Scattering of plane P waves by canyons containing multiple circular-arc-shaped layers*

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Abstract By expressing the wave functions in the form of Fourier-Bessel series, the analytical solution for the two-dimension scattering problem of plane P waves by the cylindrical canyon topography that contains arbitrary number of circular-arc-shaped layers is presented firstly. And then, the convergence of the proposed series solution with the truncation number of terms is discussed, which demonstrates that the analytical solution can converge even for very high frequencies of the incident P wave. Finally, using this solution, the influences that are imposed on the stationary ground motion by the number and the sequence of alluvial layers, as well as the stiffness of soft interlayer contained in the canyon, are studied.

Key words: scattering; canyon containing multiple layers; analytical solution; plane P wave

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1 Introduction

The local irregular topographies significantly affect the features of seismic ground motion, which in turn influence the intensity and distribution of seismic damage. Therefore, the scattering and diffraction of seismic waves by local topographies have been intensively studied in the community of earthquake engineering, and can be theoretically probed by numerical or analytical method. Generally, the numerical method is a powerful tool for solving practical complicated problems (Boore et al, 1971; Harmsen and Harding, 1981; Sánchez-Sesma et al, 1993; Sánchez-Sesma and Luzón, 1995; Bakır et al, 2002). However, as to these topographies with presumed regular geometries and simple mechanical properties, compared with the numerical method, the analytical one can present the solution validly in a very broad frequency band, which is beneficial for revealing the physical mechanism of the wave scattering problem. And most importantly, the analytical solutions for the regular problems can be used to check and calibrate the precision of these solutions obtained by numerical methods. Therefore, the deduction of analytical solutions for the regular problems is requisite due to both the theoretical and the practical reasons.

As to the regular canyon topographies, the methods for expressing the wave functions in the form of Fourier-Bessel series (Trifunac, 1973) and Mathieu series (Wong and Trifunac, 1974) were introduced firstly to solve analytically the two-dimensional scattering problems of plane SH wave by the semi-circular and the semi-elliptical canyon, respectively. The former has found its applications in different scattering problems for the cylindrical canyons with circular-arc sections (Cao and Lee, 1989, 1990; Yuan and Liao, 1994; Liang et al, 2003; Li and Zhao, 2005). As mentioned by Liang et al (2003), the real canyon topographies usually contain alluvial layers due to different geological factors. However, as to the scattering of plane P wave, because of the complexity of the problems, only the analytical solution for the canyon with just one surface circular-arc-shaped layer is deduced (Liang et al, 2002). As an extension of this study, this paper firstly deduces the analytical solution for the scattering problem of plane P wave by the canyon containing arbitrary number of circular-arc-shaped layers. Then the convergence of the proposed series solutions is discussed, and the influence imposed on the ground motion by the layering of the deposits in the canyon is also studied.

2 The model

The model of the canyon that contains multiple circular-arc-shaped layers is shown in Figure 1. There are altogether $L-1$ layers overlying the half-space, and

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the boundaries between two adjacent layers are all shallow circular arcs, whose centers are all located at the point of O_1 . The half-width, the depth, and the radius of each arc are denoted by a_l , h_l , and b_l , respectively, where $l=1, 2, \dots, L$. The height of O_1 is h_0 . As to the mechanical properties of medium, the shear-wave velocity, the dilatation-wave velocity, the mass density, the Poisson's ratio, and the shear modulus are denoted by $c_s^{(l)}$, $c_p^{(l)}$, $\rho^{(l)}$, $\nu^{(l)}$, and $\mu^{(l)}$, respectively, where $l=1, 2, \dots, L$ and the L -th medium is just the half-space medium.

Figure 1 The model of multi-layered canyon.

The incident P wave is harmonic plane wave with circular frequency ω and incident angle γ_p , its potential function is denoted by

$$\Phi^i(x, y) = \exp[ik_p^{(L)}(x \sin \gamma_p - y \cos \gamma_p)], \quad (1)$$

where i is the imaginary unit, i.e., $i = \sqrt{-1}$, and the time factor of $\exp(-i\omega t)$ is omitted in this paper for brevity. In equation (1), $k_p^{(L)}$ is the circular wave number of P wave in the half-space. Similarly,

$$k_p^{(l)} = \frac{\omega}{c_p^{(l)}} \quad k_s^{(l)} = \frac{\omega}{c_s^{(l)}} \quad (2)$$

are the circular wave number of P-wave and S-wave in the l -th medium.

In the stationary case, the potential functions of P and SV waves satisfy the following equations:

$$\begin{cases} \nabla^2 \Phi + k_p^2 \Phi = 0 \\ \nabla^2 \Psi + k_s^2 \Psi = 0 \end{cases} \quad (3)$$

where the symbols Φ and Ψ represent the potentials of P wave and SV wave, respectively. The boundary conditions of the problem include the zero-stress conditions

$$\left. \begin{Bmatrix} t_{yy}^{(l)}(x, y) \\ t_{xy}^{(l)}(x, y) \end{Bmatrix} \right|_{|x| > a_l, y=0} = 0 \quad l=1, 2, \dots, L, \quad (4a)$$

$$\left. \begin{Bmatrix} t_{rr}^{(l)}(r_1, \theta_1) \\ t_{r\theta}^{(l)}(r_1, \theta_1) \end{Bmatrix} \right|_{r_1=b_l} = 0, \quad (4b)$$

and the continuity conditions at the l -th arc

$$\left. \begin{Bmatrix} u_r^{(l-1)}(r_1, \theta_1) \\ u_\theta^{(l-1)}(r_1, \theta_1) \end{Bmatrix} \right|_{r_1=b_l, |r_1 \sin \theta_1| \leq a_l} = \left. \begin{Bmatrix} u_r^{(l)}(r_1, \theta_1) \\ u_\theta^{(l)}(r_1, \theta_1) \end{Bmatrix} \right|_{r_1=b_l, |r_1 \sin \theta_1| \leq a_l}, \quad (5a)$$

$$\left. \begin{Bmatrix} t_{rr}^{(l-1)}(r_1, \theta_1) \\ t_{r\theta}^{(l-1)}(r_1, \theta_1) \end{Bmatrix} \right|_{r_1=b_l, |r_1 \sin \theta_1| \leq a_l} = \left. \begin{Bmatrix} t_{rr}^{(l)}(r_1, \theta_1) \\ t_{r\theta}^{(l)}(r_1, \theta_1) \end{Bmatrix} \right|_{r_1=b_l, |r_1 \sin \theta_1| \leq a_l}, \quad (5b)$$

where $l=2, 3, \dots, L$, and the symbols t and u denote the stress and the displacement, respectively.

3 Potential functions of scattering waves

The approximate method proposed by Cao and Lee (1989) will be adopted here, that is, an arc with very long radius would be used to simulate approximately the flat ground surface, with its radius being R and its center at O_2 , as shown in Figure 2. Accordingly, in the coordinate system r_2 - θ_2 , the zero-stress condition (4a) is transformed into

$$\left. \begin{Bmatrix} t_{rr}^{(l)}(r_2, \theta_2) \\ t_{r\theta}^{(l)}(r_2, \theta_2) \end{Bmatrix} \right|_{r_2=R, |r_2 \sin \theta_2| > a_l} = 0 \quad l=1, 2, \dots, L. \quad (4c)$$

Figure 2 The scattering waves.

In the half-space, if the canyon did not exist, there were only the free-field waves, which include the incident P wave Φ^i , the reflected P wave Φ^r , and the reflected SV wave Ψ^r , as shown in Figure 2. In the coord-

dinate system r_1 - θ_1 , the potential function of free-field P wave, i.e., $\Phi^{i+r} = \Phi^j + \Phi^r$, and that of the free-field SV wave, i.e., Ψ^r , possess the following forms of Fourier-Bessel series:

$$\begin{cases} \Phi^{i+r}(r_1, \theta_1) \\ \Psi^r(r_1, \theta_1) \end{cases} = \sum_{n=0}^{\infty} \begin{Bmatrix} J_n(k_p^{(L)} r_1)(A_{21,n}^{(L)} \cos n\theta_1 + B_{21,n}^{(L)} \sin n\theta_1) \\ J_n(k_s^{(L)} r_1)(C_{21,n}^{(L)} \sin n\theta_1 + D_{21,n}^{(L)} \cos n\theta_1) \end{Bmatrix}, \quad (6)$$

where $J_n(x)$ is the first kind of Bessel function, and the coefficients $A_{21,n}^{(L)}$ through $D_{21,n}^{(L)}$ can be easily determined and have definite expressions (Zhang, 2008).

3.1 Scattering waves in the half-space

Besides the above free-field waves, in the half-space, there are scattering waves due to the canyon, which include the cylindrical traveling waves radiated from O_1 , represented by $\Phi_1^{(L)}$ and $\Psi_1^{(L)}$, and the cylindrical standing waves residing within the big circle, represented by $\Phi_3^{(L)}$ and $\Psi_3^{(L)}$, as shown in Figure 2. In the coordinate system r_1 - θ_1 and r_2 - θ_2 , the above scattering waves have the following potentials:

$$\begin{cases} \Phi_1^{(L)}(r_1, \theta_1) \\ \Psi_1^{(L)}(r_1, \theta_1) \end{cases} = \sum_{n=0}^{\infty} \begin{Bmatrix} H_n^{(1)}(k_p^{(L)} r_1)(A_{11,n}^{(L)} \cos n\theta_1 + B_{11,n}^{(L)} \sin n\theta_1) \\ H_n^{(1)}(k_s^{(L)} r_1)(C_{11,n}^{(L)} \sin n\theta_1 + D_{11,n}^{(L)} \cos n\theta_1) \end{Bmatrix}, \quad (7)$$

$$\begin{cases} \Phi_3^{(L)}(r_2, \theta_2) \\ \Psi_3^{(L)}(r_2, \theta_2) \end{cases} = \sum_{m=0}^{\infty} \begin{Bmatrix} J_m(k_p^{(L)} r_2)(A_{32,m}^{(L)} \cos m\theta_2 + B_{32,m}^{(L)} \sin m\theta_2) \\ J_m(k_s^{(L)} r_2)(C_{32,m}^{(L)} \sin m\theta_2 + D_{32,m}^{(L)} \cos m\theta_2) \end{Bmatrix}, \quad (8)$$

where $H_n^{(1)}(x)$ denotes the first kind of Hankel function, and $A_{11,n}^{(L)}$ through $D_{32,m}^{(L)}$ are unknown coefficients to be determined. By the Graf's addition formula (Abramowitz and Stegun, 1972), the above wave functions can be transformed in the other coordinate systems as

$$\begin{cases} \Phi_1^{(L)}(r_2, \theta_2) \\ \Psi_1^{(L)}(r_2, \theta_2) \end{cases} = \sum_{m=0}^{\infty} \begin{Bmatrix} J_m(k_p^{(L)} r_2)(A_{12,m}^{(L)} \cos m\theta_2 + B_{12,m}^{(L)} \sin m\theta_2) \\ J_m(k_s^{(L)} r_2)(C_{12,m}^{(L)} \sin m\theta_2 + D_{12,m}^{(L)} \cos m\theta_2) \end{Bmatrix}, \quad (9)$$

$$\begin{cases} \Phi_3^{(L)}(r_1, \theta_1) \\ \Psi_3^{(L)}(r_1, \theta_1) \end{cases} =$$

$$\sum_{n=0}^{\infty} \begin{Bmatrix} J_n(k_p^{(L)} r_1)(A_{31,n}^{(L)} \cos n\theta_1 + B_{31,n}^{(L)} \sin n\theta_1) \\ J_n(k_s^{(L)} r_1)(C_{31,n}^{(L)} \sin n\theta_1 + D_{31,n}^{(L)} \cos n\theta_1) \end{Bmatrix}, \quad (10)$$

where the relations between the coefficients $A_{12,m}^{(L)}$ through $D_{31,n}^{(L)}$ and $A_{11,n}^{(L)}$ through $D_{32,m}^{(L)}$ be found in Zhang (2008).

3.2 Scattering waves in the alluvial layer

In the l -th alluvial layer, where $l=1, 2, \dots, L-1$, exist two kinds of cylindrical standing waves, represented by the pair of $\Phi_2^{(l)}$ and $\Psi_2^{(l)}$, and the pair of $\Phi_3^{(l)}$ and $\Psi_3^{(l)}$, respectively, as shown in Figure 2, where the symbols $\Phi_1^{(l)}$ and $\Psi_1^{(l)}$ denote the outgoing cylindrical waves radiated from the center of O_1 . In the coordinate systems r_1 - θ_1 and r_2 - θ_2 , the potential functions of these scattering waves possess the following forms of Fourier-Bessel series:

$$\begin{cases} \Phi_1^{(l)}(r_1, \theta_1) \\ \Psi_1^{(l)}(r_1, \theta_1) \end{cases} = \sum_{n=0}^{\infty} \begin{Bmatrix} H_n^{(1)}(k_p^{(l)} r_1)(A_{11,n}^{(l)} \cos n\theta_1 + B_{11,n}^{(l)} \sin n\theta_1) \\ H_n^{(1)}(k_s^{(l)} r_1)(C_{11,n}^{(l)} \sin n\theta_1 + D_{11,n}^{(l)} \cos n\theta_1) \end{Bmatrix}, \quad (11)$$

$$\begin{cases} \Phi_2^{(l)}(r_1, \theta_1) \\ \Psi_2^{(l)}(r_1, \theta_1) \end{cases} = \sum_{n=0}^{\infty} \begin{Bmatrix} J_n(k_p^{(l)} r_1)(A_{21,n}^{(l)} \cos n\theta_1 + B_{21,n}^{(l)} \sin n\theta_1) \\ J_n(k_s^{(l)} r_1)(C_{21,n}^{(l)} \sin n\theta_1 + D_{21,n}^{(l)} \cos n\theta_1) \end{Bmatrix}, \quad (12)$$

$$\begin{cases} \Phi_3^{(l)}(r_2, \theta_2) \\ \Psi_3^{(l)}(r_2, \theta_2) \end{cases} = \sum_{m=0}^{\infty} \begin{Bmatrix} J_m(k_p^{(l)} r_2)(A_{32,m}^{(l)} \cos m\theta_2 + B_{32,m}^{(l)} \sin m\theta_2) \\ J_m(k_s^{(l)} r_2)(C_{32,m}^{(l)} \sin m\theta_2 + D_{32,m}^{(l)} \cos m\theta_2) \end{Bmatrix}, \quad (13)$$

where $A_{11,n}^{(l)}$ through $D_{32,m}^{(l)}$ are unknown coefficients to be determined. By the Graf's addition formula (Abramowitz and Stegun, 1972), the above potential functions can be expressed in the other coordinate systems as

$$\begin{cases} \Phi_1^{(l)}(r_2, \theta_2) \\ \Psi_1^{(l)}(r_2, \theta_2) \end{cases} = \sum_{m=0}^{\infty} \begin{Bmatrix} J_m(k_p^{(l)} r_2)(A_{12,m}^{(l)} \cos m\theta_2 + B_{12,m}^{(l)} \sin m\theta_2) \\ J_m(k_s^{(l)} r_2)(C_{12,m}^{(l)} \sin m\theta_2 + D_{12,m}^{(l)} \cos m\theta_2) \end{Bmatrix}, \quad (14)$$

$$\begin{cases} \Phi_2^{(l)}(r_2, \theta_2) \\ \Psi_2^{(l)}(r_2, \theta_2) \end{cases} =$$

$$\sum_{m=0}^{\infty} \left\{ \begin{matrix} J_m(k_p^{(l)} r_2)(A_{22,m}^{(l)} \cos m\theta_2 + B_{22,m}^{(l)} \sin m\theta_2) \\ J_m(k_s^{(l)} r_2)(C_{22,m}^{(l)} \sin m\theta_2 + D_{22,m}^{(l)} \cos m\theta_2) \end{matrix} \right\}, \quad (15)$$

and

$$\begin{aligned} & \left\{ \begin{matrix} \Phi_3^{(l)}(r_1, \theta_1) \\ \Psi_3^{(l)}(r_1, \theta_1) \end{matrix} \right\} = \\ & \sum_{n=0}^{\infty} \left\{ \begin{matrix} J_n(k_p^{(l)} r_1)(A_{31,n}^{(l)} \cos n\theta_1 + B_{31,n}^{(l)} \sin n\theta_1) \\ J_n(k_s^{(l)} r_1)(C_{31,n}^{(l)} \sin n\theta_1 + D_{31,n}^{(l)} \cos n\theta_1) \end{matrix} \right\}, \quad (16) \end{aligned}$$

where the relations between the coefficients $A_{12,m}^{(l)}$ through $D_{31,n}^{(l)}$ and $A_{11,n}^{(l)}$ through $D_{32,m}^{(l)}$ can refer to Zhang (2008).

After expressing the potential functions of the scattering waves in the form of Fourier-Bessel series, the forms of Fourier-Bessel series for the displacement and the stress can be determined based on fundamental theory of elastodynamics, which have been presented by Zhang (2008) and are omitted here.

4 Solutions for the unknown coefficients

4.1 Application of zero-stress conditions

Taking into account the fact that the free-field wave functions yield zero stresses on the free surface, the zero-stress condition (4c) results in the following relations:

$$\begin{pmatrix} A_{31,n}^{(l)} \\ B_{31,n}^{(l)} \\ C_{31,n}^{(l)} \\ D_{31,n}^{(l)} \end{pmatrix} = \sum_{j=0}^{\infty} \begin{pmatrix} F_{nj}^{(2)+}(k_p^{(l)} D) A_{11,j}^{(l)} \\ F_{nj}^{(2)-}(k_p^{(l)} D) B_{11,j}^{(l)} \\ F_{nj}^{(2)+}(k_s^{(l)} D) C_{11,j}^{(l)} \\ F_{nj}^{(2)-}(k_s^{(l)} D) D_{11,j}^{(l)} \end{pmatrix} + \sum_{j=0}^{\infty} \begin{pmatrix} F_{nj}^{(1)+}(k_p^{(l)} D) A_{21,j}^{(l)} \\ F_{nj}^{(1)-}(k_p^{(l)} D) B_{21,j}^{(l)} \\ F_{nj}^{(1)+}(k_s^{(l)} D) C_{21,j}^{(l)} \\ F_{nj}^{(1)-}(k_s^{(l)} D) D_{21,j}^{(l)} \end{pmatrix} \quad (17a)$$

$$\begin{pmatrix} A_{31,n}^{(L)} \\ B_{31,n}^{(L)} \\ C_{31,n}^{(L)} \\ D_{31,n}^{(L)} \end{pmatrix} = \sum_{j=0}^{\infty} \begin{pmatrix} F_{nj}^{(2)+}(k_p^{(L)} D) A_{11,j}^{(L)} \\ F_{nj}^{(2)-}(k_p^{(L)} D) B_{11,j}^{(L)} \\ F_{nj}^{(2)+}(k_s^{(L)} D) C_{11,j}^{(L)} \\ F_{nj}^{(2)-}(k_s^{(L)} D) D_{11,j}^{(L)} \end{pmatrix}, \quad (17b)$$

where $l=1, 2, \dots, L-1$, and the definition for the function $F_{nj}^{(1)\pm}(x)$ and $F_{nj}^{(2)\pm}(x)$ can refer to Zhang (2008).

In addition, from the zero-stress condition (4b), the relations among the unknown coefficients in the first alluvial layer can be obtained as

$$\begin{pmatrix} A_{11,n}^{(l)} \\ C_{11,n}^{(l)} \end{pmatrix} = \mathbf{S}_{1,n}^{(l)} \left(\begin{pmatrix} A_{21,n}^{(l)} \\ C_{21,n}^{(l)} \end{pmatrix} + \begin{pmatrix} A_{31,n}^{(l)} \\ C_{31,n}^{(l)} \end{pmatrix} \right) \quad (18a)$$

and

$$\begin{pmatrix} B_{11,n}^{(l)} \\ D_{11,n}^{(l)} \end{pmatrix} = \mathbf{S}_{2,n}^{(l)} \left(\begin{pmatrix} B_{21,n}^{(l)} \\ D_{21,n}^{(l)} \end{pmatrix} + \begin{pmatrix} B_{31,n}^{(l)} \\ D_{31,n}^{(l)} \end{pmatrix} \right), \quad (18b)$$

where

$$\begin{cases} \mathbf{S}_{1,n}^{(l)} = -[\mathbf{T}_{1,n}^{(3)+}(b_1)]^{-1} \mathbf{T}_{1,n}^{(1)+}(b_1) \\ \mathbf{S}_{2,n}^{(l)} = -[\mathbf{T}_{1,n}^{(3)-}(b_1)]^{-1} \mathbf{T}_{1,n}^{(1)-}(b_1) \end{cases} \quad (19)$$

and the expressions for $\mathbf{T}_{1,n}^{(3)\pm}(x)$ and $\mathbf{T}_{1,n}^{(1)\pm}(x)$ can refer to Zhang (2008).

4.2 Application of continuity conditions

By the same recursive deduction process as given by Zhang (2008), the continuity conditions, i.e., equations (5a) and (5b), can yield the relations among different unknown coefficients in the l -th layer as

$$\begin{pmatrix} A_{11,n}^{(l)} \\ C_{11,n}^{(l)} \end{pmatrix} = \mathbf{S}_{1,n}^{(l)} \left(\begin{pmatrix} A_{21,n}^{(l)} \\ C_{21,n}^{(l)} \end{pmatrix} + \begin{pmatrix} A_{31,n}^{(l)} \\ C_{31,n}^{(l)} \end{pmatrix} \right) \quad (20a)$$

and

$$\begin{pmatrix} B_{11,n}^{(l)} \\ D_{11,n}^{(l)} \end{pmatrix} = \mathbf{S}_{2,n}^{(l)} \left(\begin{pmatrix} B_{21,n}^{(l)} \\ D_{21,n}^{(l)} \end{pmatrix} + \begin{pmatrix} B_{31,n}^{(l)} \\ D_{31,n}^{(l)} \end{pmatrix} \right). \quad (20b)$$

Starting from the relation matrices $\mathbf{S}_{1,n}^{(1)}$ and $\mathbf{S}_{2,n}^{(1)}$ given by equation (19) for $l=1$, the relation matrices $\mathbf{S}_{1,n}^{(l)}$ and $\mathbf{S}_{2,n}^{(l)}$ for $l=2, 3, \dots, L$ can be recursively worked out. Combined these relation matrices with equation (17), the unknown coefficients of scattering waves can be determined in success by the order of $l=L, L-1, \dots, 1$. The detailed process has been given by Zhang (2008) and is omitted here for brevity.

After all of the unknown coefficients of the potential functions of the scattering waves have been solved, the scattering displacement and stress fields can be determined accordingly. Therefore, the deduce of the analytical solution has been completed.

5 Result analyses

5.1 Convergence and precision of series solution

The infinite series of the unknown coefficient solutions need to be truncated. The convergence of the series solution can be judged according to the following criteria. Supposing the series is truncated by the finite number of N , the displacement of a specific spatial point (x, y) can be calculated, which is denoted by $u(x, y; N)$. Let $e(x, y; N)$ be the difference between the displacements calculated at the successive truncation numbers, that is,

$$e(x, y; N) = |u(x, y; N+1) - u(x, y; N)|. \quad (21)$$

As to any calculation case, if exists a number, N_c , when $N \geq N_c$,

$$e(x, y; N) \leq \varepsilon, \quad (22)$$

then the truncation number N_c is defined as the convergence number, which means that the results calculated at this truncation number approach to the true solutions for the given problem with satisfactory precisions. In equation (22), the parameter ε controls the calculation precision, which is set to be 10^{-6} in this study. Referred to Liang et al (2002), the radius of the big arc that simulates the flat surface, R , takes the value of $10^2 b_L$, which is sufficient for the convergence of the solution.

In this paper, the following non-dimension parameter will be used to describe the frequency of incident wave:

$$\eta = \frac{2a_L}{\lambda_s^{(L)}} = \frac{\omega a_L}{\pi c_s^{(L)}}, \quad (23)$$

which is the ratio between the canyon width, $2a_L$, and the wavelength of the S wave in the half-space, $\lambda_s^{(L)}$. Moreover, the shorter the wave length of incident wave $\lambda_p^{(L)}$, the higher the incident frequency ω .

Figure 3 presents the variation of $e(x, y; N)$ with the truncation number N , where Figure 3a is for the incident frequency $\eta=2.0$ and Figure 3b for $\eta=20.0$. The incident angle $\gamma_p=0^\circ$, which means the P wave is incident vertically. The canyon contains three alluvial layers, which is indexed by C0-1 in Table 1 where the corresponding geometrical and mechanical properties of the canyon are presented. Table 1 also defines the other types of canyons that would be studied later.

Table 1 Geometrical and mechanical parameters of different canyons studied in this paper

Analysis purpose	Canyon index	Geometry $h_1:h_2:h_3:h_4:a_1$	Mechanical parameters*	
			$\beta_1:\beta_2:\beta_3:\beta_4$	$\rho_1:\rho_2:\rho_3:\rho_4$
Study of convergence	C0-1		200:300:400:500	1.9:2.0:2.1:2.2
Influence of the number of layers	C1-1		300:300:300:500	2.0:2.0:2.0:2.2
	C1-2		200:300:400:500	1.9:2.0:2.1:2.2
Influence of the sequence of layers	C2-1		150:300:400:500	1.8:2.0:2.1:2.2
	C2-2	3.0:4.0:5.0:6.0:6.0	300:150:400:500	2.0:1.8:2.1:2.2
	C2-3		300:400:150:500	2.0:2.1:1.8:2.2
Influence of the stiffness of the soft interlayer	C3-1		300:100:400:500	2.0:1.7:2.1:2.2
	C3-2		300:150:400:500	2.0:1.8:2.1:2.2
	C3-3		300:200:400:500	2.0:1.9:2.1:2.2

* The Poisson's ratio of all media is set to be 0.25. Shear wave velocity is denoted hereafter by β .

Figure 3 The convergence of series solution with the truncation number N of terms. The figure presents the variation of difference between successive truncation numbers. Dotted line indicates the numerical precision adopted in this study.

Figure 3 is for the y -direction displacements of the spatial point that is located at the center of the canyon bed, i.e., $x=0, y=-h_1$ in equation (21) or equation (22). As to $\eta=2.0$, when the truncation number $N \geq 13$, $e < 10^{-6}$, the convergence number N_c for the incident frequency of 2.0 is 13; while for $\eta=20.0$, which is a very high incident frequency, the convergence number N_c is 61. Generally, the higher the incident frequency, the larger the truncation number needed for convergence. And it can be seen that even for $\eta=20.0$, the analytical solution

proposed in this paper can still converge.

To demonstrate the precision of the proposed solution, the results given by this paper would be compared with those by Liang et al (2002), which considered only one surface layer overlying the half-space. Two canyons will be discussed. The firstly one is that studied by Liang et al (2002), with $h_1:h_2:a_1=3:4:3$, $\beta_1:\beta_2=200:800$, and $\rho_1:\rho_2=1.6:2.0$. The second one is the same as the first one, except that the surface layer is divided into three layers whose mechanical properties are same, with

$h_1:h_2:h_3:h_4:a_1=3:3.3:3.6:4:3$, $\beta_1:\beta_2:\beta_3:\beta_4=200:200:200:800$, and $\rho_1:\rho_2:\rho_3:\rho_4=1.6:1.6:1.6:2.0$. The Poisson's ratio of all media is 0.25. And the non-dimensional frequency of the incident P wave is equal to 1.0, i.e., $\eta=1.0$, with the incident angle being $\gamma_p=0^\circ$, representing the vertical incidence. The spatial variation curves of the surface displacement amplitudes of the second canyon computed in

this paper are given in Figure 4 by the scatter circulars, with these of the first canyon computed by the algorithm proposed by Liang et al (2002) given by the solid lines. It can be seen that these two results are same, which illustrates the precision of the proposed results for the multi-layered canyon is satisfactory.

Figure 4 Comparison of the proposed solution in this paper with that obtained by Liang et al (2002).

5.2 Influence of layering of alluvial media on ground motion

The number of the alluvial layers in the canyon would influence the ground motion, especially for high incident frequencies. To evaluate such effects, consider the canyons indexed by C1-1 and C1-2 in Table 1, from which it can be seen that the geometries of the deposits in these two canyons are same, however, C1-1 has only one layer, while the alluvial media in C1-2 are divided into three layers. The values of the mechanical parameters of the single deposit in C1-1 are just the arithmetic

means of the corresponding values of the deposits in C1-2, as listed in Table 1. Figures 5 and 6 give the variations of the displacement amplitudes on the ground surface for the incident frequencies of 1.0, 4.0, and 16.0, respectively. Figure 5 is for the vertical incidence, i.e., $\gamma_p=0^\circ$, while Figure 6 for the inclined incidence with $\gamma_p=45^\circ$. From the geometry given in Table 1, the thickness of all alluvial layers is same, $h_2-h_1=h_3-h_2=h_4-h_3=a_1/6$, while a_4 is about $1.6a_1$. Therefore, for $\eta=1.0$, the shear wavelength in the half-space, i.e., $\lambda_s^{(L)}$, is equal to

Figure 5 Influence of the number of alluvial layers on the ground motion in x-direction (a) and y-direction (b) for the vertical incidence of P wave with $\gamma_p=0^\circ$.

Figure 6 Influence of the number of alluvial layers on the ground motion in x-direction (a) and y-direction (b) for the inclined incidence of P wave with $\gamma_p=45^\circ$.

$3.2a_1=2a_4$, representing the low-frequency incidence; for $\eta=4.0$, $\lambda_s^{(L)}=0.8a_1$, which falls between $h_2=0.67a_1$ and $h_3=0.83a_1$, represents the moderate-to-high-frequency incidence; while for $\eta=16.0$, $\lambda_s^{(L)}=0.2a_1$, which is only a little more than the thickness of alluvial layers ($=a_1/6\approx0.17a_1$), represents the very-high-frequency incidence. From Figures 5 and 6, it can be seen that for the low-frequency incident wave, i.e., $\eta=1.0$, the wavelengths of the incident wave and the scattering waves are too long to be sensitive to the layering of alluvial media in the canyon. As a result, the displacements of C1-1 and C1-2 are almost the same for $\eta=1.0$. As the incident frequency increases, the wavelength decreases, and thus the incident wave begins to be more sensitive to the layering of alluvial media. Therefore, the variation of the displacement amplitude of the single-layered canyon (C1-1) is distinctly different from that of the three-layered canyon (C1-2). Furthermore, in the central part of canyon, i.e., $-0.5\leq x/a_4\leq 0.5$, the displacement amplitude of C1-2, i.e., the three-layered canyon, are higher than that of C1-1 in general.

Next consider the influence of the sequence of alluvial layers on the ground motion. Three types of three-layered canyons would be studied here, i.e., the canyons indexed by C2-1, C2-2, and C2-3 in Table 1. The geometries of these three canyons are the same. C2-1 is the normally alluvial canyon, while C2-2 is the first kind of abnormally alluvial canyon, with its soft interlayer being the second layer. The sketch of C2-2 is shown in Figure 7, where the soft interlayer and its outcrops on the surface are marked. C2-3 is the second kind of abnormally alluvial canyon, with its third layer being the soft interlayer. Figure 8 gives the distribution of displacement amplitude for these three canyons, with the incident angle $\gamma_p=0^\circ$, i.e., the vertical incidence. In Figure 8, the x -axis denotes the location on the surface of canyon by the non-dimension parameter x/a_4 , the y -axis represents the frequency of incident wave by the parameter η whose value increases from 0.01 to 20.0 with the increment of 0.01, while different colors and their shades describe the values of the displacement amplitude. In the central part of canyon surface, i.e., $-0.5\leq x/a_4\leq 0.5$, especially for the y -direction displacement, the displacement amplitude of C2-1 is higher than the two abnormally alluvial canyons in most frequency bands. While in the region of the outcrop of soft interlayer in the canyon C2-2, i.e., $-0.88\leq x/a_4\leq -0.76$ or $0.76\leq x/a_4\leq 0.88$, its displacement amplitude is higher

than the other canyons in most frequency bands, and the same phenomenon can also be found for the displacement amplitude of canyon C2-3 in its soft-interlayer outcrop region, i.e., $-1.0\leq x/a_4\leq -0.88$ or $0.88\leq x/a_4\leq 1.0$. The values in these regions are darkened by color for distinction. Such phenomenon might be due to the fact that the soft interlayer in the canyon could absorb or trap much energy of the scattering waves, which in turn causes the high displacement amplitude in the region where it outcrops on the surface. Figure 9 gives the displacement amplitude distribution for the inclined incidence of $\gamma_p=45^\circ$, which shows the similar phenomenon to Figure 8, except that for the inclined incidence, the displacement amplitude in the left part of canyon, from which the P wave is incident, is higher than the right part in most frequency bands, as indicated by the shade of color in Figure 9.

Figure 7 Sketch of the first kind of abnormally alluvial canyon C2-2. The figure marks the soft interlayer as well as its surface outcrops.

Finally the influence of the stiffness of the soft interlayer on the ground motion will be discussed. Consider only the first kind of abnormally alluvial three-layered canyon, with the soft interlayer being the second layer. Three such canyons are presented in Table 1, i.e., the canyons indexed by C3-1, C3-2, and C3-3, with the same geometry. The soft interlayer in C3-1 is the softest, while that in C3-3 is the hardest, and that in C3-2 is the middle, as indicated by the mechanical parameters given in Table 1. The variations of displacement amplitudes for these three canyons excited by the vertically incident P waves, i.e., $\gamma_p=0^\circ$, are shown in Figure 10, where the incident frequency $\eta=0.5, 2.0$. While the corresponding results for the inclined incidence with $\gamma_p=45^\circ$ are shown in Figure 11. It can be seen that the stiffness of the soft interlayer in the abnormally alluvial canyon has distinct influence on the ground motion. For the low frequency,

Figure 8 Distribution of displacement amplitude in x -direction (a) and y -direction (b) for the vertical incidence ($\gamma_p=0^\circ$). In the figure, the x -axis denotes the surface location, the y -axis represents the incident frequency η .

Figure 9 Distribution of displacement amplitude in x -direction (a) and y -direction (b) for the inclined incidence ($\gamma_p=45^\circ$). In the figure, the x -axis denotes the surface location, the y -axis represents the incident frequency η .

$\eta=0.5$, in the region of $-1 < x/a_4 < 1$, the amplitude of C3-1 is the highest, that of C3-3 the smallest, and that of C3-2 the middle. While for the high frequency, $\eta=2.0$, in the region of $-0.5 < x/a_4 < 0.5$, the highest amplitude occurs in the canyon C3-3, the lowest in C3-1, and the middle in C3-2. While in the soft outcrop region, i.e.,

$-0.88 < x/a_4 < -0.76$ or $0.76 < x/a_4 < 0.88$, for $\eta=2.0$ and vertical incidence, the displacement amplitude of C3-1 is the highest, followed successively by that of C3-2 and C3-3. Such observation is valid only in the right soft outcrop region of $0.76 < x/a_4 < 0.88$ for the inclined incidence, as shown in Figure 11.

Figure 10 Influence of the stiffness of soft interlayer in the canyon on ground motion in x -direction (a) and y -direction (b) for the vertical incident P wave with $\gamma_P=0^\circ$.

6 Conclusions

In this paper, by expanding the potential functions of the scattering waves in the form of Fourier-Bessel series, the analytical solution for the two-dimension scattering problem of plane P wave by the canyon topography containing arbitrary number of circular-arc-shaped layers is deduced. To obtain the final displacement or stress field, the infinite series solution need to be truncated into finite one, therefore the convergence of solution with the truncation number of terms is an important issue related to the proposed analytical method. The discussion for the convergence of the series solution shows that the proposed solution can converge even for the incident waves with very high frequencies.

In addition, the proposed solution is used to analyze the influence of the layering of alluvial media in the canyon on the ground motion. It has been shown that the layering properties of alluvial media in the canyon, such as the number and the sequence of the alluvial layers, and the stiffness of the soft interlayer in the abnormally alluvial canyon, significantly affect the properties of the ground motion. And the soft interlayer in the canyon can absorb or trap much energy of scattering waves, resulting in the high displacement amplitude in the outcrop region of such soft layer.

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References

Abramowitz M and Stegun I A (1972). *Handbook of Mathematical Functions*,

Figure 11 Influence of the stiffness of soft interlayer in the canyon on ground motion in x -direction (a) and y -direction (b) for the inclined incident P wave with $\gamma_P=45^\circ$.

with Formulas, Graphs, and Mathematical Tables. Dover Publication, New York, 355–389.

- Bakır B S, Özkan M Y and Cılız S (2002). Effects of basin edge on the distribution of damage in 1995 Dinar Turkey earthquake. *Soil Dynam Earthq Eng* **22**: 335–345.
- Boore D M, Lerner K H and Aki K (1971). Comparison of two independent methods for the solution of wave-scattering problems: response of a sedimentary basin to vertically incident SH waves. *J Geophys Res* **76**: 558–569.
- Cao H and Lee V W (1989). Scattering of plane SH waves by circular cylindrical canyons with variable depth-to-width ratio. *European J Earthq Eng* **2**: 29–37.
- Cao H and Lee V W (1990). Scattering and diffraction of plane P waves by circular cylindrical canyons with variable depth-to-width ratio. *Soil Dynam Earthq Eng* **9**: 141–150.
- Harmsen S C and Harding S T (1981). Surface motion over sedimentary valley for incident plane P and SV waves. *Bull Seism Soc Amer* **71**: 655–670.
- Li W H and Zhao C G (2005). Scattering of plane SV waves by cylindrical canyons in saturated porous medium. *Int J Soil Dynam* **25**: 981–995.
- Liang J W, Yan L J and Lee V W (2002). Scattering of incident plane P waves by a circular-arc canyon with a covering layer. *Acta Mechanica Solida Sinica* **23**(4): 397–411 (in Chinese with English abstract).
- Liang J W, Zhang Y S, Gu X L and Lee V W (2003). Scattering of plane SH waves by a circular-arc layered canyon. *Journal of Vibration Engineering* **16**(2): 158–165 (in Chinese with English abstract).
- Sánchez-Sesma F J and Luzón F (1995). Seismic response of three-dimensional alluvial valleys for incident P, S and Rayleigh waves. *Bull Seism Soc Amer* **85**: 269–284.
- Sánchez-Sesma F J, Ramos-Martínez J and Campillo M (1993). An indirect boundary element method applied to simulated the seismic response of alluvial valleys for incident P, S and Rayleigh waves. *Earthq Eng Struct Dynam* **22**: 279–295.
- Trifunac M D (1973). Scattering of plane SH waves by a semi-cylindrical canyon. *Earthq Eng Struct Dynam* **1**: 267–281.
- Wong H L and Trifunac M D (1974). Scattering of plane SH-waves by a semi-elliptical canyon. *Earthq Eng Struct Dynam* **3**: 157–169.
- Yuan X and Liao Z (1994). Scattering of plane SH waves by a cylindrical canyon of circular-arc cross-section. *Int J Soil Dynam* **13**: 407–412.
- Zhang Y S (2008). Analytical solution for the stationary response of alluvial valleys containing multiple circular-arc layers to incident plane P waves. *Chinese J Geophys* **51**(3): 869–880 (in Chinese with English abstract).