Pervasion degree of convective mixing and investigations of mantle mixing*

Rongshan Fu^{1,2,3,*} Jianhua Huang^{1,2,3} Jun Liu^{1,2,3} and Xiaohua Chang^{1,2,3}

¹ National Geophysical Observatory at Mengcheng, Mengcheng 233500, China

² Key Laboratory of Crust-Mantle Materials and Environments, Chinese Academy of Science, Hefei 230026, China

³ School of Earth and Space Science, University of Science and Technology of China, Hefei 230026, China

Abstract Geodynamic studies have shown that mantle convection is like a giant blender to make the original heterogeneous mantle mixing and homogenizing. However, some models, especially from geochemical data show that the modern mantle may still contain a number of reservoir bodies with different chemical composition. Then, the modern mantle is homogeneous? Authors have defined a box replacement degree of convective mantle mixing and pervasion degree of convective mantle mixing (that equals to initial density of tracing elements divided by final density of tracing elements) to investigate the mantle mixing. The previous results have shown that after four billion years the mantle is basic uniform and the box replacement of the convective mantle is more than 80% in steady-state convection models. This paper calculates and discusses the pervasion degree of convective mantle mixing in detail. For the initial state we will set some $10^\circ \times 10^\circ$ (spacing 0.25°) tracing boxes, which includes 1 681 tracer, at the top and bottom of the mantle, and then track the motions of these tracers, and investigate the convective mixed pervasions. The results show that at the initial stage though the convective mixing pervasions are very different from mode to mode, after running for some time, the convective mixing pervasions for most modes are going to be a constant and the tracers are more evenly distributed in the whole mantle.

Key words: mantle mixing; mantle convection; box replacement degree; pervasion degree

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1 Introduction

Mantle mixing, as a basic problem of the evolution of the Earth, particularly of the mantle, is faced in the studies on geophysics, geochemistry and geodynamics over the past two decades. Moreover, it can not be avoided in the study of the mantle thermodynamics. Analysis of the incompatible tracing elements in the magma rocks from the mantle showed that these rocks are divided into two categories. One is from the mid-oceanic ridge basalt (MORB), in which the mantle rock is interpreted as the result of partial melting, showing a loss of some elements, called depleted mantle (DMM). And the other is called ocean-island basalt (OIB), like Hawaii's island, interpreted as from the deeper mantle plume. Comparing with the MORB, it is relative enrichment of elements and geochemists think it

comes from an original mantle or primitive mantle (PM). Many studies simply use these geochemical data to define the difference between the upper and lower mantle so as to explain the structure of the mantle convection (Hofmann and White, 1982; Hoffman and McKenzie, 1985; Hofmann, 1997; Alle'gre et al, 1986; O'Nions, 1987; Loper, 1985; Grand et al, 1997; van der Hilst et al, 1997; Monnereau and Quere, 2001; Gurnis and Davies, 1986). But with the geochemical data accumulation this simple division can not explain new observations, so some geochemists suggest that there are many chemical reservoirs in the mantle (Hofmann and White, 1982; Hofmann, 1997). Some studies suppose that there are enormous reservoirs with different chemical composition in the mantle. These chemical reservoirs could preserve in a long geological time going with differentiated mantle from the original Earth (Hart and Zindler, 1986; Ten et al, 1998; Becker et al, 1999). Based on these observations and theoretical speculation, the modern mantle may still maintain the status of non-uniform. This raises a question that modern mantle is evenly? In the

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* Corresponding author. e-mail: ffrs@ustc.edu.cn

processes of mantle evolution, convection will play what role in mantle mixing?

Over a period of time, numerical simulation methods are employed to investigate the mantle mixing. The study in 2D or 3D space domain shows that on the conditions of existed toroidal field even a steady convection is also able to generate effective mixing (Ferrachat and Ricard, 1998; Schmalzl et al, 1996). van Keken and Zhong (1999) studied the mantle mixing problem in a 3D spherical region on the conditions of the modern mantle convection and took the revised Luyaponov index to evaluate the degree of mantle mixing. The results show that the main factors to control the mantle mixing efficiency are the driving force of the slabs (in particular circum-Pacific region) and the toroidal components of modern plate movement. Relatively speaking, the Earth's mantle has been mixed effectively after a long-term evolution and could not keep some isolated blocks in it (van Keken and Ballentine, 1998, 1999). Meantime, some regional or specific characteristics of the mixing problem have also been discussed (Walzer and Hendel, 1999; Kellogg et al, 2002; Farnetani et al, 2002).

In order to improve the accuracy of the Luyaponov index the grid points of the computed models are going to be more and more. However, grid refinement not only needs large-scale parallel computing, but also is difficult to provide a complete mantle mixing framework. At the same time, Luyaponov index is to determine the separation degree of the arbitrarily small initial fragments in a convective system only, but could not give the replacements for whole-mantle material (the replacement degrees of the convective system). As for this case, we put forward a more intuitive way to investigate the mantle mixing, which is to define the box replacement degree of convective mantle mixing and pervasion degree of convective mantle mixing, and compute the box replacement degree of convective mantle mixing in detail, and conclude that the Earth's mantle is mixed very well passing four billion years (Fu et al, 2007). Assuming that there is a stable convection system in the mantle driven by plate motions and density anomalies as well, this paper calculates and analyzes the pervasion degree of convective mantle mixing by the same method.

2 Pervasion degree of convective mantle mixing and mantle convection model

This study uses a stable convection model developed by Fu et al (2005, 2007), in which the mantle con-

vection is driven by the modern plate movement and the density anomaly of the mantle (Figure 1). If the mantle is divided into upper and lower mantle at 660 km depth ($r=660/R_{\text{LMB}}$) and their average viscosity are ν_1 and ν_2 respectively (Figure 1), the radial component W_1 and W_2 of the velocity can be written as

$$W_1 = \sum_l \sum_m \{ [(E_l^m r^l + F_l^m r^{l+2} + G_l^m r^{-(l+1)} + H_l^m r^{-(l-1)}) \cdot \cos m\phi + (\bar{E}_l^m r^l + \bar{F}_l^m r^{l+2} + \bar{G}_l^m r^{-(l+1)} + \bar{H}_l^m r^{-(l-1)}) \cdot \sin m\phi] + \sum_k \frac{g\alpha R_1^4 l(l+1)}{\nu_1 \beta_k^4 \sqrt{r}} \cdot Q_{1/2}(\beta_k r) [{}_k C_l^m \cos m\phi + {}_k D_l^m \sin m\phi] \} P_l^m(\cos \theta) \quad (1.1)$$

for $R_{660}/R_1 \leq r \leq R_{\text{LMB}}/R_1$,

and

$$W_2 = \sum_l \sum_m \{ [(K_l^m r^l + L_l^m r^{l+2} + M_l^m r^{-(l+1)} + N_l^m r^{-(l-1)}) \cdot \cos m\phi + (\bar{K}_l^m r^l + \bar{L}_l^m r^{l+2} + \bar{M}_l^m r^{-(l+1)} + \bar{N}_l^m r^{-(l-1)}) \cdot \sin m\phi] + \sum_k \frac{g\alpha R_1^4 l(l+1)}{\nu_2 \beta_k^4 \sqrt{r}} \cdot Q_{1/2}(\beta_k r) [{}_k C_l^m \cos m\phi + {}_k D_l^m \sin m\phi] \} P_l^m(\cos \theta) \quad (1.2)$$

for $R_{\text{CMB}}/R_1 \leq r \leq R_{660}/R_1$,

where θ and ϕ are latitude and longitude; R_1 is taken as R_{LMB} , R_{CMB} and R_{660} , which are the radius of the bottom of lithosphere (LMB), core-mantle boundary (CMB) and

Figure 1 Scheme of mantle convection model and the mantle viscous structure. (a) Driving forces of mantle convection; (b) Viscous structure of the mantle where the value 10, 100, 1000 is viscosity ratio ν_2/ν_1 between lower and upper mantle; (c) Initial position and tracing elements of modes (see Table 1 in the text).

660 km discontinuous, respectively; g is the gravity acceleration and α is thermal expansion coefficient. The first term of the right hand of equation (1) is the general solution, where $E_l^m, F_l^m, G_l^m, H_l^m, \bar{E}_l^m, \bar{F}_l^m, \bar{G}_l^m, \bar{H}_l^m, K_l^m, L_l^m, M_l^m, N_l^m, \bar{K}_l^m, \bar{L}_l^m, \bar{M}_l^m$ and $\bar{N}_l^m$ are the constants determined by the boundary conditions. The second one is the special solution corresponding to the density anomaly of the mantle, in which it expands to the spherical Bessel function as

$$Q_{1/2}(\beta_k r) = J_{-1/2}(\beta_k \eta) J_{1/2}(\beta_k r) - J_{1/2}(\beta_k \eta) J_{-1/2}(\beta_k r), \quad (2)$$

where β_k is the zero points of the spherical Bessel function in the region of $1-\eta$, where η is expressed by R_{CMB}/R_1 .

On certain conditions, we can understand the convective process in a non-uniform material system as a homogenization process of the system. In other words, the convective mixing process for a particular anomaly region in a system is an anomaly material pervasion process to the whole system. If put M tracing elements into a small box with a volume ΔV at the initial time, the density of tracing elements is

$$\rho_0 = \frac{M}{\Delta V}. \quad (3)$$

When the convection is operating, we can calculate the distributed volume V of the M tracing elements, so we have the density of tracing elements is

$$\rho = \frac{M}{V}. \quad (4)$$

Then we define the pervasion degree of convective mantle mixing as

$$K = \frac{\rho}{\rho_0} = \frac{V}{\Delta V}. \quad (5)$$

3 Model computation and analysis

The viscosity ratio of the upper mantle and lower mantle is set to be 1:100. The driving forces of the mantle convection will take three models: by the plate motions only, by the mantle density anomaly only and by above two driving forces synchronously. In view of calculating time on computer, we only select the plate motions as the driving force of the mantle convection. The plate motion model is Nuvel1 and the plate movements are expanded to 20 degree and order spherical harmonic functions for poloidal and toroidal field, respectively. We have shown that the results of this driving force model do not affect the final results for discussing the mantle mixing (Fu et al, 2007).

In order to discuss the pervasion degree of the mantle mixing in detail, we calculate eight test box modes at 100 km depth (MTOP-A, B, C, D) and 100 km above CMB (MBOT-A, B, C, D), respectively. The positions of these boxes are listed in Table 1. The volume of the test box is $10^\circ \times 10^\circ \times 0.625$ km and every box contains $41 \times 41 = 1681$ tracing elements and its gap is 0.25° . In the computation the time interval is taken as 500 thousand years. We calculate 8 000 steps for four billion year. For each test box mode the computation time approximately is eleven hours. We can track each tracing element to investigate the box replacement degree and the pervasion degree of convective mantle mixing. If dividing the mantle into $36 \times 15 \times 9 = 4860$ blocks with size $10^\circ \times 10^\circ \times 300$ km, the box replacement degree of the convective mantle mixing is listed in Table 1, too. We find that except MBOT-C, the box replacement degree for each mode is over 45%, which shows that the paths of 1 681 tracing elements for most modes have swept more than 50% mantle blocks.

Fu et al (2007) defined an apparent density to discuss the pervasion degree of convective mantle mixing

Table 1 Computation models and their box replacement degree of convective mantle mixing

Mode	Position of box	Box replacement degree		
		Whole mantle	Upper mantle	Lower mantle
MTOP-A	45°N–55°N, 89°E–99°E	75.32%	83.64%	72.94%
MTOP-B	45°N–55°N, 269°E–279°E	53.15%	62.80%	50.40%
MTOP-C	35°S–45°S, 89°E–99°E	48.11%	62.15%	44.10%
MTOP-D	35°S–45°S, 269°E–279°E	60.29%	74.80%	56.14%
MBOT-A	45°N–55°N, 89°E–99°E	65.27%	74.24%	62.70%
MBOT-B	45°N–55°N, 269°E–279°E	46.51%	59.51%	42.80%
MBOT-C	35°S–45°S, 89°E–99°E	16.95%	23.68%	15.02%
MBOT-D	35°S–45°S, 269°E–279°E	60.05%	72.18%	56.58%

conveniently. In this case, the definition of an equivalent volume is $\delta V = \Delta \theta \cdot \Delta \phi \cdot \Delta r$ (degree $\times$ degree $\times$ spacing of united ratio). The pervasion process of mode MTOP-A is illustrated in Figure 2. This figure shows that how the 1 681 initial tracing elements limited in a small box expand in the whole mantle gradually. Because we use a rectangular coordinate system to describe the problem in a spherical coordinate system, so there presents a big spacing region in Figure 2 for one billion year. In fact, in the spherical coordinate system longitude 0 degree is the same as longitude 360 degree. Figure 3 shows the distribution of the 1 681 tracing elements for the eight initial position models after operating four billion years. In this figure may see intuitively, except MBOT-C, the 1 681 tracing elements for each test box almost flood in the whole mantle, even each group of tracing elements at the different initial positions correspond to different convective mixing processes. The identical region in the mantle possibly contains many tracing elements from different initial boxes. It means that the convection process causes the material from the different initial point in the mantle to be mixed. Meanwhile, we find a special region in the mantle, where the distribution of tracing elements is circular-like. This is a special phenomenon and it infers that the convection streamline

will be closed in this region.

Figure 4 give the apparent density changes with time for the eight initial test box modes. It can be seen in terms of different modes, that the dependence on the apparent density evolution is very different. At the beginning of the process, the apparent density shows a great difference, and for some models the apparent density fluctuates with time. But over time, their apparent densities gradually tend to a value of a stable region between 0.006 7 and 0.007 2. Table 2 lists the apparent density and the pervasion degree of convective mantle mixing after four billion years running of the system for the eight initial position modes. It can be seen that the pervasion degree of convective mantle mixing is stable with the values between 4.0×10^{-7} and 4.3×10^{-7} .

Table 2 Apparent density of the eight modes

Mode	Initial density ρ_0	Final density ρ	Pervasion degree $K/10^{-8}$
	$/(10^5 \text{ count}/(1^\circ \times 1^\circ \times 1 \text{ km}))$	$/(10^{-3} \text{ count}/(1^\circ \times 1^\circ \times 1 \text{ km}))$	
MTOP-A	1.689 1	67.63	0.400 3
MTOP-B	1.689 1	69.90	0.413 8
MTOP-C	1.689 1	67.50	0.399 6
MTOP-D	1.689 1	67.42	0.399 1
MBOT-A	1.689 1	68.54	0.405 8
MBOT-B	1.689 1	68.60	0.406 1
MBOT-C	1.689 1	71.92	0.425 8
MBOT-D	1.689 1	67.48	0.399 5

Figure 2 Pervasion process of the mode MTOP-A.

Figure 3 Distributions of 1 681 tracing elements for the eight initial position modes after four billion years.

For a more detailed understanding the degree of the mantle mixing in this case, we divide the mantle into 4 860 boxes, too. We calculate the contributions of the 13 448 (8×1 681) tracing elements to the mantle mixing after four billion years running of the system. That is to

count how many empty boxes in the mantle, where are not any tracing elements. In this case, there are 1 680 empty boxes, accounting for 30.5% of the mantle boxes. Perhaps the reason for too many empty boxes is that there is too small number of tracing element for us to estimate

Figure 4 Apparent density changes with time for the eight modes.

the actual degree of mantle mixing. In order to increase the number of the tracing elements, we select 2 689 600 ($8 \times 1\,681 \times 200$) tracing elements in the last 100 million years and 5 397 200 ($8 \times 1\,681 \times 400$) tracing elements in the last 200 million years. If adding up the number of the empty boxes for the eight modes one by one, the ratio of empty boxes to all boxes of the mantle is 13% (for 200 time points) and 9.7% (for 400 time points), respectively (see Table 3). In other words, nearly 87% and 90.3% of the mantle boxes contain the tracing elements from the eight initial test boxes. It can be expected that this proportion will increase if increasing the number of initial test boxes. From this statistics we can suggest that through four billion years the mantle convective systems can transfer the tracing elements from different location (perhaps with differences in composition) to spread in the whole mantle.

Table 3 Accumulation of empty boxes and the ratio of empty boxes to all mantle boxes

Mode	Case for 200 time points		Case for 400 time points	
	Number of mpty boxes	Percent of empty boxes	Number of empty boxes	Percent of empty boxes
MTOP-A	1 930	39.7%	1 474	30.3%
+MTOP-B*	1 469	30.2%	1 096	22.6%
+MTOP-C	1 209	24.9%	882	18.1%
+MTOP-D	895	18.4%	600	12.3%
+MBOT-A	759	15.6%	563	11.6%
+MBOT-B	696	14.3%	512	10.5%
+MBOT-C	689	14.2%	510	10.9%
+MBOT-D	634	13.0%	469	9.7%

*+MTOP-B means to plus this mode box.

4 Discussion and conclusions

In this paper, a simple model is employed to discuss the problem of the mantle mixing by counting the

pervasion degree in a convective system. The main conclusion is that on the model conditions the abnormal material limited in small regions, which are located at the top or bottom of the mantle, has been transported to the whole mantle, and its pervasion degree of the mantle mixing is to be a stable value between 4.0×10^{-7} and 4.3×10^{-7} after four billion years running of the system. The ratio of the empty boxes, where are not any tracing elements, is 13% (for 200 time points and 2 689 600 tracers) and 9.7% (for 400 time points and 5 397 200 tracers), respectively. This calculation results can be deduced that the abnormal mantle blocks, which were generated in the early stages of mantle formation, may disappear passing about four billion years. In other words, the mantle has basically been mixed. We understand that above conclusions are only in a very simple model. In fact, there is a most important implicit assumption in the computation model, that is, the surviving time of the abnormal material boxes in this model is 100 million years or 200 million years and there is not any new abnormal material box to generate in the mantle. If some abnormal blocks, on certain conditions, can remain isolated and not involved in convection, or the abnormal material of these blocks can supply from the mantle, the above conclusions will not hold.

Corresponding to the above calculated results, there is a very important question. Whether or not there are still large (at least greater than 500 km) abnormal blocks in the modern mantle (besides the region of lithosphere and core-mantle boundary)? It will require more extensive and detailed study of the seismic observations. However, there is not any report about it. The fact that the continental crust could not return to the mantle has been proved by geological observations or by theoretical

calculations, so since the early differentiation of the Earth, the continental crust has floating over the mantle. There is only a very small amount of crustal material to be carried into the mantle, even the deep mantle, in the process of the deep subduction of the continental crust, but most of the crustal blocks, which have been driven in the mantle, will still return to the Earth's crust at last. The study of the deep mantle near the CMB shows much more complex problems. Seismic tomography proves that there are some anomalies in the region of the core-mantle boundary, including low-speed high-density thermo-chemical anomaly body (Ni et al, 2002; Ni and Helmberger, 2003; Wang and Wen, 2007), very lower velocity region of S-wave (Williams and Garnero, 1996; Williams et al, 1998; Wen et al, 2001; Lay et al, 2004) and the accumulation bodies from subductive plates (Garnero, 2004) and so on. So there still remain lots of questions. For example, these abnormal regions or blocks are the participations in mantle convection? How do they evolve? What is the role of them in the mantle mixing? However, we recognize that because of the simple model in this paper is not a substitute for the real mantle, so when facing the reality, we all need to constantly remind ourselves: there is only one Earth!

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