break rocks in the in-situ fault drilling WFSD-1 in Wenchuan, of which part was fractured in the Wenchuan earthquake, and part has pre-existed. Tian et al. (2009) studied the seismic velocity anomalies of Longmenshan fault zone after Wenchuan $M_S8.0$ earthquake and proposed that it is caused by the rock break of the fault during the after-shocks of Wenchuan $M_S8.0$ earthquake. More and more

evidences show that the earthquake faulting is not along plane without width, but a break interlayer with definite width. According to Ding et al. (2004), in the rupture process of Kunlun earthquake ($M_S8.1$) on November 14, 2001, the rupture fault in deep is much wider than that on ground surface. The width of rupture on ground surface along a measuring line is only 2 m, but it approaches to 100 m in the depth of 30 m. As well known, it is found that there is a lot of break rock and fault gouge in the fault zone. The fault gouge forms mylonite under high temperature and high pressure.

The above phenomena show that the earthquake faulting is quite complicated and it can not be interpreted simply as 2-D shear slipping of fault plane, but a 3-D fracture. The 3-D fracture means natural fracture of 3-D crack (or fault), e.g., the surface or interior crack (fault) are often modeled as semi-ellipse or ellipse crack.

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1.1 Several experiments of 3-D fracture

The earliest experiment of 3-D fracture is Hertz contact that is formed by compressing a cylindrical steel indenter of the diameter, 1.59 mm, 3.13 mm and 6.35 mm, respectively, on the block of Silicate glass (Roesler, 1956). Geometry and loading are shown in Figure 1a. Conical crack extends from the edge of contact area, the angle between the generatrix of the cone and the symmetry axis is about 68.5° (Figure 1b).

The geometry and loading for mode I and mode III fracture (Sommer, 1969) are shown in Figure 2a. A crack is formed on a circular specimen made up of AR-

glass by using a razor blade. The torsion and tensile forces are applied to the specimen at the same time. The tensile loading is acted by the high pressure water filling into the gap of the crack. One can observe double lances-shape initial growth (Figure 8b) and a lot of fracture extending to the outer edge of the specimen (Figure 2b) by using electron microscope.

The geometry and loading for mode III (anti-plane-shear) fracture (Knauss, 1970) are shown in Figure 3a. The material of specimen is Solithane 113 glass composed of a two-component cross-linked polymer in the 50/50 composition. The crack was produced with a

Figure 1 Cone-shaped fracture surface at the edge of contact area of glass specimen (Roesler, 1956). (a) Geometry; (b) View of crack extension.

Figure 2 Modes I and III fracture (Sommer, 1969). (a) Geometry where $1''=1$ inch (25.40 mm); (b) Extending of crack.

0.254 mm thick hollow ground jeweler saw blade and the bottom of the crack was extended 0.254 mm with a razor blade. Brass flat stock was glued to the flanks of the crack for applying the shear force to the specimen in an instron tester. These bars were kept parallel to each other during the loading. Multiple para-helical-shaped fractures appear at the front of cracks, as shown in Figure 3b.

1.2 Normal stress fracture criterion

Figures 2b and 3b show that the 3-D crack does not extend along its own plane but generate multiple twisted and turned tensile fracture surfaces. There are two intersecting lines between the fracture surface and the original crack plane. As well known, there is only one intersecting line between two planes. In such way, it is obvious that these initial fractures are not planes

Figure 3 Pure mode III (anti-shear) fracture (Knauss, 1970). (a) Geometry; (b) Close-up view of crack extension.

but curved surfaces.

Summarizing these 3-D fracture experimental results, Palaniswamy and Knauss (1978) proposed the normal stress criterion: (1) The fracture extends along the direction perpendicular to the maximal principle stress σ_1 ; (2) The initial fracture starts when σ_1 near the front of crack approaches to critical value.

1.3 Imitation of the initial fracture surfaces

Palaniswamy and Knauss (1978) proposed a method to imitate the 3-D fracture surfaces by using double Euler angles (2EA) (Figure 4). Suppose the orthogonal coordinates of the point P is (x, y, z) , and its cylinder coordinate is (r, θ, z) ($-\pi \leq \theta \leq +\pi$). z -axis is the tangent line of the crack front, and x - z -plane coincides with the original crack plane (Figure 4). They computed the maximum of normal stress σ_N in the plane obtained by rotating the x - z plane about the z -axis (locally tangent to the crack front) by an angle δ and then

Figure 4 Local coordinate system at the front of a crack and definition of orientation of crack extension plane (Palaniswamy and Knauss, 1978).

rotating about the new x -axis (say, x' axis) by an angle ϕ to get the resulting plane (Figure 4). However, they did not give the results of imitation. In fact, the imitating result by using the 2EA method is only a plane but not a curved surface.

Li (1990, 1991) proposed a method of composition of the first principal differential planes (CFPDP) and successfully imitated the 3-D initial curved surface. But he did not point out the defect of the 2EA method presented by Palaniswamy and Knauss (1978), and did not illustrate the difference of CFPDP method from the 2EA method. In such case, the theory of 3-D fracture needs to be further improved.

2 Experiments of 3-D crack propagation under compression

As most of the faults are under compression and belong to fracture of mode II and III, it is necessary to introduce the experimental results and to have a comprehension of mode II and mode III fracture (Teng et al., 1987, Yin et al., 1989, 1991).

2.1 Geometry and loading description

The non-penetrated (surface of semi-ellipse) crack is pre-generated in the center of a plate and is oblique to the direction of uniaxial compression loading (Yin et al., 1988; Li, 1991). The crack surfaces are under shear loading and are closed in glass and organic glass when the specimen is pressed. No lubrication oil is filled into the crack surfaces in group A of silicate glass specimen, but in group B of silicate glass specimen, some engine oil is filled into the crack surfaces.

2.2 Geometry and principle of the experiment

An orthogonal coordinate system is set up, as shown in Figure 5. Suppose the x -axis is along the major axis of the semi-ellipse, the y -axis is along the minor axis of the semi-ellipse and both are perpendicular to the surface of the plate. The z -axis is perpendicular to

Figure 5 Geometry of 3-D fracture of surface crack under uni-axial press. The left is the local side looking of crack. h is the thickness of the plate of the specimen, a is the length of half major axis and b is the length of half minor axis of the ellipse.

the crack surface. The origin point O is located in the centers of the semi-ellipse. x -, y - and z -axis consists of a right hand coordinate system (Figure 5). The angle β between the direction of the semi-ellipse and the press loading σ is called crack angle. For uniaxial loading, the boundary conditions are

$$\begin{aligned}\sigma_{xx} &= -\sigma \cos^2 \beta, & \sigma_{zz} &= -\sigma \sin^2 \beta, \\ \tau_{xz} &= -\sigma \sin \beta \cos \beta,\end{aligned}\quad (1)$$

where σ is positive for tensile. For simplicity, the friction along the crack surfaces is supposed as uniform. So the valid shear stress along the crack surface is

$$\tau_e = \tau_{xz} - f\sigma_{zz}, \quad (2)$$

where f is frictional coefficient. The stress intensity factors are (China Academy of Aeronautics, 1981, 1993)

$$K_{II} = M_2 \tau_e \frac{\sqrt{\pi b}}{E(k)}, \quad K_{III} = M_3 \tau_e \frac{\sqrt{\pi b}}{E(k)}, \quad (3)$$

where $E(k)$ is second kind of complete ellipse integral,

$$\begin{aligned}E(k) &= \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \varphi} d\varphi, \\ k^2 &= 1 - \frac{b^2}{a^2}.\end{aligned}\quad (4)$$

M_2 and M_3 can be consulted in Figure 6. From the above principle, one can know that mode III fracture may be generated at $\Omega=0$ and $M_2=0$, mode II fracture may be generated at $\Omega=\pm 90^\circ$ and $M_3=0$, and fracture of modes II and III with different ratios of K_{II}/K_{III} may be generated at the other Ω , where $K_{II}/K_{III}=M_2/M_3$.

Figure 6 M_2 and M_3 values at the crack front when the crack in the surface of a plate that is under uniform shear loading (China Academy of Aeronautics, 1981, 1993).

The geometry of our experiment is referred to China Academy of Aeronautics (1981, 1993). In this handbook, the geometry of a crack in the surface of a plate is listed into the section named “3-D crack”, including $b/h=0.2, 0.6, 0.8$, $b/a=0.1, 0.2$. In our experiment, the cases including $h=5$ mm, $b/h \approx 0.6$, $b/a \approx 0.15$ are all contained in the handbook. In such way, there is no doubt that the geometry of our experiment belongs to 3-D crack.

2.3 Observation of experiments in glass specimen

When the loading approaches to critical value, the front of original crack start to grow, one can observe the following phenomena.

1) The first extension appears in the middle of the crack (edge of short axis), then the extensions spread to the tips, at last extensions appear at the tips of major axis.

2) Hundreds of discrete initial fractures appear at the length of 25 cm, as shown in Figure 7a. The initial extensions merge into each other in the process of

Figure 7 3-D initial extensions from the front of surface crack in glass specimen.

propagation, its number decreases gradually, but the distance keeps equal. The value of distance equals roughly the radius of the fracture surface, as shown in Figure 7b.

3) The shapes of the fracture surfaces are like parhelical, but the sizes are different. Where the extending in the middle of the original crack is the shortest, the extending at the tips of major axes are the longest.

4) The shape of every crack extension is referred to parhelical but one side is longer than another. These extensions array directionally and appear as en echelon or saw-tooth in the surface of plate as shown in Figures 7b and 8. In Figure 8a, the crack with $\beta \approx 39^\circ$ in group A breaks out and extends like bean valve caused by stick-slip between the surfaces of crack. In Figures 8b–8d, the crack of group B grows steadily with the loading increasing slowly and uniformly. In Figure 8d, $\beta \approx 69.2^\circ$, the location of main fracture surface is not at the tip of the major axis but near the middle section.

Figure 8 Extensions generated from the front of surface crack in glass specimen. (a) Group A, $\beta \approx 39^\circ$; (b) Group B, $\beta \approx 45^\circ$; (c) Group B, $\beta \approx 15^\circ$; (d) Group B, $\beta \approx 69.2^\circ$.

It seems that it is related to the asperity in the crack surfaces.

In Figure 8b, one can see many spoke shaped stripes that start from the initial points similar to that in Figure 3b. Lawn and Wilshaw (1975) thought that it is a kind of branch phenomenon and discussed its mechanism. In the extension surfaces in Figure 8, one can see many interfered stripes of light, which can be seen as the results of interference of the incident light

and the reflect light between the micro-opened crack surfaces.

2.4 Analysis of experimental results

2.4.1 Order of extension initial

From equation (3), we have $K_{II}/K_{III} = M_2/M_3$. In Figure 6, it can be seen that $M_2|_{\Omega=\pm 90^\circ} \ll M_3|_{\Omega=0^\circ}$, thus $K_{II}|_{\Omega=\pm 90^\circ} \ll K_{III}|_{\Omega=0^\circ}$. In such way, in different points at the crack edge, the critical loading values are different. At the point $\Omega=0$, the critical condition is

approached firstly, and the mode III fracture is initiated. Then at $0^\circ < |\Omega| < 90^\circ$, the fractures with different ratios of K_{II}/K_{III} are initiated one by one. At the point $\Omega = \pm 90^\circ$, the critical condition is approached at last, and the mode II fracture is initiated. Here the extensions are the longest and are called main fracture.

2.4.2 Condition of fracture initiation

Substituting equations (1) and (2) into equation (3), we have

$$K_J = h_j [\sin 2\beta \cos \beta - f(1 - \cos 2\beta)] \quad (J = \text{II, III}; j = 1, 2), \quad (5)$$

where $h_j = -[M_j/2E(k)]\sigma\sqrt{\pi b}$. In critical condition, $|K_J| = K_{Jc}$, $\sigma = \sigma_c$, and then $h_j = h_c$, substituting the above-mentioned into equation (5), we have

$$|h_c| = \frac{K_{Jc}}{\sin 2\beta - f(1 - \cos 2\beta)}. \quad (6)$$

In equation (6), only β is variable. It can be proved that there is a minimum of h_c at $\beta = \beta_m$, as shown in Figure 9, denoted as h_{cm} . That means the fracture initiates easiest when $\beta = \beta_m$.

$$\tan 2\beta_m = \frac{1}{f} \quad \text{or} \quad \beta_m = \frac{1}{2} \arctan \frac{1}{f} \quad (7)$$

Figure 9 Graph of h_c as a function of β .

The relationship between f and β_m are shown in Table 1. Besides, there are two points at $\beta_{\infty 1}$ and $\beta_{\infty 2}$ in which h_c is infinite.

$$\beta_{\infty 1} = 0, \quad \beta_{\infty 2} = \arctan \frac{1}{f}. \quad (8)$$

That means h_c becomes infinite when $\beta \rightarrow \beta_{\infty 1}$ (decreases to approach) or $\beta \rightarrow \beta_{\infty 2}$ (increases to approach), as

shown in Figure 9. It is very difficult to make the crack start to extend. Comparing equation (8) with equation (7), one can see that $\beta_{\infty 2} = 2\beta_m$ (Figure 9).

Table 1 Relationship between f and β_m

f	0.8	0.7	0.6	0.5	0.4	0.3
β_m	25°40'	27°30'	29°31'	31°43'	34°06'	36°39'

Through experiment, Yin et al. (1988) obtained $f \approx 0.76$ for group A and $f \approx 0.56$ for group B between the surfaces of crack in silicate glass. According to Mohr criterion, the pore pressure of lubrication oil offsets part of normal stress acting on the crack surface. If the pore pressure is not included in, the friction coefficient in group B is low. From Table 1, we can see that $\beta_{\infty 2} \approx 52^\circ$ for group A, and $\beta_{\infty 2} \approx 60^\circ$ for group B. Obviously, for the specimen showed in Figures 8a–8c, β is smaller than $\beta_{\infty 2}$, whereas for the specimen in Figure 8d, β ($\approx 69.2^\circ$) is much larger than $\beta_{\infty 2} \approx 60^\circ$. The direction of maximal principle stress is near perpendicular to the crack surface. The shear stress is very low, but the extensions still generate, except that the location of main fracture is stochastic instead of at the tip of the origin crack. From Table 1, in this places, if only $f < 0.35$, $\beta_{\infty 2}$ can be near 70° and this part becomes sliding section. Despite of the crack angle is very large, and then the shear stress is very low, the static friction force can still be overcome, and therefore the main fractures appear at the tip of the sliding section.

The experimental phenomenon shown in Figure 8d can be used to interpret similar case of earthquake fault at plate margin. An interesting fact is that for fault located at the plate margin the shear stress is very low, even is much lower than that of intraplate faults. The results of stress measurements in the drilling hole (depth of 304.8 m) at Cajon Pass near the San Andreas fault and many earthquake source mechanisms show that the direction of maximal principal stress is almost perpendicular to the fault. This result shows that the shear stress in the fault is very low (Ruff, 2002). In recent years, a common recognition has been obtained that the earthquakes with low stress occurred at the margin of plate should be resulted from effect of pore pressure of water from a lot of observation data especially from the deep drilling hole (Ruff, 2002).

2.5 Experimental research of acoustic emission (AE) of 3-D fracture in rock specimen

In 1999, Institute of Geophysics, China Earthquake Administration, cooperated with A F Ioffe Physical-Technical Institute, Russian Academy of Science, stud-

ied experimentally decimeter scale rock fracture. The SDAE-8 AE real time sampling and locating system (provided by Pro. Kuksenko) and Changjiang-500 testing machine are used (Li et al., 2000, 2009).

Figures 10a–10c show the location of AE events during the process of extension of the non-penetrated notch of marble, where $2a$ is the length of the major axis

of semi-ellipse. Most of the AE events centralize along the arc-like line of the notch front. These AE events disperse gradually as the loading increase slowly, indicating that multiple extensions appear on the points in arc-like line front of the notch. As contrast, the AE events always centralize at the two tips of the penetrated notch of the rock specimen, as shown in Figures 10d–10f.

Figure 10 (a–c) Location of AE during the process of propagation of non-penetrated notch in a plate of marble No. 991018 with size 400 mm×250 mm×41 mm, length of the major axis $2a=172$ mm and crack angle $\beta=30^\circ$. (d–f) Contrast of penetrated notch in another plate of marble No. 991015 with size 399 mm×248 mm×39 mm, length of the notch $2a=143$ mm and $\beta=29^\circ$.

3 Theoretical research of 3-D initial fracture

3.1 Method of evaluating σ_{Nmax} by using three Euler angles (3EA)

There are two reasons for that the 2EA method is not quite successful. Firstly, the size of differential plane

is enlarged arbitrarily; secondly, only two Euler angles of δ and ϕ are considered while the variations of ψ -angle is neglected (Figure 11). In fact, $\psi=0$ is always default in their method. In this way, only one plane but not curved surface solution is obtained.

We set up the coordinate just as the same as Figure 4. According to Lawn and Wilshaw (1975), the components of stress near the front of crack are

$$\left\{ \begin{array}{l} \sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3}{2}\theta \right) - \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3}{2}\theta \right) + o(r^{-1/2}) \\ \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3}{2}\theta \right) + \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \cdot \cos \frac{3}{2}\theta + o(r^{-1/2}) \\ \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \cdot \cos \frac{3}{2}\theta + \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3}{2}\theta \right) + o(r^{-1/2}) \\ \tau_{xz} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} + o(r^{-1/2}), \quad \tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} + o(r^{-1/2}) \\ \sigma_{zz} = \begin{cases} 0 & \text{for plane stress} \\ 2\nu \left(\frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} - \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \right) + o(r^{-1/2}) & \text{for plane strain} \end{cases} \end{array} \right. , \quad (9)$$

where K_I , K_{II} , K_{III} are the stress intensity factors, ν is Poisson ratio.

Next we will discuss the 3EA method. This paper takes $0 \leq \psi \leq 2\pi$ (Figure 11) in general situations. In order to search for the maximum of σ_N , we need to rotate x - z -plane about O -point by using three Euler angles δ , ϕ and ψ (3EA) as the following steps.

Figure 11 Coordinate of the crack front.

1) Rotating the xyz coordinate system about y -axis by an angle ψ , the x - z -plane turns to be x_1 - z_1 -plane in $x_1y_1z_1$ coordinate system.

2) Rotating the $x_1y_1z_1$ coordinate system about z_1 -axis by an angle δ , the x_1 - z_1 -plane turns to be x_2 - z_2 -plane in $x_2y_2z_2$ coordinate system.

3) Rotating the $x_2y_2z_2$ coordinate system about the x_2 -axis by an angle ϕ , the x_2 - z_2 -plane turns to be x_3 - z_3 -plane in $x_3y_3z_3$ coordinate system. Then the stress tensor in $x_3y_3z_3$ coordinate is

$$\sigma_{i_3j_3} = \alpha_{23} [\alpha_{12} (\alpha_{01} \sigma_{ij} \alpha_{01}^T) \alpha_{12}^T] \alpha_{23}^T. \quad (10)$$

where

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{pmatrix},$$

$$\alpha_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix},$$

$$\alpha_{12} = \begin{pmatrix} \cos \delta & \sin \delta & 0 \\ -\sin \delta & \cos \delta & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\alpha_{01} = \begin{pmatrix} \cos \psi & 0 & \sin \psi \\ 0 & 1 & 0 \\ -\sin \psi & 0 & \cos \psi \end{pmatrix},$$

then

$$\sigma_N = \sigma_{y_3y_3}. \quad (11)$$

Based on the specified value of ψ , $\sigma_{N\max}$ can be deduced from the solution of the following equations:

$$\frac{\partial \sigma_N}{\partial \delta} = 0, \quad \frac{\partial \sigma_N}{\partial \phi} = 0. \quad (12)$$

Suppose δ_0 and ϕ_0 are roots of the above equations. Substituting δ_0 and ϕ_0 into the left side of the following formulae, one can verify whether the following inequalities are satisfied.

$$\begin{aligned} \frac{\partial^2 \sigma_N}{\partial \delta^2} \Big|_{\substack{\phi=\phi_0 \\ \delta=\delta_0}} &< 0, \quad \frac{\partial^2 \sigma_N}{\partial \phi^2} \Big|_{\substack{\phi=\phi_0 \\ \delta=\delta_0}} < 0, \\ \left[\frac{\partial^2 \sigma_N}{\partial \delta^2} \frac{\partial^2 \sigma_N}{\partial \phi^2} - \left(\frac{\partial^2 \sigma_N}{\partial \delta \partial \phi} \right)^2 \right] \Big|_{\substack{\phi=\phi_0 \\ \delta=\delta_0}} &> 0. \end{aligned}$$

Now, the normal direction of x_3 - z_3 -plane is

$$\begin{aligned} \mathbf{n}_s &= \mathbf{O} \mathbf{y}_3(x, y, z) = (l, m, n) = \\ &(-\cos \phi \sin \delta \cos \psi - \sin \phi \sin \psi, \cos \phi \cos \delta, \\ &-\cos \phi \sin \delta \sin \psi + \sin \phi \cos \psi). \end{aligned} \quad (13)$$

From the geometry in Figure 11, one can get

$$\tan \theta = \sec \psi \cdot \tan \delta, \quad r = \frac{R \sin \delta}{\sin \theta}. \quad (14)$$

From the above equations, we can draw that if $\psi \neq 0$, then $\theta \neq \delta$, whereas, only if $\psi = 0$, then $\theta = \delta$.

Let $\psi = 0$ in equation (14), then $\delta = \theta$, the conditions are reduced to the situation treated by Yin et al. (1991). It can be seen that the 3EA method proposed by Palaniswamy and Knauss (1978) can only deduce a special differential plane with $\psi = 0$ that is called zero line.

3.2 Method of evaluating maximal normal stress with directional vector (NSDV)

3.2.1 Maximal normal stress

The first step of the normal stress with directional (NSDV) method is to rotate the differential plane M at the point $P(r, \theta_0, z)$ near the initial point to find a normal direction $\mathbf{n} = (l, m, n)$, in which the normal stress σ_N is maximum. The differential plane is denoted as $M_{\max}$, its normal direction is $\mathbf{n}_{\max}(l_0, m_0, n_0)$.

According to the Cauchy equation,

$$\sigma_N = l^2 \sigma_{xx} + m^2 \sigma_{yy} + n^2 \sigma_{zz} + 2lm\tau_{xy} + 2mn\tau_{yz} + 2nl\tau_{zx}. \quad (15)$$

When the conditions

$$\frac{\partial \sigma_N}{\partial l} = 0, \quad \frac{\partial \sigma_N}{\partial m} = 0, \quad \frac{\partial \sigma_N}{\partial n} = 0 \quad (16)$$

are satisfied, σ_N will become $\sigma_{N\max}$. With the constraint of the condition $l^2 + m^2 + n^2 = 1$, only the 1st and 2nd equations are needed to be solved. Thus, the roots l_0, m_0, n_0 are obtained and then substituted into the left of the following formulae so as to verify whether the following inequalities are satisfied.

$$\left. \frac{\partial^2 \sigma_N}{\partial l^2} \right|_{\substack{l=l_0 \\ m=m_0 \\ n=n_0}} < 0, \quad \left. \frac{\partial^2 \sigma_N}{\partial m^2} \right|_{\substack{l=l_0 \\ m=m_0 \\ n=n_0}} < 0, \\ \left[\frac{\partial^2 \sigma_N}{\partial l^2} \frac{\partial^2 \sigma_N}{\partial m^2} - \left(\frac{\partial^2 \sigma_N}{\partial l \partial m} \right)^2 \right] \bigg|_{\substack{l=l_0 \\ m=m_0 \\ n=n_0}} > 0. \quad (17)$$

3.2.2 Geometry of constraint condition

Suppose line OP is one of the initial extensions that are straight segments. Then the direction of OP is

$$OP = \mathbf{R}_P = \{x_P - 0, y_P - 0, z_P - 0\} = \{r \cos \theta_0, r \sin \theta_0, z_P(r, \theta_0)\}, \quad (18)$$

where $0 \leq r \leq r_m$, $-\pi \leq \theta_0 \leq \pi$. Naturally $\mathbf{R}_P \perp \mathbf{n}_{\max}(l_0, m_0, n_0)$, i.e., $\mathbf{R}_P \cdot \mathbf{n}_{\max} = 0$, that means

$$l_0 r \cos \theta_0 + m_0 r \sin \theta_0 + n_0 z_P(r, \theta_0) = 0. \quad (19)$$

That is just the algebraic form of $M_{\max}$. Equation (19) gives the necessary geometry condition that $M_{\max}$ extends through O -point. In such way, equation (19) is called geometry constraint equation.

3.2.3 Construction of initial fracture surface

The second step of the NSDV method is to construct the initial fracture surface. From equation (19) $z_P(r, \theta_0)$ can be obtained, and then is substituted into equation (18), we have

$$\mathbf{R}_P = \{x_P, y_P, z_P\} = r \left\{ \cos \theta_0, \sin \theta_0, \frac{-(l_0 \cos \theta_0 + m_0 \sin \theta_0)}{n_0} \right\}, \quad (20)$$

and $\mathbf{R}_P$ is also the expression of $M_{\max}$ that is coincident with OP .

Equation (20) holds to $-\pi \leq \theta \leq \pi$. That is to say, equation (20) shows the set of all differential planes extending from the point O in the range of $-\pi \leq \theta_0 \leq \pi$. It is just this set that constructs the total initial fracture surface extending from the point O .

The third step of the NSDV method is to locate the edge of initial fracture surface. We compute the isoline of $\sigma_1 = \sigma_m$ as the edge of initial fracture surface, where σ_m is anti-tensile strength of the medium.

3.2.4 Identities of NSDV and 3EA methods

As mentioned above, the NSDV method and 3EA method can be used independently. From equation (13), we get the relationship between the normal direction vector of $M_{\max}$ after rotating three times and the three Euler angles are

$$\begin{cases} l = -\cos \phi \sin \delta \cos \psi - \sin \phi \sin \psi \\ m = \cos \phi \cos \delta \\ n = -\cos \phi \sin \delta \sin \psi + \sin \phi \cos \psi \end{cases}. \quad (21)$$

From Figure 12, we can deduce $\tan \psi = z_P/x_P$. By using equations (14), (20) and the 2nd equation of (21), the reverse transformation of equation (21) is

$$\begin{cases} \psi = -\arctan \frac{l + m \tan \theta}{n} \\ \delta = \arctan \frac{n \tan \theta}{\sqrt{n^2 + (l + m \tan \theta)^2}} \\ \phi = \arccos \left[m \sqrt{1 + \frac{n^2 \tan^2 \theta}{n^2 + (l + m \tan \theta)^2}} \right] \end{cases}. \quad (22)$$

Figure 12 Illustration of three Euler angle method.

Thus it is clear that 3EA method is equivalent to NSDV method except that the steps are different. In 3EA method, the geometry constraint condition is restricted at first, then the maximum of σ_N is searched. In NSDV method, the orders are just contrary.

The NSDV method is still complicated in calculation, but it is a key step, from which the method of composition of first principal differential planes (CFPDP) can be deduced easily.

3.3 Method of composition of first principal differential planes (CFPDP)

3.3.1 Basis and main points

The CFPDP method was proposed by Li (1990, 1991) based on the well-known law that the maximal principal stress σ_1 is just the maximal normal stress σ_{Nmax} (Timoshenko and Gooder, 1970; Qian and Ye, 1956) at any point in the continuous medium. So, we only need change from computing the maximal normal stress, which is the first step of NSDV method, to computing the principal stress and the first principal direc-

tion at an arbitrary point near the front of the crack and keep the other steps unchanged. That means CFPDP method is identical to the NSDV method. Specially, CFPDP method brings a great simplification in calculation.

The key of CFPDP method lies in whether the following proposition is satisfied: the first principal differential plane M_1 with normal direction $\mathbf{n}_1(l_0, m_0, n_0)$ may intersect with the original crack front (i.e., z -axis in Figure 11) in the definition area of $-\pi \leq \theta \leq +\pi$. Li (1991) and Li et al. (2009) proved that only if $K_{III} \neq 0$, the geometry constraint condition given by equation (19) can always be satisfied. In such way, the proposition can be tenable under the condition of $K_{III} \neq 0$.

In the following sections, we will introduce some results by using CFPDP method.

3.3.2 Pure mode III (anti-plane shear) crack

Substituting $K_I = K_{II} = 0$, $K_{III} \neq 0$ into equation (9), we can obtain analytic solutions of initial fracture surface. The maximal principal stress is $\sigma_1 = K_{III}/\sqrt{2\pi r}$, the first principal direction is $\mathbf{N}_1 = \sqrt{2}/2(\sin \theta/2, -\cos \theta/2, -1)$, and the initial fracture surface $\mathbf{R}_P$ is

$$\mathbf{R}_P = r \left(\cos \theta, \sin \theta, -\sin \frac{\theta}{2} \right) \quad (0 < r < r_m, -\pi \leq \theta \leq +\pi). \quad (23)$$

This is a wide angle cone (Figure 13a). The edge of r is located, where $K_{IIIc} = K_{Ic}$, K_{Ic} is the toughness of the medium, σ_m is the strength of anti-tensile of the medium, so r_0 is a constant. There are two intersect lines between the fracture surface and the original crack planes. The angles between the two lines and the x -axis are $\pm 135^\circ$ respectively (Figure 13a). The above results

Figure 13 Initial fracture surface of pure mode III crack. (a) Theoretical simulation. (b) Observation of Experiment (Knauss, 1970).

confirm the experimental results of Knauss (1970) (Figure 13b) quite well. It can be tested and verified that these results are all equal to the results by using the vector methods.

3.3.3 Mode I (tensile) and mode III (anti-plane shear) crack

Substituting $K_{II}=0$, $K_I K_{III} \neq 0$ into equation (9), we can find the numerical computing results of the initial fracture surface exhibit symmetrical distortion. The

larger is the ratio K_I/K_{III} , the flatter, the longer and the narrower the initial fracture surface becomes. The results of numerical calculation are showed in Figure 14a, which is drawn by adjusting the view angle. A great improvement is achieved compared with the results of Li (1990, 1991). The initial extension shape like double lances is observed as in the experiment of Sommer (1969) (Figure 14b).

Figure 14 Initial fracture surface of mode I and mode III crack (Poisson ratio $\nu=0.25$). (a) Theoretical imitation (Li, 2009); (b) Observation of experiment (Sommer, 1969).

3.3.4 Mode II (in-plane shear) and mode III (Anti-plane shear) crack

Substituting $K_I=0$, $K_{II} K_{III} \neq 0$ into equation (9), we can find the numerical computing results of the initial fracture surface show asymmetrical distort with respect to x - z -plane. $r_m(\theta)$ has one maximum at $\theta=-180^\circ$ and a minimum at $\theta=+180^\circ$ (Figures 15a and 15c), which is consistent with the experimental results (see section 2.3 and Figure 15b) quite well.

Figure 15a corresponds to the places near the crack

tip of specimen ($\Omega \approx 80^\circ$) when $K_{II}/K_{III}=1$, and Figure 15b corresponds to the places near $\Omega \approx 80^\circ$ when $K_{II}/K_{III}=1/5$. The extended length of asymmetric distortion for the initial fracture has a positive relation with the ratio of K_{II}/K_{III} .

3.3.5 Mode I (tensile) and mode II (in-plane shear) crack

Substituting $K_{III}=0$, $K_I K_{II} \neq 0$ into equation (9), and transforming it into the polar coordinate system, we have

$$\begin{cases} \sigma_{rr} = \frac{1}{2\sqrt{2\pi r}} \left[K_I(3 - \cos \theta) \cdot \cos \frac{\theta}{2} + K_{II}(3 \cos \theta - 1) \cdot \sin \frac{\theta}{2} + o(r^{-1/2}) \right] \\ \sigma_{\theta\theta} = \frac{1}{2\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I(1 + \cos \theta) - 3K_{II} \sin \theta + o(r^{-1/2}) \right] \\ \tau_{r\theta} = \frac{1}{2\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \sin \theta + K_{II}(3 \cos \theta - 1) + o(r^{-1/2}) \right] \end{cases} \quad (24)$$

Figure 15 Initial fracture surface of mode II and mode III crack. (a) Theoretical simulation, $K_{II}/K_{III}=1$; (b) Observation of experiment; (c) Theoretical simulation, $K_{II}/K_{III}=1/5$.

According to tensile stress criterion in polar system, $\sigma_{\theta\theta}$ should be principal stress, i.e., $\tau_{r\theta}=0$. If only the single term of stress is kept, the fracture angle is

$$\theta_0 = -2\arctan \frac{\sqrt{1 + 8(K_{II}/K_I)^2} - 1}{4K_{II}/K_I}. \quad (25)$$

For mode II crack, $\theta_0 = \arccos(1/3) = -70^\circ 32'$ (Erdogan and Sih, 1963). By using the above results, one can interpret the experimental observation of contact of cylindrical steel indenter with the block of glass (Roesler, 1956). It can be seen that for the force of the indenter edge to the surface of glass, the direction of displacement of inner part of micro-arc line segment is down, i.e., perpendicular to the surface of the specimen. In such way, the fracture is mode II. The axis symmetry makes the shape of extending route become circular cone. The angle between generatrix and the normal direction is just the fracture angle θ_0 . The angle from the experimental observation is $\theta_0 \approx -68.5^\circ$ which has a 2° deviation from the theoretical results. We think that the reasons for the deviation are ways of loading and some terms neglected in theoretical resolution.

3.4 Fractal construction of 3-D fractures

If the edge of crack near point O is flat, generally, the fracture surface extending from point $O'(0, 0, z_0)$ is

$$\mathbf{R}'_P = \left\{ r \cos \theta, r \sin \theta, z_0 - \frac{r(l_0 \cos \theta + m_0 \sin \theta)}{n_0} \right\}. \quad (26)$$

Comparing equation (26) with equation (20), $\mathbf{R}'_P$ is just $\mathbf{R}_P$ moving z_0 from point- O . This means that the shape of $\mathbf{R}'_P$ is the same as that of $\mathbf{R}_P$. As z_0 is arbitrary, any point at the edge of the original crack can become starting point of extension. In such way, infinite initial fracture surfaces can be generated with the same shape. These surfaces are coalesced and restricted each other, which results in finite initial fracture surfaces in finite length along original crack edge.

The observation of experiments in glass shows that the group of the same mode of 3-D fractures generated from the edge of crack exhibit self-similar characteristic. The number of fractal dimension is 1.0, which is deduced as follows.

Let's denote the characteristic scale of the fracture at a level i as r_i . According to the definition of fractal (Xie, 1993), the number of fracture with characteristic scale r_i is $N_i = c/r_i^D$, where c is constant, D is the number of fractal dimension. Thus at level j , the number of fracture with the characteristic scale r_j is $N_j = c/r_j^D$.

$$\frac{N_i}{N_j} = \left(\frac{r_j}{r_i} \right)^D \quad (27)$$

On the other hand, the experiment results show

that the distance d of the fractures is positively proportional to r , i.e., $d \propto r$, so, $N=L/d \propto L/r$. In such way, $N_i \propto L/r_i$, $N_j \propto L/r_j$. Thus, we have $N_i/N_j = r_j/r_i$. Comparing this with equation (27), we get $d=1.0$ at once. Noting that here 1.0 means fraction dimension, which is different from the meaning of ordinary 1 dimension. The latter one is integer.

The fractal construction of 3-D fractures shows that the multiple fractures may occur in different scales, which can be as large as hundreds of kilometer of faults, or as small as the millimeter scale of the crystals of the rock. Superposition of the multiple fractures with the different scales forms the great and small break rock-body, rock mass and fault gauge that are often be seen.

4 Other application of 3-D fracture to faulting

4.1 Fracture of an ellipse crack acted by homogenous shear stress

The slipping section between the asperities in the buried fault plane is often modeled as ellipse crack in an infinite body. Under the pressure in the crust, the shear stress in the fault plane may be treated as nearly homogenous. Thus the initial extensions at the edge of ellipse crack may be calculated as 3-D fracture of modes II and III.

Denote the length of half major axis of the ellipse as a , the half minor axis as b , the homogenous shear stress acted on the crack plane as τ , and the angle between the direction of τ and the major axis of the ellipse as γ (Figure 16). According to China Academy of Aeronautics (1993), the stress intensity factors at point A with coordinate $x=a\cos\theta$, $y=b\sin\theta$ at the edge of the ellipse are

Figure 16 An ellipse crack acted by homogenous shear stress.

$$K_{II}(\theta) = \frac{\tau\sqrt{\pi b}k^2}{\Theta} \left(\frac{1}{C} \sin \gamma \sin \theta + \frac{k'}{B} \cos \gamma \cos \theta \right), \quad (28)$$

$$K_{III}(\theta) = \frac{(1-\nu)\tau\sqrt{\pi b}k^2}{\Theta} \left(-\frac{1}{B} \cos \gamma \sin \theta + \frac{k'}{C} \sin \gamma \cos \theta \right), \quad (29)$$

where ν is Poisson ratio,

$$\begin{aligned} \Theta &= \left(\sin^2 \theta + \frac{b^2}{a^2} \cos^2 \theta \right)^{1/4}, \\ B &= (k^2 - \nu)E(k) + \nu k'^2 K(k), \\ C &= (k^2 + \nu k'^2)E(k) - \nu k'^2 K(k), \\ k^2 &= 1 - \frac{b^2}{a^2}, \quad k'^2 = \frac{b^2}{a^2}. \end{aligned}$$

$K(k)$ and $E(k)$ are the 1st kind and 2nd kind total ellipse integral, respectively, $E(k)$ is shown in equation (4),

$$K(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}.$$

Especially when $\gamma=0$, the direction of τ is parallel to x -axis. For convenience to compare with Figure 6, we denote $\theta=90^\circ+\Omega$. Thus the stress intensity factors can be simplified as

$$K_{II} = \left(\frac{\pi b^3}{a} \right)^{1/2} \frac{k^2 \tau \sin \Omega}{B(a^2 \cos^2 \Omega + b^2 \sin^2 \Omega)^{1/4}} \quad (30)$$

and

$$K_{III} = (\pi ab)^{1/2} \frac{(1-\nu)k^2 \tau \cos \Omega}{B(a^2 \cos^2 \Omega + b^2 \sin^2 \Omega)^{1/4}}, \quad (31)$$

then

$$\frac{K_{II}}{K_{III}} = \frac{b \tan \Omega}{a(1-\nu)} \quad (32)$$

It is obvious that for any value of Ω , the value of K_{II} , K_{III} and the ratio K_{II}/K_{III} can be determined, and the shape of initial fracture surfaces can be calculated by using the method of principal planes in this paper. These theoretical solutions of different ratios K_{II}/K_{III} refer to section 3.3.4. The whole shape of fracture distribution in the area $-90^\circ \leq \Omega \leq 90^\circ$ with different normal stresses is similar to the experimental results shown in Figure 8. For example, at $\Omega=\pm 90^\circ$, $K_{III}=0$, the fracture belongs to mode II,

$$K_{II} = \left(\frac{\pi}{a} \right)^{1/2} \frac{b k^2 \tau}{B} \quad (\Omega = \pm 90^\circ). \quad (33)$$

At $\Omega=0$, the fracture belongs to mode III,

$$K_{III} = (\pi b)^{1/2} \frac{-(1-\nu)k^2\tau}{B} \quad (\Omega = 0^\circ). \quad (34)$$

From the above equations, it is obvious that under the condition of $a \gg b$, we have $|K_{II}|_{\Omega=\pm 90^\circ} \ll |K_{III}|_{\Omega=0^\circ}$. In such way, the critical loading values are different in different points at the crack edge. At the point of $\Omega=0$, the critical condition is approached firstly, and the pure mode III fracture is initiated. Then at the points $0^\circ < |\Omega| < 90^\circ$, the fractures with different ratio of K_{II}/K_{III} are initiated one by one. At the point $\Omega=\pm 90^\circ$ the critical condition is approached to at last, and mode II fracture is initiated. Here the extensions are the longest and called main fracture. In such way, there are no essential differences of the fractures between the half section of ellipse crack, i.e., $-90^\circ \leq \Omega \leq +90^\circ$, with the surface crack shaped semi-ellipse, just only some difference in the quantity of ratio of K_{II}/K_{III} . The distribution in the section of $90^\circ < \Omega < 270^\circ$ and the section of $-90^\circ < \Omega < +90^\circ$ are symmetric.

These theoretical solutions can verify the guess of 3-D fracture mode of fault proposed by Scholz (1990).

4.2 EDA model of anisotropy of medium

From the fractures of modes II and III of 3-D crack under pressure, it is shown that the directions of initial extensions are the same roughly. This phenomenon called directional arrangement of fissures. Extensive Dilatancy Anisotropy (EDA), is the important mechanism of anisotropy caused by evolution of micro-cracks, and important theoretical basis of precursors of S wave splitting and earth resistivity.

4.3 Tectonophysics

The 3-D structure of fault is the important study object of tectonophysics. For instance, the en echelon array faults in the ground surface, the tensile fractures form tension basin together with filling (Atkinson, 1987). The 3-D propagation of fault is described as “obviously bending, simply and independent propagation of small shear fault”, and “cone or helical shear fracture” (Gezuofusiki, 1975), as shown in Figure 17. The above description of tensile fissures can be interpreted as 3-D fracture of anti-plane shear fault.

Figure 17 En echelon array of tensile fissures of shear fault (Gezuofusiki, 1975), where the black arrows indicate the direction of fault dislocation.

4.4 3-D fractures in rock

Cox and Scholz (1988) observed some 3-D fractures under shear stress in rock specimen by using microscope. Wu (1991) presented a lot of evidences of 3-D fatigue fractures of rock in field and indoor slice of quartz. He proposed that these fatigue fractures are the integral of tidal force action. From these pictures, we can see that the shapes of rock fatigue fracture are quite similar to the quasi-helical 3-D fractures in the experimental observations.

4.5 The multiple earthquake events

The 3-D fracture model can be used to interpret the non-coplanar of the sources distribution of sub-events of large earthquake (Tseng and Shi, 1978). If the sub-events do not occur in transient, but one by one in a longer time, they may form foreshock or parts of after-shock. In Figure 18, the 3-D locations of the foreshocks of $M_S 5.6$ Xiuyan-Haicheng earthquake on November 29, 1999 (Zhang et al., 2001) are shown as the series number of time. It is also shown that the five $M_L \geq 4$ foreshocks occurred at the outer edge of a helical surface, and the main shock is located in the inner edge. The shape of the helical is just similar to the theoretical result and the experimental observation of pure mode III fracture.

Figure 18 The 3-D locations of the fore-shocks of M_s 5.6 Xiuyan-Haicheng earthquake on November 29, 1999 (Zhang et al., 2001). (a) Figure of latitude profile (master view). (b) Figure of longitude profile (left view). (c) Figure of platform.

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