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Denoising of seismic data via multi-scale ridgelet transform*

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Abstract Noise has traditionally been suppressed or eliminated in seismic data sets by the use of Fourier filters and, to a lesser degree, nonlinear statistical filters. Although these methods are quite useful under specific conditions, they may produce undesirable effects for the low signal to noise ratio data. In this paper, a new method, multi-scale ridgelet transform, is used in the light of the theory of ridgelet transform. We employ wavelet transform to do sub-band decomposition for the signals and then use non-linear thresholding in ridgelet domain for every block. In other words, it is based on the idea of partition, at sufficiently fine scale, a curving singularity looks straight, and so ridgelet transform can work well in such cases. Applications on both synthetic data and actual seismic data from Sichuan basin, South China, show that the new method eliminates the noise portion of the signal more efficiently and retains a greater amount of geologic data than other methods, the quality and consecutiveness of seismic event are improved obviously as well as the quality of section is improved.

Key words: **ridgelet transform; multi-scale; random noise; sub-band decomposition; complex Morlet wavelet**
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1 Introduction

Random noise is a stubborn enemy in seismic exploration and to suppress it is an important work (Li, 1993). Denoising is the premise and foundation for high-resolution seismic processing, imaging, inversion of lithology parameters and attributes analysis. Because of complicated surface including water system in plain, outcrop in carbonate area and in sand-shale area, the old stratum age, poor signal reflection, as well as complicated subsurface structures, these complicated geologic conditions and interferences will lead to low signal to noise ratio. So we can not pick up the first-breaks precisely, can not obtain high-accuracy velocity analysis, can not effectively imaging for pre-stack seismic data. Li (2005) discussed the problems such as investigation of surface structure, shot receiving factor, interference wave and observation system design, and proposed the corresponding countermeasure and technology according to exploration practice. It can increase the profile data's quality significantly with the optimization of seismic acquisition technology. However, the complex surface and subsurface geological structure are still the difficulties for seismic acquisition, processing and interpretation.

Wavelet transform has been successfully used in many scientific fields such as image compression, image denoising, signal processing, computer graphics, and pattern recognition (Leblanc and Morris, 2001; Liu et al, 2007b; Gao et al, 2006), which are largely attributed to the capability of wavelet time-frequency analysis. When 2-D wavelet is used to process image data, it can only detect information in horizontal, vertical, diagonal, and several other limited directions, so the linear characteristics of the signals can not be well described. To the shortcomings of wavelet transform, on the foundation of wavelet theory, Candès et al put forward ridgelet transform (Candès and Donoho, 1999; Donoho, 2001), which could process signals that contain linear characteristics and many changes in the direction. As an improvement to wavelet analysis, ridgelet transform can get a better result in seismic data processing (Zhang et al, 2007, 2008;

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Bao et al, 2007), but it also has its limitations. Candès and Donoho (1999), Arivazhagan et al (2006) and Zhang et al (2007) pointed out when used to process the seismic data, the ridgelet transform could only deal with the linear events, while the actual seismic form also exist curving characteristic and many other characteristics.

In response to this limitation of ridgelet transform, we firstly block the signals on the basis of multi-scale decomposition, and then process every block signal in different scales by ridgelet transform. The idea is that, in sufficiently fine scale, a curving singularity can look straight, so ridgelet analysis works well in it.

2 The method and principle

2.1 Ridgelet transform

To overcome the weakness of wavelet in higher dimension, Candès and Donoho (1999) developed an alternative system of multi-resolution analysis, called ridgelet, which can effectively deal with line-like phenomena in two dimension.

The ridgelet transform in $\mathbf{R}^2$ can be defined as follows.

Pick a smooth univariate function ψ: $\mathbf{R}\to\mathbf{R}$ with sufficient decay and vanishing mean, $\int_{\mathbf{R}}\psi(x)\mathrm{d}x=0$. For each $a>0$, each $b\in\mathbf{R}$ and each $\theta\in[0, 2\pi)$, define a bivariate function $\psi_{a,b,\theta}$: $\mathbf{R}^2\to\mathbf{R}^2$ by

$$\psi_{a,b,\theta}(x_1,x_2)=\frac{1}{\sqrt{|a|}}\psi\left(\frac{x_1\cos\theta+x_2\sin\theta-b}{a}\right). \quad (1)$$

This function is constant along "ridges", i.e., $x_1\cos\theta+x_2\sin\theta=$const. Transverse to these ridges it is a wavelet; hence the name ridgelet. Given an integrable bivariate function $f(x)$, define its ridgelet transform as

$$R_f(a,b,\theta)=<f,\psi_{a,b,\theta}>=\int_{\mathbf{R}^2}\overline{\psi}_{a,b,\theta}(x)f(x)\mathrm{d}x. \quad (2)$$

Our hypotheses on ψ guarantee that $K_\psi=\int_{\mathbf{R}}(|\psi(\omega)|^2/|\omega|^2)\mathrm{d}\omega<\infty$, and if suppose further that ψ is normalized we have $K_\psi=\int_{\mathbf{R}}(|\psi(\omega)|^2/|\omega|^2)\mathrm{d}\omega=1$, so that the exact reconstruction formula is

$$f(x)=\frac{1}{4\pi a^3}\int_0^{2\pi}\int_{-\infty}^{+\infty}\int_0^{+\infty}R_f(a,b,\theta)\psi_{a,b,\theta}(x)\mathrm{d}a\mathrm{d}b\mathrm{d}\theta. \quad (3)$$

The ridgelet transform is similar to the 2-D continuous wavelet transform except that the point parameters are replaced by line parameters. In 2D, points and lines are related by Radon transform, thus the wavelet and ridgelet transforms are linked via the Radon transform. The Radon transform of an object f is the collection of line integrals indexed by (θ, ρ) ($\theta\in[0, 2\pi)$), which is given by

$$R(\rho,\theta)=\int_{\mathbf{R}}\int_{\mathbf{R}}f(x_1,x_2)\delta(\rho-x_2\sin\theta-x_1\cos\theta)\mathrm{d}x_1\mathrm{d}x_2, \quad (4)$$

where δ is the dirac distribution.

2.2 Ridgelet transform based on multiscale analysis

The literatures (Candès and Donoho, 1999; Arivazhagan et al, 2006; Zhang et al, 2007; Zhang and Liu, 2007) had pointed out the ridgelet's limitation when it is used to process seismic data. For the complicated seismic records, it may be applicable to use the calculus based on the idea of separation. The idea is that, at sufficiently fine scale, a curving singularity looks straight, so ridgelet transform can work well in such cases. The multi-scale ridgelet transform refers to the ridgelet transform with certain length and width of the block at certain scales; it can also be said tower structure by a series of window-ridgelet transform, we can call it local ridgelet transform.

In this paper, we use wavelet analysis to do sub-band decomposition. For the signal f, do the following decomposition (Mallat, 1989; Mallat and Huang, 1992):

$$f=A_N+\sum_{j=1}^{N}D_j, \quad (5)$$

where A_N is N-order approximation, and D_j is j-order detail.

We now present a sketch of the algorithm: ① apply the wavelet algorithm with N scales; ② set $n_1=2^M$, where n_1 is the block number at 1 scale; ③ for j=1, 2, ···, N, decompose the subband D_j with a block number n_j and apply the ridgelet transform to each block; ④ set $n_j=2^{M-j+1}$, where n_j is the block number at j scale.

Considering the random noise included in the details, we note that the coarse description of the data A_N is not processed. We use the default value $2^M=2^N$ in our implementation. Finally, Figure 1 gives an overview of the organization of the algorithm.

2.3 Non-linear thresholding

Donoho (1995) pioneered a denoising scheme by using hard thresholding and soft thresholding. Hard thresholding sets any coefficient less than the threshold as zero, which is given by

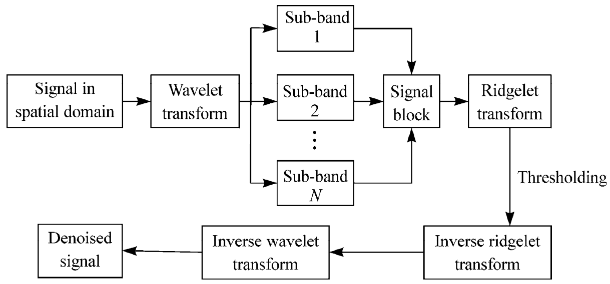

Figure 1 Flow chart of algorithm.

$$\hat{R}_f(a,b,\theta)=\begin{cases} R_f(a,b,\theta) & \left|R_f(a,b,\theta)\right| \ge t \\ 0 & \left|R_f(a,b,\theta)\right| < t \end{cases}; \tag{6}$$

while soft thresholding is subtracted from any coefficient which is greater than the threshold, that is

$$\hat{R}_f(a,b,\theta)=\begin{cases} R_f(a,b,\theta)-t & \left|R_f(a,b,\theta)\right| \ge t \\ 0 & \left|R_f(a,b,\theta)\right| < t \end{cases}. \tag{7}$$

From Figure 2, we can see the hard threshold function is discontinuous at $|R_f|=t$, due to this discontinuity at the threshold, the hard-thresholding function is known to yield abrupt artifacts in the denoised signal, especially when the noise level is significant, which will make noise cannot be effectively reduced. The soft threshold function has permanent bias and it loses part of edge information in reconstruction due to neglecting singularity detecting. To avoid the discontinuity caused by using the hard-thresholding model and the biased estimation caused by using the soft-thresholding model, the non-linear thresholding is adopted as following:

$$\hat{R}_f(a,b,\theta)=\begin{cases} R_f(a,b,\theta) & \left|R_f(a,b,\theta)\right| \ge t_1 \\ R_f(a,b,\theta)\cdot\varphi(t,t_1) & t \le \left|R_f(a,b,\theta)\right| < t_1 \\ 0 & \left|R_f(a,b,\theta)\right| < t \end{cases}, \tag{8}$$

where $\varphi(t, t_1)$ is the weight function damping from 1 to 0 at interval $(|t|, |t_1|)$.

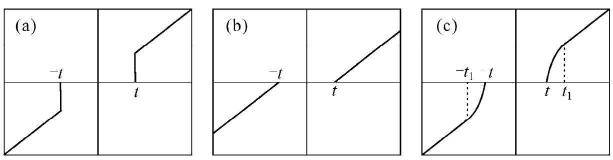

Figure 2 The relationship between original coefficient and thresholding coefficient. In this paper, x-axis represents original ridgelet coefficient and y-axis represents thresholding ridgelet coefficient. Both t and t_1 are the threshold. (a) Hard threshold; (b) Soft threshold; (c) Non-linear threshold.

3 Synthetic model data

First, we formulated the functions of the signal-to-noise ratio ($R_{\rm SN}$) and the mean square error ($r_{\rm MSE}$). Suppose x is original signal and y is noisy signal, they are all 2-D signals with M rows and N columns. Then we have

$$R_{\rm SN}=\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}x_{ij}^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}(y_{ij}-x_{ij})^2} \quad \text{and} \quad r_{\rm MSE}=\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}(x_{ij}-y_{ij})^2}{M\cdot N}.$$

A pure Ricker wavelet distribution of synthetic data is generated (Figure 3a), and the dominant frequencies of the two waveforms are 35 Hz and 25 Hz. Adding random noise to the pure data, the result (its signal-to-noise ratio is 0.1) is shown in Figure 3b, which is the data set that we used to test the denoising methods.

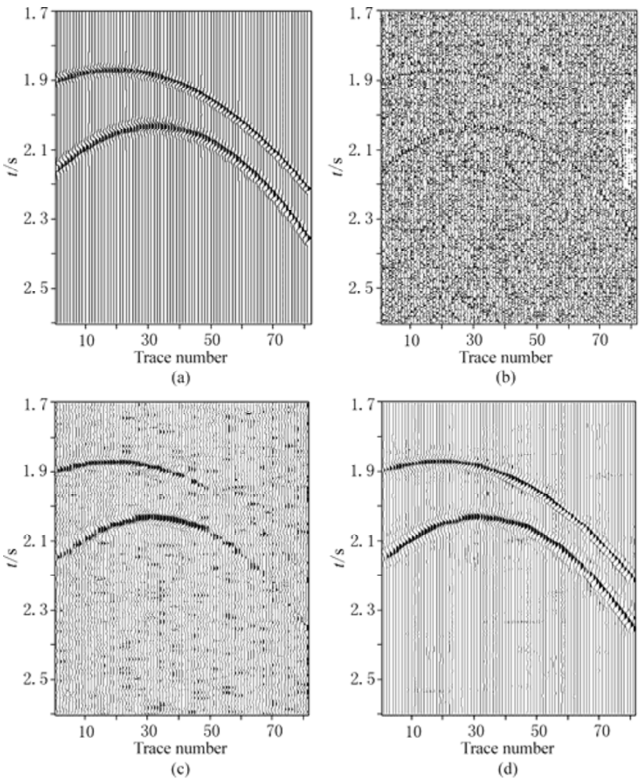

Figure 3 Processing for synthetic data in different transform domains. (a) Pure data; (b) Noisy data with signal-to-noise ratio of 0.1; (c) Denoising by wavelet transform; (d) Denoising by multi-scale ridgelet transform.

The first denoising procedure to be attempted is wavelet transform and its result is in Figure 3c. The random noise does remove to some extent, but the form of the resulting time series has been corrupted. Specifically, the waveforms are quite different. Some waveforms have been reduced, which is more evident at the tail of the form. Further, considerable noise still exists in the de-noised data throughout the entire map. The objective of the exercise is to recover the pure waveform from the denoised data. The result of wavelet transform on the synthetic data shows apparently that some noises still exist and some reflection waves have been suppressed. Applying the multi-scale ridgelet transform to the data in Figure 3b, we get the waveform shown in Figure 3d. This process has nearly eliminated all the random noise, and the quality of section is improved. Furthermore, the reconstruction of the signal is better than the result by wavelet transform, and the waveforms are preserved accurately.

Figure 4 gives the results of spectrum analysis by complex Morlet wavelet (Sheng et al, 2007; Liu et al, 2007a; Lu et al, 2002) for the synthetic data (50th trace). The dominant frequencies of the two wavelets are 35 Hz and 25 Hz, respectively (Figure 4a). The spectral analysis of the noised data (Figure 4b) indicates that random noise in the entire frequency band has obscured the useful signal. Figure 4c is the spectral analysis for Figure 3c, the random noise has been suppressed, but it loses some useful signal (we can see the signal of 35 Hz has been suppressed), so this method may lead to information distortion. Figure 4d is the spectral analysis for Figure 3d, the multi-scale ridgelet transform can get better noise suppression, while effective reflected wave is well maintained; the result has better information fidelity.

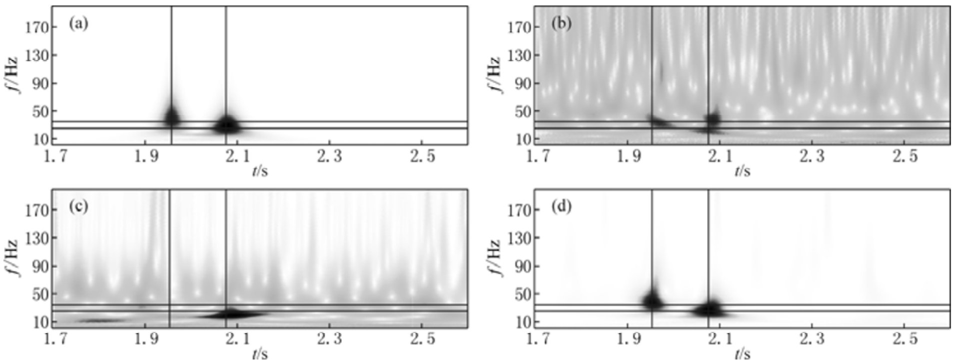

Figure 4 Spectrum analysis of the 50th trace corresponding to the subfigures in Figure 3.

The denoising ability of the two methods mentioned above is efficiently assessed by calculating both the mean-square error (MSE) and the signal-to-noise ratio (SNR) of the data. A summary of the results is listed in Table 1. As regards to the elimination of noise and retention of significant signals, it is evident that the multi-scale ridgelet transform method provides significantly better result. In fact, the MSE of the multi-scale ridgelet transform result is nearly one-half of the wavelet transform's. Furthermore, it is also clear that the SNR of the multi-scale ridgelet transform result is nearly about 1.64 times of the wavelet transform's.

Table 1 Comparison of the pure Ricker wavelet signals with the denoised synthetic data by two transforms

	Mean-square error	Signal-to-noise ratio
Noisy data	0.326 0	0.102 0
Wavelet transform	0.010 0	3.091 5
Multi-scale ridgelet transform	0.005 2	5.059 0

4 Field data processing

We processed seismic data from an oil field in Sichuan basin, South China by the new method in this paper.

Figure 5a shows the original seismic section with much interference from the survey area and its signal-to-noise ratio is low. Figure 5b shows the section processed by wavelet transform. We can see that wavelet thresholding method can reduce noise to some extent, but it does not detect the seismic events effectually, so it will lose some event information. Besides, the event's continuity of the profile is not good.

Figure 5c gives the processing result by multi-scale ridgelet transform. It is clear that the noise is greatly reduced, the quality and continuity of seismic events are improved obviously, and the signal to noise ratio of seismic section is improved perfectly. Figure 6 gives the spectral analysis on a random column in Figures 5a, 5b and 5c. In Figure 6c, the bandwidth of dominant frequency and useful information have been reserved well and its effect of random noise attenuation at relatively low-frequency and relatively high-frequency is better than wavelet transform method's. The multi-scale ridgelet transform can suppress the random noise drastically with the precondition of maintenance for the effective seismic wave.

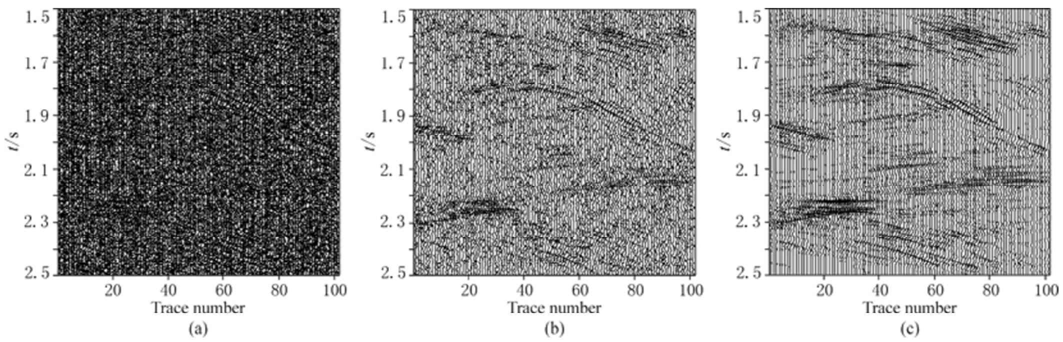

Figure 5 Origin seismic data (a) and denoised results by wavelet transform (b) and multi-scale ridgelet transform (c).

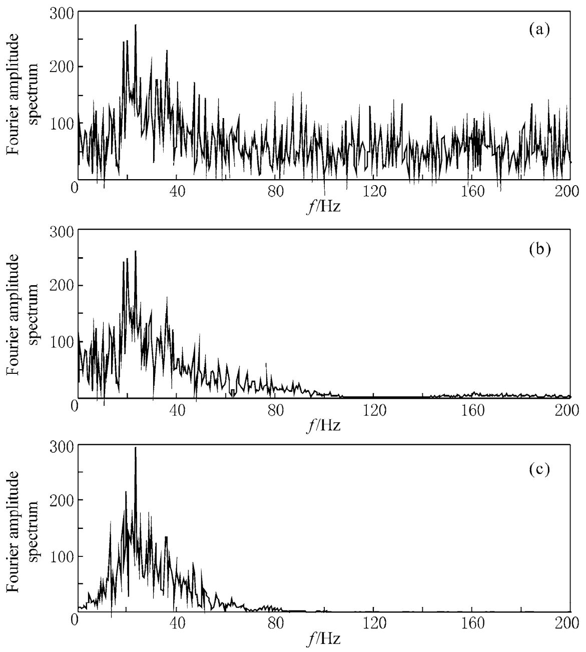

Figure 6 Spectrum analysis of the 50th trace corresponding to the subfigures in Figure 5.

5 Conclusions

In every phase of the seismic data processing, such as the estimate of residual static corrections, velocity analysis, migration, etc, as a prerequisite, the seismic data must be of a certain signal to noise ratio. On the basis of multi-scale wavelet analysis, this paper proposes a new wavelet transform. In sufficiently fine scale, a curving singularity looks straight, so ridgelet works well in it. In other words, this can overcome the limitation of ridgelet transform that can only handle the seismic signals containing linear events. Comparison of multi-scale ridgelet transform denoising with wavelet denoising shows the former can significantly eliminate noise and increase retention of geologic signals, therefore the quality and consecutiveness of seismic events are improved obviously; meanwhile the signal to noise ratio and resolution of seismic section are improved.

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