Spatio-temporal characteristics of strong aftershocks of the $M_s8.0$ Wenchuan earthquake*

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Abstract Based on Gutenberg-Richter's relation, Bath's law, Omori's law and Wells's relation of rupture scale, this paper forecasts the temporal decay, total number, possible area and greatest magnitude of strong aftershocks (greater than or equal to $M6.0$) of the $M_s8.0$ Wenchuan earthquake by using the magnitude and statistical parameters of earthquakes in California area of USA. The number of strong aftershocks, the parameters of Gutenberg-Richter's relation and the modified form of Omori's law are validated based on the relocation data of aftershock sequence of the $M_s8.0$ Wenchuan earthquake. Moreover, the spatio-temporal characteristics and wave energy release of the strong aftershocks ($M \geq 6.0$) are analyzed. The result shows that strong aftershocks may occur at the end of local drop and sharp drop on the wave energy release curve.

Key words: Wenchuan earthquake; strong aftershock; wave energy release; statistic of aftershocks

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1 Introduction

Aftershocks would occur naturally after a main shock and may last one more years. And the strong aftershocks could make the condition of damaged buildings worse, landslides occurring continually, roads blocked again. Up to now, it is impractical to forecast when and where the strong aftershock will occur, but any way trying to reveal the pattern of the strong aftershocks is meaningful.

This paper forecasts the temporal decay of aftershock activity, the number of strong aftershocks ($M \geq 6.0$), the magnitude of the greatest aftershock, and possible area of the aftershocks of the $M_s8.0$ Wenchuan earthquake based on Gutenberg-Richter's relation (Gutenberg and Richter, 1954), Bath's law (Bath, 1965), Omori's law (Omori, 1894) and the relation of rupture scale, with the magnitude (Wells and Coppersmith, 1994) and statistical parameters of some earthquakes, such as Landers, Northridge, Hector Mine and San Simeon earthquakes, occurred in California area (Kisslinger and Jones, 1991; Shcherbakov et al, 2005). The number of the strong af-

tershocks, the parameters of Gutenberg-Richter's relation and the modified form of Omori's law are confirmed based on the relocation data of 1 364 aftershocks with $M \geq 3.0$ of the Wenchuan earthquake (Huang et al, 2008; Wu et al, 2009). Moreover, spatio-temporal characteristics and wave energy release of the strong aftershocks with $M_s \geq 6.0$ are analyzed, and the result shows that strong aftershock may occur at the end of local drop and sharp drop on the wave energy release curve.

2 A-prior forecast of aftershocks

The occurrence of aftershocks meets four statistical relations. The first is Gutenberg-Richter frequency-magnitude scaling, which states that aftershocks satisfy Gutenberg-Richter (G-R) frequency-magnitude scaling just as all earthquakes do. It is essential to note that the validity of the G-R scaling is related to a fractal scaling between the number of earthquakes and their rupture areas. Thus, aftershock distributions are scale invariant in terms of the frequency and the rupture-area. The G-R relation (Gutenberg and Richter, 1954) is

$$\log_{10} N(\geq m) = a - bm, \quad (1)$$

where $N(\geq m)$ is the cumulative number of earthquakes with magnitude greater than m in a specified region and in a time window. The constant b varies from region to

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region, but is generally in the range of 0.8–1.2 (Frohlich and Davis, 1993). The constant a is a measure of the regional level of seismicity and gives the logarithm of the number of earthquakes with magnitude greater than and equal to zero.

Another important scaling law concerning aftershocks is Bath's law. This law states that it is a good approximation to assume that the difference in magnitude between the main shock and its largest aftershock is a constant independent of the magnitude of the main shock (Bath, 1965). That is

$$\Delta m = M_{ms} - M_{as}^{\max}, \quad (2)$$

where M_{ms} is the magnitude of the main shock, $M_{as}^{\max}$ is the magnitude of the largest aftershock, and Δm is taken as a constant approximately, and typically $\Delta m=1.2$.

A third scaling relation, the modified form of Omori's law, describes the temporal decay of aftershock activity and is given in the form (Utsu et al, 1995; Shcherbakov et al, 2004)

$$f(t) = \frac{K}{(c+t)^p}, \quad (3)$$

where $f(t)$ is occurrence rate of aftershocks with magnitude greater than m ; t is the time that has elapsed since the main shock; K , p and c are parameters. This law manifests temporal correlations of aftershock sequences, which can be viewed as complex relaxation processes occurring after main shocks. In his original formulation, Omori (1894) introduced equation (3) with $p=1$, which is usually quite a good approximation.

A fourth scaling relation, Wells's relation between rupture scale and magnitude (Wells and Coppersmith, 1994), describes positions of aftershocks to occur. Regression equations of subsurface rupture length (L_{RD}) and downdip rupture width (W_R) with respect to moment magnitude (all type) are

$$\log_{10} L_{RD} = -2.44 + 0.59M \quad \sigma = 0.16 \quad (4)$$

and

$$\log_{10} W_R = -1.01 + 0.32M \quad \sigma = 0.15. \quad (5)$$

Based on the four statistical relations mentioned above, and the regression parameters from Landers, Northridge, Hector Mine and San Simeon earthquakes in California (Table 1) (Kisslinger and Jones, 1991; Robert et al, 2005), a-prior forecast of the aftershocks of the $M_S8.0$ Wenchuan earthquake will be carried out as following.

Table 1 Partial regression parameters of several earthquakes in California

Earthquake	M_{ms}	Δm	b
Landers	7.3	1.0	0.98
Northridge	6.7	0.8	0.91
Hector Mine	7.1	1.3	1.01
San Simeon	6.5	1.7	1.00

Note: M_{ms} is the magnitude of main shock, Δm is the difference in magnitude between the main shock and its largest aftershock, and b is the slope of the straight line G-R relation, see equation (1).

Presumably aftershocks satisfy the G-R scaling for the same reason that all earthquakes do. However, no generally accepted theory is available for the explanation of the scale-invariant nature of this distribution. Actually, aftershock activity is directly related to the causative fault, but those research results are very limit. So, b of the causative fault (Beichuan-Yingxiu fault) of the Wenchuan earthquake equals mean of the four earthquakes mentioned previously, i.e., $b=0.975$. And $\Delta m=1.2$ is adopted according to the Bath's law. So, magnitude of the largest aftershock is $M6.8$, $N(M \geq 6.8)=1$. Then $a = 6.63$ in equation (1) and $N(M \geq 3)=5070$. Assumed $c=0.05$ in equation (3) with original formulation of

Omori's law, i.e. $p=1$ and $\int_0^T \frac{K}{(0.05+t)} dt = 5070$, one

can get that $K=570$ when $T=365$ days. The forecast results are

$$\log_{10} N(\geq m) = 6.63 - 0.975m, \quad (6)$$

$$M_{ms} - M_{as}^{\max} = 1.2, \quad (7)$$

and

$$f(t) = \frac{570}{0.05+t}. \quad (8)$$

According to the $M_S8.0$ main shock, its subsurface rupture length is about 200 km, projected width of the downdip rupture width is about 25 km, which is under the assumption of the dip angle is 45 degrees, and the aftershocks will occur on the narrow area. Figure 1 is the result of a-prior forecast of the aftershocks ($M \geq 3.0$), and the main points are:

1) Magnitude of the largest aftershock would be $M6.8$ occurring only once;

2) Aftershocks with magnitude greater than or equal 6.0 may occur six times;

3) The frequency of aftershocks with magnitude greater than or equal 3.0 may be very high; it may occur 2300 times in ten days after the main shock;

Figure 1 A-prior forecast of the aftershocks of the Wenchuan earthquake based on G-R relation (a) and Omori law (b).

- 4) Aftershocks would occur in a narrow area of 200 km long and 25 km wide;
- 5) It is unknown when and where the strong aftershocks will occur.

3 Post statistics of the aftershocks

The relocation data of 1364 aftershocks with $M \geq 3.0$ of the $M8.0$ Wenchuan earthquake from Huang et al

(2008) and Wu et al (2009) are shown in Figure 2. For the sake of convenience, the local plane coordinates are defined in Figure 2 and local initial time is set as 00:00:00 (Beijing Time), May 12, 2008, the unit is day (in Figure 3d and afterward figures). By the least-squares fits of equation (1) with the data of $M \geq 3.0$ aftershocks, the G-R relation on the causative fault is expressed as

Figure 2 Distribution of the main shock and aftershocks of the Wenchuan earthquake.

$\log_{10} N(\geq m) = 5.575 - 0.798m \quad \sigma = 0.147. \quad (9)$

By August of 2008, the number of the aftershocks with $M\geq 6.0$ is 6 (Table 2), which equals the result of the prior forecast of the aftershocks. Away from $M6.0$, the difference is getting large (Figure 4).

The magnitude of the largest aftershock is 6.3, the difference in magnitude between the main shock and its largest aftershock is $\Delta m=1.7$, a little larger than 1.2 based on Bath’s law. Figure 4 shows comparison between G-R relations by prior forecast and post statistics.

Figure 3 Spatio-temporal distribution of the aftershocks of the $M_s8.0$ Wenchuan earthquake where N is the frequency of aftershock occurrence. (a) Magnitude distribution along the causative fault; (b) Density distribution along the causative fault; (c) Density distribution across the causative fault; (d) Magnitude distribution along time.

Table 2 Catalogue of the $M\geq 6.0$ aftershocks of Wenchuan earthquake (Huang et al, 2008)

No.	Origin time (Beijing Time)		t_i/d	Lat. /°N	Long. /°E	Depth /km	M	y /km	Reference site
	a-mo-d	h:min:s							
①	2008-05-13	04:08:48	1.172	31.461	103.829	21.2	6.0	102.9	Yuejiashan-Wenzhen
②	2008-08-01	16:32:42	81.689	32.095	104.620	14.3	6.1	205.6	Pingtong-Doukou
③	2008-05-25	16:21:51	13.681	32.628	105.363	15.2	6.3	297.2	Shangma
④	2008-08-05	17:49:15	85.742	32.799	105.470	11.1	6.2	317.6	Yaodu-Qingmichuan
⑤	2008-07-24	15:09:28	73.631	32.834	105.489	12.7	6.3	321.6	Yaodu-Qingmichuan
⑥	2008-07-24	03:54:43	73.162	32.824	105.501	13.5	6.0	321.7	Yaodu-Qingmichuan

Note: y denotes the coordinate of y axis in Figure 2.

Figure 4 Comparison between G-R relations by prior forecast and post statistics.

By global optimization method fits of equation (3)

with the data from 7 days to 107 days after the main shock (the record between 0–6 days seems incomplete), the modified Omori’s law is

$$f(t) = \frac{649}{(0.012 + t)^{1.079}} \quad \sigma = 7.316. \quad (10)$$

From Figure 5, the result of the prior forecast shows the number of the aftershocks is getting large with time and the trend of two curves based on the prior forecast and post statistics is the same. But the actual situation is different, the frequency of the aftershocks at the 80th day increases abruptly (Figure 3d), which is obviously related to seismo-geology condition (barriers), stress state, dislocation distribution and so on.

Figure 5 Comparison between Omori laws.

It is clear in Figure 2 that the aftershocks occur in a narrow area. Its length is 316 km which is much longer than the result based on the prior forecast, and its width is less 25 km on most part of the fault.

In general, the result of the prior forecast of the aftershocks is reasonable based on limited parameters and assumptions. For engineering seismology, when and where these aftershocks with $M \geq 6.0$ will occur is very important. Following section will discuss frequency, magnitude, and wave energy release of aftershocks.

4 Characteristics of strong aftershocks

Adopted time window is 5 days and span window is 100 km, i.e., the time range for density and magnitude analysis is 5 days before occurrence of a strong aftershock and the span is 50 km away from the position of a strong aftershock, respectively. Detail of the six aftershocks with $M \geq 6.0$ is listed in Table 2 and Figure 6.

The wave energy released by aftershocks is defined as

$$E(T, Y) = \sum_{t_i} \sum_{y_i} E(m_i) |m_i \geq 3.0$$

$$t_i \in (T - \Delta T, T], \quad y_i \in (Y - 0.5\Delta D, Y + 0.5\Delta D], \quad (11)$$

where, $\log_{10} E(m_i) = 4.8 + 1.5m_i$, $E(m_i)$ is the wave energy (in J) radiated by the source, $\Delta T = 5$ d and $\Delta D = 50$ km. If $T - 5 < 0$, $t_i \in (0, T]$; if $Y - 25 < 0$, $y_i \in (0, Y + 0.5\Delta D]$; and if $Y + 25 > 345$, $y_i \in (Y - 0.5\Delta D, 345]$.

4.1 Aftershock ① with $M6.0$ occurred at 04:08:48 on May 13, 2008

The aftershock occurred at $t = 1.172$ days, $y = 102.9$ km, between Yuejiashan and Wenzhen. Statistical results show that before the strong aftershock occurred, the density nearby 25 km was very low and the frequency three hours before the shock was declining. And there is a segment dentate drop on the curve of

Figure 6 Spatio-temporal distribution of the six strong aftershocks with $M \geq 6.0$.

Figure 7 Statistical results of the strong aftershock with $M6.0$ occurred on May 13, 2008 (① in Table 2). (a) Magnitude distribution in $y_i \in (52, 153)$, y in unit of km; (b) Density distribution in $y_i \in (52, 153)$; (c) Magnitude distribution in $t_i \in (0, 1.172)$, t in unit of day; (d) Frequency distribution in $t_i \in (0, 1.172)$; (e) Wave energy release.

wave energy release (Figure 7e). Then strong aftershock ① occurred at the end.

4.2 Aftershock ② with $M6.1$ occurred at 16:32:42 on August 1, 2008

The aftershock occurred at $t = 81.689$ days, $y = 205.6$ km, between Pingtong and Doukou. Analysis domain is $y_i \in (155, 256)$, $t_i \in (71.7, 81.7)$. Wave energy release (Figure 8a) shows that the aftershock occurred at $t_i = 81.789$ days. Before the strong aftershock occurred, the density nearby 37 km was very low and the frequency was declining three days before the aftershock (there are 12 aftershocks with the largest magni-

tude of 3.8 in the range of 0–100 km and 0–10 days). At 197–205 km is a segment drop on the wave energy release curve. Then strong aftershock ② occurred at one end of the segment.

4.3 Aftershock ③ with $M6.3$ occurred at 16:21:51 on May 25, 2008

The aftershock occurred in Shangma at $t=13.681$ days, $y=297.2$ km. Statistical result is shown in Figure 8b. Analysis domain is $y_i \in (247, 345)$, $t_i \in (3.7, 13.7)$. Wave energy release shows the aftershock occurred at $t_i=13.7$ days. Before the strong aftershock occurred, the density nearby 50 km was high, and the frequency was declining one day before the shock, and there is a segment drop (280–300 km) on the wave energy curve. Then strong aftershock ③ occurred at the middle point of the sharp drop segment.

4.4 Aftershock ④ with $M6.2$ occurred at 17:49:15 on August 5, 2008

The aftershock occurred at $t=85.742$ days, $y=317.6$ km, between Yaodu and Qingmuchuan. Statistical result is shown in Figure 8c. Analysis domain is taken as $y_i \in (267, 345)$, $t_i \in (75.7, 85.7)$. The aftershock occurred at $t_i=85.742$ days. Before the strong aftershock occurred, the density nearby 40 km was low, and the frequency one day before the aftershock was declining and there is a segment drop (300–325 km) on the wave energy

release curve. Then strong aftershock ④ occurred at the end of the sharp drop segment.

4.5 Aftershocks ⑤ with $M6.3$ at 15:09:28 and ⑥ with $M6.0$ at 03:54:43 on July 24, 2008

The aftershock ⑤ ($M6.3$) occurred at $t=73.631$ days, $y=321.6$ km, and ⑥ ($M6.0$) at $t=73.162$ days, $y=321.7$ km. Both occurred between Yaodu and Qingmuchuan. Statistical result is shown in Figure 8d. Analysis domain is $y_i \in (272, 345)$, $t_i \in (63.6, 73.6)$. Wave energy release curve shows the aftershocks occurred at $t_i=73.162$ and 73.631 days). Before the strong aftershocks occurred, the density 90 km nearby was low, and the frequency 10 days before the aftershock was declining and there is a segment drop (300–325 km) on the curve of wave energy release. Then two strong aftershocks ⑤ and ⑥ occurred at the middle of the sharp drop segment.

In general, the spatial distribution of the aftershocks greater than or equal to 6.0 is approximately uniform, although it is not at the end of the surface faulting caused by the main shock, the distribution in temporal domain is in chaos (Figure 6). From the wave energy release curves, strong aftershocks occurred at the end or the middle point of a sharp drop segment on the curve one day before.

Figure 8 Statistical results of wave energy release for the strong aftershocks ② (a), ③ (b), ④ (c) and ⑤ and ⑥ (d)

5 Discussion and conclusions

There exist some differences between the results based on the prior forecast and the post statistics, especially in the length of surface rupture and the largest magnitude of aftershocks. But the change tendencies are the same.

There are some obvious features about the locations of the six strong aftershocks. Some locations possess these features but no strong aftershock took place, i.e., there is not a one-to-one correspondence between the features and strong aftershocks. If the locations in

accord with these features were forecasted as candidates of strong aftershocks, casualties caused by strong aftershock would be minimized.

1) The result of the prior forecast of the aftershocks is reasonable based on limited parameters of Gutenberg-Richter's relation, Bath's law, Omori's law, Well's relation of rupture scale with the magnitude and some assumptions.

2) The number of strong aftershocks, the parameters of Gutenberg-Richter's relation and the modified form of Omori's law are validated based on the data of relocation aftershock sequence of the $M8.0$ Wenchuan

earthquake. There exist some differences between the results of the prior forecast and the post statistics, especially in the length of surface rupture and the largest magnitude of aftershock. The change tendencies are the same.

3) Spatial distribution of the aftershocks with $M \geq 6.0$ is approximate uniform, although it is not at the end of the surface faulting caused by the main shock. The temporal distribution is in chaos. From the wave energy release curves, one can find that strong aftershocks usually occur at the end or the middle point of a sharp drop segment in the wave energy release curve one day before, although data of strong aftershocks are limited.

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