Diffraction of plane P waves by a canyon of arbitrary shape in poroelastic half-space (II): Numerical results and discussion*

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Abstract This paper investigates in detail the nature of diffraction of plane P waves around a canyon in poroelastic half-space, and studies the effects of incident frequency, drainage condition, porosity, etc, on the diffraction of waves. It is shown that the surface displacement amplitudes of the drained case are close to those of the undrained case, however, the surface displacement amplitudes of the dry case are very different from those of the saturated (either drained or undrained) cases. There are large phase shift between the dry case and the saturated cases, as well as slightly longer resultant wavelengths for the undrained case than those for the drained case and longer resultant wavelengths for the drained case than those for the dry case. For small porosity the surface displacement amplitudes for the saturated cases are almost identical to those for the dry case; while for large porosity, the effect of drainage condition becomes significant, and the surface displacement amplitudes for the undrained case are larger than those for the drained case. As the incident frequency increases, the effect of porosity becomes significant, and more significant for the undrained case than that for the drained case. As the porosity increases, the pore pressures increase significantly but their oscillations become smoother. As the incident frequency increases, the pore pressures become more complicated.

Key words: poroelastic half-space; canyon; diffraction; plane P wave; surface displacement; pore pressure

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1 Introduction

Liang and Liu (2009) presented an indirect boundary integration equation method for diffraction of plane P waves by two-dimensional canyons of arbitrary shape in poroelastic half-space. This paper investigates in detail the nature of diffraction of plane P waves around a canyon in poroelastic half-space, and studies the effects of incident frequency, drainage condition, porosity, etc, on the diffraction of waves.

2 Numerical results and discussion

To reveal the nature of diffraction of plane waves around a canyon in poroelastic half-space, this paper studies a semi-cylindrical canyon for brevity, and the effect of canyon shape on the diffraction will be discussed in a future paper.

The properties of poroelastic half-space (Lin et al, 2005) are used here: Poisson's ratio $\nu=0.25$, the critical porosity $n_{cr}=0.36$, the critical bulk modulus of the solid frame $K_{cr}=200$ MPa, the bulk modulus of the solid grain $K_g=3.6\times10^4$ MPa, the bulk modulus of fluid $K_f=2\times10^3$ MPa, the mass density of the solid grain $\rho_g=2.65\times10^3$ kg/m³, the mass density of fluid $\rho_f=1\times10^3$ kg/m³, porosity $n=0.1, 0.3, 0.34$ and 0.36 , respectively, and the bulk modulus of the solid frame $K_{dry}=2.6055\times10^4, 6.167\times10^3, 2.189\times10^3$ and 200 MPa. Table 1 gives the properties of poroelastic medium for different porosities, which are transformed from the above data using the relation (Biot, 1962) between the two different Biot's models. The linear hysteretic damping ratio $\zeta=0.001$ is used, and the effect of damping on the diffraction will be discussed in another paper.

Figure 1 illustrates the frequency-domain horizontal ($|U_x|/|A_p|$) and vertical ($|U_y|/|A_p|$) surface displacement amplitudes ($|A_p|$ denotes the amplitude of incident P waves) around a canyon in a dry poroelastic half-space, drained poroelastic half-space and undrained poroelastic

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Table 1 Properties of poroelastic medium for different porosities

n	λ^*	M^*	ρ^*	m^*	α	$c_{\alpha 1}$ /km·s ⁻¹	$c_{\alpha 2}$ /km·s ⁻¹	c_{β} /km·s ⁻¹
0.10	1.00	1.17	0.20	22.13	0.28	4.417	0.568	2.517
0.30	1.00	1.64	0.46	3.35	0.83	2.670	0.806	1.355
0.34	1.00	4.08	0.48	2.77	0.94	2.041	0.675	0.827
0.36	1.00	42.2	0.49	2.55	0.99	1.686	0.256	0.254

Note: n is porosity. λ^* , M^* , ρ^* and m^* are dimensionless poroelastic parameters defined in equation (15) in Liang and Liu (2009). α is a Biot's parameter describing compressibility of the two-phased material. $c_{\alpha 1}$, $c_{\alpha 2}$ and c_{β} are P-wave velocities and SV-wave velocity in poroelastic medium, respectively.

half-space, respectively, for porosity $n=0.3$, Poisson's ratio $\nu=0.25$, dimensionless incident frequency $\eta=0.5$, 1.0, 2.0 and 5.0, and incident angle $\theta_{\alpha 1}=0^\circ$, 30° , 60° and 85° , respectively. The surface displacement amplitudes are normalized by displacement intensity of the incident P waves. It can be seen from the figure that the surface displacement amplitudes of the drained case are close to those of the undrained case, which shows that the drainage condition has insignificant effect on the surface displacement amplitude; however, the surface displacement amplitudes of the dry case are very different from those of the saturated (either drained or undrained) cases. For lower incident frequencies, the peaks of the horizontal surface displacement amplitudes for the dry case are larger than those for the saturated cases, especially for larger incident angles, which is partially resulted from their free-field responses (Lin et al, 2005); and as the incident frequency increases, the difference decreases gradually. This may be explained by that the higher incident frequency may easily produce the larger pore pressure around the canyon and the larger displacement amplitudes. However, the peaks of the vertical surface displacement amplitudes of the dry case are near to those of the saturated cases. From the figure large phase shift can also be easily observed between the dry case and the saturated cases, as well as the slightly longer resultant wavelengths for the undrained case than those for the drained case and the longer resultant wavelengths for the drained case than those for the dry case. The difference may be due to the wave interference around the canyon, which is identical to that for incident SV waves (Liang et al, 2006). For nearly grazing incident waves, a standing wave pattern can be observed at $x/a < -1$ for both the dry case and the saturated cases.

Figures 2 and 3 illustrate the frequency-domain horizontal and vertical displacement amplitudes around a canyon in drained poroelastic half-space and undrained poroelastic half-space, respectively, for porosity $n=0.1$,

0.3, 0.34 and 0.36, Poisson's ratio $\nu = 0.25$, dimensionless incident frequency $\eta=0.5$, 1.0, 2.0 and 5.0, and incident angle $\theta_{\alpha 1}=0^\circ$, 30° , 60° and 85° , respectively. The surface displacement amplitudes are normalized by the displacement intensity of the incident P waves. It can be seen from these figures that the porosity has more significant effect on the horizontal surface displacement amplitudes than on the vertical ones, which is similar to their free-field response (Lin et al, 2005); the porosity also has significant effect on the phase shift of the surface displacement amplitudes, and as the porosity increases the wavelengths of the surface displacement amplitudes increase. From these figures, it can also be seen that for small porosity (e.g., $n=0.1$) the surface displacement amplitudes of the saturated cases (either drained or undrained) are almost identical to those of the dry case in Figure 1, and drainage condition has little influence on the surface displacement amplitudes; while for large porosity, the effect of drainage condition becomes significant, and the surface displacement amplitudes for the undrained case are larger than those for the drained case, which is identical to that for incident SV waves (Liang et al, 2006) and may be explained by that, for the undrained case, the pore pressure is larger since there is no energy overflow. It can also be seen that as the incident frequency increases, the effect of porosity gradually becomes significant, and more significant for the undrained case than that for the drained case.

Figure 4 illustrates the frequency-domain surface pore pressures around a canyon in undrained poroelastic half-space, for porosity $n=0.1$, 0.3, 0.34 and 0.36, Poisson's ratio $\nu=0.25$, dimensionless incident frequency $\eta=0.5$, 1.0, 2.0 and 5.0, and incident angle $\theta_{\alpha 1}=0^\circ$, 30° , 60° and 85° , respectively. The surface pore pressures are normalized by the stress intensity of the incident P waves. It can be seen from these figures that large pore pressures occur around the canyon, and the pore pressures depend on the incident angles more strongly, which may be due to the wave interference around the canyon. As the porosity increases, the pore pressures increase significantly but their oscillations become smoother, which implies that more energy is taken by the pore fluid and less by the soft solid frame for larger porosity, and larger porosity (with more pore fluid) results in smoother oscillation then. When the porosity approaches to its critical state ($n=0.36$), the modulus of the solid frame becomes very soft, and most of the energy is undertaken by the first P waves (Lin et al, 2005),

Figure 1 Frequency response of dry poroelastic, drained poroelastic, and undrained poroelastic half-space ($n=0.3$, $\nu=0.25$) for η taken as 0.5 (a), 1.0 (b), 2.0 (c), and 5.0 (d), respectively.

Figure 2 Frequency response for different porosities for drained boundary. (a) $\eta=0.5$; (b) $\eta=1.0$; (c) $\eta=2.0$; (d) $\eta=5.0$.

therefore pore pressures increase significantly, which is very different from that for incident SV waves (Liang et al, 2006). For incident SV waves most of the energy is carried by SV waves and the second P waves (Lin et al, 2005), but SV waves do not produce pore pressures.

From these figures it can also be seen that as the incident frequency increases, the pore pressures become more complicated, suggesting that the higher-frequency waves more easily stimulate pore pressures of large amplitudes and of rapid oscillations.

Figure 3 Frequency response for different porosities for undrained boundary. (a) $\eta=0.5$; (b) $\eta=1.0$; (c) $\eta=2.0$; (d) $\eta=5.0$.

Figures 5 and 6 illustrate the time-domain surface displacements of poroelastic half-space of drained boundary and undrained boundary for porosity $n=0.1, 0.3, 0.34$ and 0.36 , Poisson's ratio $\nu = 0.25$, incident wave of Ricker wavelet with the characteristic frequency

of 1.0 , and incident angle $\theta_{ai}=0^\circ$ and 30° , respectively. It can be seen from the figures that the drainage condition and the porosity both have significant effects on the time domain response, especially when the porosity is large; and the creeping waves can be observed both for drained

Figure 4 Surface pore pressure for different incident angles for undrained boundary. (a) $\eta = 0.5$; (b) $\eta = 1.0$; (c) $\eta = 2.0$; (d) $\eta = 5.0$.

Figure 5 Time response for drained boundary with porosity n of 0.1 (a), 0.3 (b), 0.34 (c) and 0.36 (d) for incident angles of 0° (upper) and 30° (bottom).

Figure 6 Time response for undrained boundary with porosity n of 0.1 (a), 0.3 (b), 0.34 (c) and 0.36 (d) for incident angles of 0° (upper) and 30° (bottom).

and undrained cases, which is similar to the elastic (dry) case (Kawase, 1988). In general, the nature in time domain is consistent with those in frequency domain.

3 Conclusions

This paper investigates in detail the nature of diffraction of plane P waves around a canyon in poroelastic half-space, and studies the effects of incident frequency, drainage condition, porosity, etc, on the diffraction of waves. Some useful conclusions are obtained.

The surface displacement amplitudes of the drained case are close to those of the undrained case, however, the surface displacement amplitudes of the dry case are very different from those of the two saturated (either drained or undrained) cases, and as the incident frequency increases the difference decreases gradually. There are large phase shift between the dry case and the saturated cases, as well as the slightly longer resultant wavelengths for the undrained case than those for the drained case and the longer resultant wavelengths for the drained case than those for the dry case, and the difference is identical to that for incident SV waves.

The porosity has more significant effect on the horizontal surface displacement amplitudes than on the vertical ones, and the porosity also has significant effect on the phase shift of the surface displacement amplitudes, and as the porosity increases the wavelengths of the surface displacement amplitudes increase. For small porosity the surface displacement amplitudes of the saturated cases are almost identical to those of the dry case, and drainage condition has little influence on the surface displacement amplitudes; while for large porosity, the effect of drainage condition becomes significant, and the surface displacement amplitudes for the undrained case are larger than those for the drained case, which is identical to that for incident SV waves. As the incident frequency increases, the effect of porosity gradually becomes significant, and more significant for the undrained case than that for the drained case.

The large pore pressures occur around the canyon, and the pore pressures depend strongly on the incident angles. As the porosity increases, the pore pressures increase significantly but their oscillations become smoother. When the porosity approaches to its critical state, the modulus of the solid frame is very soft, and

most of the energy is undertaken by the first P waves, therefore, pore pressures increase significantly, which is very different from that for incident SV waves. As the incident frequency increases, the pore pressures become more complicated, and the higher-frequency waves more easily stimulate pore pressures of large amplitudes and of rapid oscillations.

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