Review on localized boundary integral equation: Discrete wavenumber method for 2D irregular layers*

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Abstract The pioneer study of simulating the wave field in media with irregular interface belongs to Aki and Larnar. Since that many numerical methods on the subject have been developed, such as pure numerical techniques, ray method and boundary method. The boundary method based on boundary integral equation is a semi-analytical method which is suitable to modeling wave field induced by irregular border. According to the property of the applied Green's function the boundary methods can be sorted into space domain boundary method and wavenumber domain boundary method. For both of them it is necessary to solve a large equation, which means much computation is needed. Thus, it is difficult for the boundary methods to be applied in simulating wave field with high frequency or in large range. To develop a new method with less computation is meaningful. For this purpose, localized boundary integral equation, i.e., discrete wavenumber method is proposed. It is rooted in the Bouchon-Campillo method, an important wavenumber domain boundary method. Firstly the force on interface is separated into two parts: one is on flat part and the other on irregular part of the interface. Then Fourier transform is applied to identify their relation, the unknown distributes only on irregular part. Consequently computation efficiency is dramatically improved. Importantly its accuracy is the same as that of Bouchon-Campillo.

Key words: Bouchon-Campillo; irregular interface topography; Fourier transform; *lobIE-DWM*

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1 Introduction

Simulating the wave field in media with irregular interface is to an important issue in seismology. For this issue, pioneering work should be attributed to Aki and Larnar (1970). They proposed a fast method with an assumption of Rayleigh Anta. But it only fits to simulating wave field due to slow dip interface (Cao et al, 2004). This method was also extended to calculate the irregular topography of the P-SV scattering wave field (Bouchon, 1973, 1976, 1979; Bouchon and Aki, 1977; Bard and Bouchon, 1980a, b, 1985) and irregular interface scattering two-dimensional P-SV wave field (Kohketsu, 1987; Kohketsu et al, 1991; Takenaka et al, 1996a), as well as the 3D wave field scattered irregular interface (Horike et al, 1990; Uebayashi et al, 1992). Using wave function expansion method, Trifunac (1971,

1973) gave the two-dimensional semi-circular canyon SH wave field analytical solution, which became an important theoretical test tool for a variety of SH-wave field numerical simulation methods. Wave function expansion method can also present the two-dimensional P-SV scattering wave field analytical solution with the arc, semi-circular or circular interface (Lee and Karl, 1992, 1993; Moeen-Vaziri and Trifunac, 1988, Liang et al, 2001).

Apart from the above approaches, other two categories of method, pure numerical simulation method (such as finite element, finite difference, etc.) and boundary integral equation method, were also important. Pure numerical simulation method including finite element (Smith, 1975; Koketsu et al, 2004), finite difference (Boore, 1970, 1972; Boore et al, 1971; Drake, 1972; Fuyuki and Matsumoto, 1980; IDlan, 1977; Graves, 1996; Levander, 1998; Moczo, 1989; Coutant et al, 1995), pseudo spectral method (Kolsloff and Baysal, 1982, Kolsloff et al, 1984; Furumura et al, 1998; Wang et al, 2001), spectral element method (Komatitsch, 2000;

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Komatitsch and Tromp, 2002; Komatitsch and Vilotte, 1998) etc. They are applicable to all kinds of complicated geological structure of the wave field, but for a limited computational domain, wave field on its borders must be specially handled, such as adding a variety of absorbing boundary conditions, otherwise, their results would be affected by the artificial border.

The boundary method, which was applied to calculating the irregular topography of the SH-wave scattering problems by Wong and Jennings (1975) for the first time based on Banaugh and Goldsmith (1963) formula, has been greatly developed. Green's function is the nuclear of the boundary integral equation, which can be represented in space domain or wave number domain. Spatial domain form is complex (Aki and Richards, 1980) and has singularity, while the wave number domain form is simple and has no singularity. Thus it is widely used in seismic exploration, micro-earthquakes, source inversion, fault rupture, stress calculation and seismic wave propagation simulation in 2D/3D media. Based on different Green's function the boundary method can be sorted into spatial method (Sanchez-Sesma and Campillo, 1991, 1993; Sanchez-Sesma and Luzon, 1995; Gil-Zepeda et al, 2002) and wave number method (Bouchon, 1979, 1985, 1989, 1996a, b; Campillo and Bouchon, 1985; Gaffet and Bouchon, 1989; Kawase, 1988; Kawase and Aki, 1989, 1990; Chen, 1990, 1995, 1999; Cao, 2003; Cao et al, 2004; Takenaka, 1996b).

This boundary method usually involves solving a large full vector of linear equations, so it needs a large quantity of calculation. How to reduce the calculation of volume is the key to make it more practical method in use. Kawase (1988), Kawase and Aki (1989) (also Gaffet and Bouchon, 1991) used wave number domain half-space Green's function to limit the displacement and stress within the undulating surface, thus significantly reducing the number of unknowns. Unfortunately, for the uplift of the terrain, half-space Green's function is non-existent. This method must be plus a virtual interface within the uplift region which makes the problem to be a complex layer problem. Furthermore the half-space Green's function only fit to half-space problem. Campillo (1987) used layered Green's function to simulate underground drifts scattered wave field, and the layered media Green's function limited the unknown quantity to irregular at the interface. But there is no simple mathematical expression of layered Green's

function, which needs special calculation, so that efficiency is not particularly evident. Bouchon (1996a, b) made the coefficient matrix to bandwidth by setting the unknown coefficient variable to zero which is less than a threshold value. However, this treatment reduces the calculation accuracy (Yokio and Sanchez-Sesma, 1998). We must be careful to apply it. On the basis of Bouchon-Campillo discrete wave number method, we proposed a different new approach, localized boundary integral equation, i.e., discrete wave number method (*loBIE-DWM*), while retaining the high precision under the premise of improving computational efficiency.

2 Thought of *loBIE-DWM*

The discrete wave number domain boundary method, no matter how specific operations, must include the handling of the cyclical and imaginary frequency, which are the two most basic elements of the generalized discrete wave number method.

2.1 Introduction of the calculation of cycle length L

Figure 1 shows a medium with a line power source. Displacement and stress of excitation source in the medium can be expressed as follows:

$$\mathbf{G}(x, z, \omega) = e^{i\omega t} \int_{-\infty}^{+\infty} f(k, z) e^{-ikx} dk, \quad (1)$$

where k is wave number, $x=[-\infty, +\infty]$.

Figure 1 Original problem with line force source.

Figure 2 introduces the medium and source of the periodic distribution. Let L the cycle length. One advantage of this approach is avoiding a singularity of $f(k, z)$. The second is to facilitate the numerical computation. When L is long enough (Figure 2), the difference between the wave field due to cycle media and that due to the source with the original problem (Figure 1) can be ignored. It is clear from physical mechanism view that the greater L is, the less the difference between them.

Introduction of the source of periodic distribution makes equation (1) become

$$\mathbf{G}(x, z, \omega) = e^{i\omega t} \int_{-\infty}^{+\infty} f_1(k, z) e^{-ikx} \sum_{m=-\infty}^{m=+\infty} e^{ikmL} dk. \quad (2)$$

Further simplified, truncate the discrete wavenum-

ber k , the above equation become

$$G(x, z, \omega) = \frac{2\pi}{L} e^{i\omega t} \sum_{n=-N}^{n=N} f_1(k_n, z) e^{-ik_n x}, \quad (3)$$

where $k_n = \frac{2\pi}{L} n$

Figure 2 Change of the problem in Figure 1.

2.2 Introduction of imaginary frequency ω_I

The introduction of cycle length L makes the original problem with a single source become a problem with numbers of sources. If one can calculate the value of $G(x, z, \omega)$ with continuous frequency and use a continuous Fourier inverse transform to get non-periodic time response, based on the time difference between different sources, adding time window can separate out a single source of the wave field. But we can only have $G(x, z, \omega)$ on the discrete frequency $\omega = 2\pi/T$ (T is time window), and get solution in time domain through the discrete Fourier transform, imaginary frequency must be introduced to eliminate the impact of source in adjacent areas, namely:

$$g(t, x, z) = \sum_{\omega_R = -M\omega}^{M\omega} G(x, z, \omega) e^{i\omega_R t} e^{\omega_I t} d\omega_R, \quad (4)$$

ω_I is a constant meeting

$$e^{\omega_I T} \ll 1,$$

and can make the signal outside the time window T decay. So, L can be selected long enough to ensure the time response within the time window T being not subjected to interference from adjacent area sources.

Usually

$$\omega_I = \left[-\frac{\pi}{T}, -\frac{2\pi}{T} \right].$$

It makes the signal attenuating in $e^{\omega_I T}$ with time within the time windows.

2.3 Theory of BIE-DWM

For topographical and multiple-layer problems the theory of loBIE-DWM is similar, though with different formulae. For simplicity here we only demonstrate its

formula for the topography problem subjected to incident P-SV waves following the procedure in Zhou and Chen (2008a, b). Other formulae can be seen in Zhou and Chen (2006a, 2009a).

2.3.1 Formulation of BIE-DWM for P-SV topographic problem

The problem considered here is the seismic wave scattering due to an irregular topography subjected to incident P-SV waves in a homogeneous isotropic medium (Figure 3). In this problem, the total seismic wave field can be written as

$$\begin{cases} \mathbf{u}(x, z) = \mathbf{u}_{in}(x, z) + \mathbf{u}_{sc}(x, z) \\ \mathbf{t}(x, z) = \mathbf{t}_{in}(x, z) + \mathbf{t}_{sc}(x, z) \end{cases} \quad (5)$$

where $\mathbf{u}$ denotes the displacement in frequency domain, and the subscripts ‘in’ and ‘sc’ denote the incident and scattered wave fields, respectively. According to Huygens’ Principle, the scattered waves can be represented by a Kirchhoff integral over the diffracting surface (see Bouchon, 1985, 1989; Campillo and Bouchon, 1985):

$$\begin{cases} \mathbf{u}_{sc}(x, z) = \int \mathbf{G}(x - x', z - z') \mathbf{q}(x', z') ds \\ \mathbf{t}_{sc}(x, z) = \int \mathbf{H}(x - x', z - z') \mathbf{q}(x', z') ds \end{cases} \quad (6)$$

where $\mathbf{G}$ is the tensor of Green’s function for P-SV waves in a 2D elastic full space, $\mathbf{q}(x')$ is the distribution of the unknown force, and the integration domain S is the irregular topographic surface. It is noted that the Green’s tensor and force distribution are also the function of ω , though it is not explicitly indicated here for compactness in mathematical presentation.

Figure 3 Ordinary topography model.

To solve this topographic scattering problem efficiently, we adopt the discrete wave number method (abbreviated as DWN) (see, e.g., Bouchon and Aki, 1977). With this method, the scattered wave field at an arbitrary discrete horizontal grid is reduced to the following finite summation over all the discrete Huygens source points (Bouchon, 1985, 1989; Compillo and Bouchon, 1985):

$$\begin{cases} u_{sc}(x_n, z, \omega) = \frac{L}{(2N+1)} \cdot \sum_{m=-N}^N G^{DWN}[x_n - x_m, z - \zeta(x_m)] \cdot \mathbf{q}(x_m) \\ t_{sc}(x_n, z, \omega) = \frac{L}{(2N+1)} \cdot \sum_{m=-N}^N H^{DWN}[x_n - x_m, z - \zeta(x_m)] \cdot \mathbf{q}(x_m), \end{cases} \quad (7)$$

where $\mathbf{G}^{DWN}$ is the discrete wave number representation of Green's tensor $\mathbf{G}$, $\{x_m, \zeta(x_m); m=0, \pm 1, \pm 2, \dots, \pm N\}$ are the locations of the Huygens point source along the irregular topographic surface, and $\mathbf{q}(x_m)$ denotes the strength of the m -th Huygens point source at the surface. L is the length of periodicity of the source-medium configuration. $(2N+1)$ is the total grids with an equal horizontal interval within one periodicity, thus $x_m = m\Delta x$, $m=0, \pm 1, \pm 2, \dots, \pm N$ and $\Delta x = L/(2N+1)$. Moreover, at the horizontal grids, Fourier transform of $\mathbf{G}^{DWN}$ and its explicit expression is given in Appendix A.

The irregular surface is a free surface on which traction should vanish, i.e.,

$$t_{in}(x_n, z_n) + \sum_{m=-N}^N \{H^{DWN}[x_n - x_m, z_n - \zeta(x_m)]\} \cdot \mathbf{q}(x_m) = 0, \quad (8)$$

where $z_n = \zeta(x_n)$ and $n=0, \pm 1, \pm 2, \dots, \pm N$. t_{in} and H^{DWN} are the tractions associated with u_{in} and G^{DWN} , respectively. Noticing that equation (8) provides exactly the number of linear equations as that of the unknowns $\mathbf{q}(x_m)$, $m=0, \pm 1, \pm 2, \dots, \pm N$, thus they could be directly applied to solving the unknowns after a minor modification on the collocation-elements of the coefficient matrix (Bouchon, 1985, 1989; Bouchon and Compillo, 1985). This is the basic idea of the BIE-DWM. Its validity and the applicability to practical problems have been demonstrated in a number of studies (Bouchon, 1985, 1989, 1996a, 1996b; Bouchon and Coutant, 1994; Campillo and Bouchon, 1985). It is obvious, however, that the computation cost of BIE-DWM directly depends on the total sampling number within one spatial periodicity, i.e., $x \in [-L/2, L/2]$. Because, on the one hand, the dimension of the coefficient matrix involved in this linear equation system is proportional to the total sampling number, while on the other hand, a proper estimation on the high frequency problem requires dense spatial sampling. Consequently, the dimension of the matrix is increased and so does the computation cost for high frequency problems.

2.3.2 Formulation of the localized BIE-DWM for P-SV topographic problem

Similar to the SH topographic problem (Zhou and Chen, 2006a), the first step for such an extension is an orthogonal decomposition of the unknown source strength vector $\mathbf{q}$, i.e.,

$$\mathbf{q} = \mathbf{q}_f \oplus \mathbf{q}_c, \quad (9)$$

where, as shown in Figure 4, $\mathbf{q}_c$ denotes the unknown source strengths of those sampling points within the corrugated part of the surface, while $\mathbf{q}_f$ denotes the unknowns of the flat part of the surface.

Figure 4 Cartoon of the orthogonal decomposition of the unknown source strength vector $\mathbf{q}$. Solid dots and open dots denote the unknown force source on the flat surface and those on the irregular surface, respectively.

Substituting equation (9) into equation (8) yields

$$\sum_{m=-N}^N H^{DWN}(x_n - x_m, 0) \mathbf{q}_f(x_m) + \sum_{m=N_a}^{N_b} H^{DWN}[x_n - x_m, 0 - \zeta(x_m)] \mathbf{q}_c(x_m) + t_{in}(x_n, 0) = 0, \quad (10)$$

where N_a and N_b are the sampling point number corresponding to the irregular part of the topography in Figure 4. Note that equation (10) holds only for $x_n \in \Omega_f$. For the range of $x_n \in \Omega_c$ (in Figure 4), equation (10) does not hold any more, namely, the right-hand-side of equation (10) is equal to a non-zero quantity whose value is unknown yet at this stage. We denote this unknown quantity with $t_{un}(x_n)$, and it obviously has the property

$$t_{un}(x_n) = \begin{cases} \text{non-zero quantity,} & x_n \in \Omega_c \\ 0, & x_n \in \Omega_f. \end{cases} \quad (11)$$

With $t_{un}(x_n)$, equation (10) can be extended to the whole assembly of horizontal sampling within one spatial periodicity, i.e.,

$$\begin{aligned} & \sum_{m=-N}^N H^{DWN}(x_n - x_m, 0) \mathbf{q}_f(x_m) + \\ & \sum_{m=N_a}^{N_b} H^{DWN}(x_n - x_m, 0 - z_m) \mathbf{q}_c(x_m) = \\ & -t_{in}(x_n, 0) + t_{un}(x_n), \end{aligned} \quad (12)$$

for $x_n \in [-L/2, L/2]$.

Then equation (12) can be executed by the discrete Fourier transform (DFT) with respect to $x_n \in [-L/2, L/2]$, and becomes

$$\begin{aligned} & \tilde{\mathbf{H}}(k_p, 0) \tilde{\mathbf{q}}_f(k_p) + \\ & \sum_{m=N_a}^{N_b} \tilde{\mathbf{H}}(k_p, -z_m) \mathbf{q}_c(x_m) \exp(ik_p x_m) = \\ & -\tilde{t}_{in}(k_p) + \frac{1}{2N+1} \sum_{m=N_a}^{N_b} t_{un}(x_m) \exp(ik_p x_m), \end{aligned} \quad (13)$$

where, $\tilde{\mathbf{H}}(k_p, z)$ is a DFT of $\{\tilde{\mathbf{H}}(x_n, z)\}$ and $\{\tilde{\mathbf{H}}(x_n, z)\}$ is given in Appendix A, and

$$\begin{aligned} \tilde{\mathbf{q}}_f(k_p) &= \frac{1}{2N+1} \sum_{m=-N}^N \mathbf{q}_f(x_m) \exp(ik_p x_m), \\ \tilde{t}_{in}(k_p) &= \frac{1}{2N+1} \sum_{m=-N}^N t_{in}(x_m, 0) \exp(ik_p x_m), \end{aligned}$$

with $k_p = 2\pi p/L$ for $p=0, \pm 1, \pm 2, \dots, \pm N$.

Equation (13) can be solved as

$$\begin{aligned} \tilde{\mathbf{q}}_f(k_p) &= -[\tilde{\mathbf{H}}(k_p, 0)]^{-1} \cdot \\ & \left\{ \sum_{m=N_a}^{N_b} \left[\tilde{\mathbf{H}}(k_p, -z_m) \mathbf{q}_c(x_m) - \frac{t_{un}(x_m)}{2N+1} \right] \right. \\ & \left. \exp(ik_p x_m) + \tilde{t}_{in}(k_p, 0) \right\}. \end{aligned} \quad (14)$$

Applying inverse DFT to equation (14) and exchanging summation order yields

$$\begin{aligned} \mathbf{q}_f(x_l) &= \\ - \sum_{n=N_a}^{N_b} & \left\{ \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \tilde{\mathbf{H}}(k_p, -z_n) \exp[ik_p(x_n - x_l)] \right\} \cdot \\ \mathbf{q}_c(x_n) & - \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \tilde{t}_{in}^0(k_p) \exp(-ik_p x_l) + \\ \sum_{m=N_a}^{N_b} & \left\{ \frac{1}{2N+1} \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \exp[-ik_p(x_l - x_m)] \right\} \cdot \\ t_{un}(x_m) & \end{aligned} \quad (15a)$$

for $x_l \in \Omega_f$; and

$$\begin{aligned} 0 &= - \sum_{n=N_a}^{N_b} \left\{ \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \tilde{\mathbf{H}}(k_p, -z_n) \exp[ik_p(x_n - x_l)] \right\} \cdot \\ \mathbf{q}_c(x_n) & - \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \tilde{t}_{in}^0(k_p) \exp(-ik_p x_l) + \end{aligned}$$

$$\begin{aligned} & \sum_{m=N_a}^{N_b} \left\{ \frac{1}{2N+1} \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \exp[-ik_p(x_l - x_m)] \right\} \cdot \\ & t_{un}(x_m) \end{aligned} \quad (15b)$$

for $x_l \in \Omega_c$. Here, we denote $\tilde{\mathbf{H}}_0(k_p) = \tilde{\mathbf{H}}(k_p, 0)$ and $\tilde{t}_{in}^0(k_p) = \tilde{t}_{in}(k_p, 0)$ for simplicity. In compact form, equations (15a) and (15b) are expressed as

$$\underline{\mathbf{Q}}_f + \underline{\mathbf{A}}_f \underline{\mathbf{Q}}_c = -\underline{\mathbf{S}}_f + \underline{\mathbf{B}}_f \underline{\mathbf{T}}_{un}, \quad (16a)$$

$$\underline{\mathbf{A}}_c \underline{\mathbf{Q}}_c = -\underline{\mathbf{S}}_c + \underline{\mathbf{B}}_c \underline{\mathbf{T}}_{un}. \quad (16b)$$

where $\underline{\mathbf{A}}_f$, $\underline{\mathbf{A}}_c$, $\underline{\mathbf{B}}_f$, $\underline{\mathbf{S}}_f$ and $\underline{\mathbf{S}}_c$ are the sub-blocks of matrices. $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$, and vector $\underline{\mathbf{S}}$, are defined as:

$$\begin{aligned} \underline{\mathbf{A}}_f &= \begin{pmatrix} \underline{\mathbf{A}}_1 \\ \underline{\mathbf{A}}_3 \end{pmatrix}, \quad \underline{\mathbf{A}}_c = \underline{\mathbf{A}}_2, \quad \underline{\mathbf{B}}_f = \begin{pmatrix} \underline{\mathbf{B}}_1 \\ \underline{\mathbf{B}}_3 \end{pmatrix}, \\ \underline{\mathbf{B}}_c &= \underline{\mathbf{B}}_2, \quad \underline{\mathbf{S}}_f = \begin{pmatrix} \underline{\mathbf{S}}_1 \\ \underline{\mathbf{S}}_3 \end{pmatrix}, \quad \underline{\mathbf{S}}_c = \underline{\mathbf{S}}_2; \end{aligned} \quad (17)$$

and

$$\underline{\mathbf{A}} = \begin{pmatrix} \underline{\mathbf{A}}_1 \\ \underline{\mathbf{A}}_2 \\ \underline{\mathbf{A}}_3 \end{pmatrix}, \quad \underline{\mathbf{B}} = \begin{pmatrix} \underline{\mathbf{B}}_1 \\ \underline{\mathbf{B}}_2 \\ \underline{\mathbf{B}}_3 \end{pmatrix} \quad \text{and} \quad \underline{\mathbf{S}} = \begin{pmatrix} \underline{\mathbf{S}}_1 \\ \underline{\mathbf{S}}_2 \\ \underline{\mathbf{S}}_3 \end{pmatrix}. \quad (18)$$

The dimensions of $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ and vector $\underline{\mathbf{S}}$, are “ $2(2N+1) \times 2(2N+1)$ ” and “ $2(2N+1) \times 1$ ”, and their block elements are defined by

$$\begin{aligned} (\underline{\mathbf{A}})_{mn} &= \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \\ & \tilde{\mathbf{H}}(k_p, -z_n) \exp[ik_p(x_n - x_m)], \end{aligned} \quad (19a)$$

$$(\underline{\mathbf{B}})_{mn} = \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \exp[ik_p(x_n - x_m)], \quad (19b)$$

$$(\underline{\mathbf{S}})_m = \sum_{p=-N}^N [\tilde{\mathbf{H}}_0(k_p)]^{-1} \tilde{t}_{in}^0(k_p) \exp(-ik_p x_m), \quad (19c)$$

where $n, m=0, \pm 1, \pm 2, \dots, \pm N$. The unknowns in equations (16a) and (16b) are

$$\underline{\mathbf{T}}_{un} = \begin{bmatrix} t_{un}(x_{N_a}) \\ t_{un}(x_{N_a+1}) \\ \vdots \\ t_{un}(x_{N_b}) \end{bmatrix}, \quad \underline{\mathbf{Q}}_c = \begin{bmatrix} q_c(x_{N_a}) \\ q_c(x_{N_a+1}) \\ \vdots \\ q_c(x_{N_b}) \end{bmatrix}, \quad \underline{\mathbf{Q}}_f = \begin{bmatrix} \underline{\mathbf{Q}}_{f,1} \\ \underline{\mathbf{Q}}_{f,2} \end{bmatrix},$$

$$\underline{\underline{Q}}_{f,1} = \begin{bmatrix} \mathbf{q}_f(x_{-N}) \\ \mathbf{q}_f(x_{1-N}) \\ \vdots \\ \mathbf{q}_f(x_{N_a-1}) \end{bmatrix} \text{ and } \underline{\underline{Q}}_{f,2} = \begin{bmatrix} \mathbf{q}_f(x_{N_b+1}) \\ \mathbf{q}_f(x_{N_b+2}) \\ \vdots \\ \mathbf{q}_f(x_N) \end{bmatrix}. \quad (20)$$

Combining equations (16a) and (16b) and eliminating the unknown vector $\underline{\underline{T}}_{in}$, we obtain a linear relationship between $\underline{\underline{Q}}_f$ and $\underline{\underline{Q}}_c$ as follows,

$$\underline{\underline{Q}}_f = -\underline{\underline{S}}_f + \underline{\underline{B}}_f(\underline{\underline{B}}_c)^{-1}\underline{\underline{S}}_c + \{\underline{\underline{B}}_f(\underline{\underline{B}}_c)^{-1}\underline{\underline{A}}_c - \underline{\underline{A}}_f\}\underline{\underline{Q}}_c. \quad (21)$$

To determine $\underline{\underline{Q}}_c$, we recast equation (8) only on the corrugated part of the surface and obtain

$$\underline{\underline{E}}_f \underline{\underline{Q}}_f + \underline{\underline{E}}_c \underline{\underline{Q}}_c = -\underline{\underline{T}}_{in}, \quad (22)$$

where

$$(\underline{\underline{E}}_f)_{nm} = \mathbf{H}^{DWN}[x_n - x_m, \zeta(x_n)],$$

for $x_n \in \Omega_c$ and $x_m \in \Omega_f$;

$$(\underline{\underline{E}}_c)_{nm} = \mathbf{H}^{DWN}[x_n - x_m, \zeta(x_n) - \zeta(x_m)],$$

for $x_n, x_m \in \Omega_c$;

$$(\underline{\underline{T}}_{in})_n = t_{in}[x_n, \zeta(x_n)],$$

for $x_n \in \Omega_c$.

Substituting equation (21) into (22) yields a linear equation system for determining the unknown $\underline{\underline{Q}}_c$ as follows,

$$\{\underline{\underline{E}}_f[\underline{\underline{B}}_f(\underline{\underline{B}}_c)^{-1}\underline{\underline{A}}_c - \underline{\underline{A}}_f] + \underline{\underline{E}}_c\}\underline{\underline{Q}}_c = -\underline{\underline{T}}_{in} + \underline{\underline{E}}_f[\underline{\underline{S}}_f - \underline{\underline{B}}_f(\underline{\underline{B}}_c)^{-1}\underline{\underline{S}}_c]. \quad (23)$$

With this linear equation $\underline{\underline{Q}}_c$ can be determined, thus

$\underline{\underline{Q}}_f$ can be consequently determined via equation (21).

Finally, substituting the solved $\underline{\underline{Q}}_c$ and $\underline{\underline{Q}}_f$ into equation (6), we can calculate the P-SV wave field scattered by the irregular surface.

Compared with the BIE-DWM proposed by Bouchon and Campillo (1985, 1989), the *lo*BIE-DWM for topographic problem of P-SV waves proposed here is more efficient in terms of computation. This is because, according to equation (23), the dimension of the matrices to be inverted in this method is proportional to the dimension of Ω_c which is only a fraction of the di-

mension of Ω_L , while the dimension of the matrix to be inverted in BIE-DWM is proportional to the dimension of Ω_L . Obviously, the more localized the corrugated part of the ground surface is, the more efficiently this new method performs.

The above description does not limit Ω_c to a continuum region. This means the formulae are suitable to the topography with several separated irregular regions. In that case $\underline{\underline{Q}}_c$ is composed of the total forces distrib-

uted on all irregular regions, while $\underline{\underline{Q}}_f$ is composed of the total forces on all flat regions. For multiple-layer problem, the above theory is also suitable with different formulae (Zhou and Chen, 2009a, b).

2.3.3 Validations of efficiency and accuracy

In terms of accuracy, the results of the *lo*BIE-DWM should be identical with those of BIE-DWM. Figure 5 displays our results (dot line) for the semi-cylindrical canyon's (with radius a) scattering problem from vertical plane P and SV, showing identical results with those of BIE-DWM (solid line). The computational parameters used in these comparisons are: $L=20a$, $\Delta x=0.2a$ and $\eta=1$. Further, the *lo*BIE-DWM performs more efficiently. Figure 6 shows the comparison of the computational times consumed by the two methods for this problem for taking different ratios of L/a in the computation, where the $\Delta x/a$ is fixed as 0.05 for all cases and the dimensionless frequency η is set to be 1. The computed results shown here were executed on a PC/P4 with 2.8 GHz clock speed. It is obvious that the *lo*BIE-DWM becomes more efficient than the BIE-DWM as the ratio of L/a increases, e.g., it is about 1 000 times faster when $L/a=40$.

3 Conclusions

Bouchon-Campillo discrete wave number method (BC) avoids the singularity of Green's function in space domain, has high calculation precision, and can simulate the wave field of a number of geological formations. But it needs much calculation. For high-frequent or large-scale problems it is not competent. To find an alternative method with less calculation is deserved. To reduce the computation, we proposed an improved method (*lo*BIE-DWM), of which the basic idea is to divide force source on the interface into two separate sections, use Fourier transform to decide their mathematical relationship, and then compose a linear equation only about force source on irregular part of interface.

Figure 5 Comparison between our results and BC due to vertical plane P and SV. The dot line is our results and the solid line is BC's results. (a) and (c) are respectively the vertical component and horizontal component due to incident P plane wave; (b) and (d) are the vertical component and horizontal component due to incident SV plane wave.

Figure 6 Ratio of CPU time between BC and our again L/a .

So it greatly reduces the computation and also can solve more problems. Its efficiency and valuation have been demonstrated in our studies (Zhou and Chen, 2006a, b, 2007, 2008 a, 2008 b, 2009a, b). With this method we can not only calculate the topography of the problem, but also solve the irregular interface issues. For the topography SH wave field calculations, the computation is only 1/10 of that of BC, while for the P-SV wave field the computation is only 1/100 of that of BC. For the layered problem the advantage is clear. It is expected that the *lo*BIE–DWM's advantage will be more remarkable with the progressing research.

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Appendix A

Green's function for displacement

$$\mathbf{G} = \begin{bmatrix} G_{xx} & G_{xz} \\ G_{zx} & G_{zz} \end{bmatrix}$$

where

$$\begin{aligned} G_{xx} &= -\frac{i}{2L\mu} \sum_{n=-N}^N \left(\frac{k_n^2}{\gamma_1} e^{-i\gamma_1|z-z_c|} + \gamma_2 e^{-i\gamma_2|z-z_c|} \right) e^{-ik_n(x-x_c)} \\ G_{xz} &= -\frac{i \operatorname{sgn}(z-z_c)}{2L\mu} \sum_{n=-N}^N k_n (e^{-i\gamma_1|z-z_c|} - e^{-i\gamma_2|z-z_c|}) e^{-ik_n(x-x_c)}; \\ G_{zx} &= G_{xz} \\ G_{zz} &= -\frac{i}{2L\mu} \sum_{n=-N}^N \left(\gamma_1 e^{-i\gamma_1|z-z_c|} + \frac{k_n^2}{\gamma_2} e^{-i\gamma_2|z-z_c|} \right) e^{-ik_n(x-x_c)} \end{aligned}$$

Appendix B

Green's function for stress

$$\mathbf{H} = \begin{bmatrix} H_{xx} & H_{xz} \\ H_{zx} & H_{zz} \end{bmatrix}$$

where

$$\begin{aligned} H_{xx} &= \frac{1}{2Lk_\beta^2} \sum_{n=-N}^N \left\{ \left[\frac{k_n}{\gamma_1} (2\gamma_1^2 - k_\beta^2) e^{-i\gamma_1|z-z_c|} - 2k_n\gamma_2 e^{-i\gamma_2|z-z_c|} \right] n_1 - \operatorname{sgn}(z-z_c) \left[2k_n^2 e^{-i\gamma_1|z-z_c|} + (2\gamma_2^2 - k_\beta^2) e^{-i\gamma_2|z-z_c|} \right] n_2 \right\} e^{-ik_n(x-x_c)} \\ H_{xz} &= \frac{1}{2Lk_\beta^2} \sum_{n=-N}^N \left\{ -\operatorname{sgn}(z-z_c) \left[2k_n^2 e^{-i\gamma_1|z-z_c|} + (2\gamma_2^2 - k_\beta^2) e^{-i\gamma_2|z-z_c|} \right] n_1 + \left[\frac{k_n}{\gamma_1} (2k_n^2 - k_\beta^2) e^{-i\gamma_1|z-z_c|} + 2k_n\gamma_2 e^{-i\gamma_2|z-z_c|} \right] n_2 \right\} e^{-i(x-x_c)} \\ H_{zx} &= \frac{1}{2Lk_\beta^2} \sum_{n=-N}^N \left\{ \operatorname{sgn}(z-z_c) \left[(2\gamma_2^2 - k_\beta^2) e^{-i\gamma_1|z-z_c|} + 2k_n^2 e^{-i\gamma_2|z-z_c|} \right] n_1 + \left[-2k_n\gamma_1 e^{-i\gamma_1|z-z_c|} + \frac{k_n}{\gamma_1} (2\gamma_2^2 - k_n^2) e^{-i\gamma_2|z-z_c|} \right] n_2 \right\} e^{-i(x-x_c)} \\ H_{zz} &= \frac{1}{2Lk_\beta^2} \sum_{n=-N}^N \left\{ \left[-2k_n\gamma_1 e^{-i\gamma_1|z-z_c|} + \frac{k_n}{\gamma_2} (2\gamma_2^2 - k_\beta^2) e^{-i\gamma_2|z-z_c|} \right] n_1 + \operatorname{sgn}(z-z_c) \left[(2k_n^2 - k_\beta^2) e^{-i\gamma_1|z-z_c|} - 2k_n^2 e^{-i\gamma_2|z-z_c|} \right] n_2 \right\} e^{-ik_n(x-x_c)} \end{aligned}$$

with $\gamma_2 = \sqrt{\frac{\omega^2}{\beta^2} - k_n^2}$, $\operatorname{Im}(\gamma_2) \leq 0$; $\gamma_1 = \sqrt{\frac{\omega^2}{\alpha^2} - k_n^2}$, $\operatorname{Im}(\gamma_1) \leq 0$; α and β are the velocity of P and S waves, respectively.