Meshing scheme in the computation of spontaneous fault rupture process^{*}

LIU Qi-ming¹⁾ (刘启明) CHEN Xiao-fei^{2),*} (陈晓非)

1) School of Earth and Space Science, Peking University, Beijing 100871, China

2) School of Earth and Space Science, University of Science and Technology of China, Hefei 230026, China

Abstract

The choice of spatial grid size has been being a crucial issue in all kinds of numerical algorithms. By using BIEM (Boundary Integral Equation Method) to calculate the rupture process of a planar fault embedded in an isotropic and homogeneous full space with simple discretization scheme, this paper focuses on what grid size should be applied to control the error as well as maintaining the computing efficiency for different parameter combinations of (D_c, T_c) , where D_c is the critical slip-weakening distance and T_c is the initial stress on the fault plane. We have preliminarily found the way of properly choosing the spatial grid size, which is of great significance in the computation of seismic source rupture process with BIEM.

Key words: seismic source dynamics; finite fault source rupture; boundary integral equation; meshing scheme

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Introduction

Numerical simulations of seismic source dynamics have great significance in understanding both the physical mechanisms and the radiation characteristics of the rupture processes. The numerical methods widely used in this field include the Finite Difference Method (FDM) (Madariaga, 1976; Andrews, 1976), the Finite Element Method (FEM) (Oglesby *et al*, 2000; Aagaard *et al*, 2001) and the Boundary Integral Equation Method (BIEM) (Fukuyama and Madariaga, 1995; Aochi *et al*, 2000). Among these methods, BIEM gets much attention for its efficiency and accuracy (Aochi *et al*, 2000; Zhang and Chen, 2006), and plays an important role in the study of seismic source dynamics.

BIEM is a semi-numerical and semi-analytical method. In principle, the integral equation which controls the changes of stress and slip on the fault plane is firstly obtained based on representation theorem, then discretized into a set of linear equations and finally solved by incorporating with a friction law. Generally speaking, the smaller the grid size is, the more accurate the simulation will be. However, the cost of simulation will increase significantly when the grid size decreases. Thus, tradeoff between efficiency and accuracy needs to be made, which means that a maximum grid size in accordance with a given error level should be found. Assuming that all the

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[✱] **Author for correspondence:** xfchen1@ustc.edu.cn

stresses are measured relatively to the residual stress and a slip-weakening law is applied, the allowed maximum grid size is dependent on basic parameters such as critical slip-weakening distance D_c , shear strength T_u and initial stress T_e . Although the determination of the grid size is important, few studies on this problem have been carried out so far, which limits the applications of BIEM. In this article, we performed twelve selected simulations with various D_c and T_e for a given error level of 2%, and revealed the dependence of maximum grid size on D_c and T_e by comparisons between different simulations.

1 Model configuration

For simplicity, we discuss the dynamic rupture processes on a finite planar fault embedded in an unbounded, homogeneous and isotropic elastic medium. As shown in Figure 1, a 6 km×6 km asperity at the center of a 24 km×24 km square fault is the rupture nucleation zone. A simple box discretization scheme (Aochi *et al*, 2000; Zhang and Chen, 2006) is applied, in which the slip rate between two adjacent time steps in any spatial grid remains a constant. A , B and C are three “observation points” used for evaluating the accuracy of the simulations with different parameters and grid sizes.

By taking partial derivatives of both sides of the displacement representation theorem in frequency domain (Aki and Richards, 2002), we can get the stress representation theorem as follows

$$\tilde{\tau}_{kl}(\mathbf{x}, \omega) = c_{klmn} \iint_{\Sigma} \Delta \tilde{u}_i(\xi, \omega) c_{ijpq} v_j \frac{\partial}{\partial \xi_m} [\tilde{G}_{np,q}(x, \xi, \omega)] dS(\xi) \quad (1)$$

Figure 1 Geometry model for this study and the three observing points

In equation (1), ξ and $\mathbf{x}$ are coordinates of source and field points, respectively. Σ represents the two sides of the fault plane, ω is the circular frequency, $\Delta \tilde{u}_i$ is the dislocation on the fault, $\tilde{G}_{np,q}$ is the spatial derivatives of the Green's function. c_{ijpq} and v_j are the elastic constants of the medium and the unit vector normal to the fault plane, respectively. After discretization of the integral equation, the problem of spontaneous rupture is transformed into a set of linear equations. Incorporated with a friction law, these equations can be solved iteratively step by step. The most widely used friction law is the slip-weakening law (Ida, 1972) shown in Figure 2. An appropriate Courant-Friedrichs-Lewy (CFL) should be applied in the simulation to keep the

computation stable (Madariaga *et al*, 1998).

In order to investigate how different combinations of D_c and T_e affect the choice of the maximum grid size, three typical values of D_c are selected in the simulation: 0.070 m (small), 0.348 m (middle) and 0.695 m (large). For a specific D_c there is a critical value $T_{ec}(D_c)$, only when $T_e > T_{ec}(D_c)$ the rupture can take place in the entire computational domain. Therefore, we will determine the $T_{ec}(D_c)$ through numerical tests first, then with this value as the starting point, we will select three other values of T_e . Other parameters remain fixed in all the simulations, such

as $CFL=0.4$, the shear strength $T_u=10$ MPa, the initial stress within the asperity $T_i=10.01$ MPa, the density $\rho=3.30$ g/cm³, Poisson's ratio $\nu=0.25$, Young's modulus $E=3.45\times 10^{10}$ Pa. We ran 12 simulations with three different D_c and four distinct T_c . For each set of instances, we apply a gradually refined meshing scheme. The computation domain shown in Figure 1 is divided into grids of 24×24 ($\Delta s=1$ km), 40×40 ($\Delta s=0.6$ km), 56×56 ($\Delta s=0.43$ km), 72×72 ($\Delta s=0.33$ km), 88×88 ($\Delta s=0.27$ km), 104×104 ($\Delta s=0.23$ km), $\dots$, $[4(6+4n)]\times [4(6+4n)]$ ($\Delta s=3/[3+2n]$), $\dots$. We define the scheme with grid of $[4(6+4n)]\times [4(6+4n)]$ as the “scheme n ” for short hereafter. These gradually refined schemes can guarantee that the “observation points” A , B and C always stay at the center of the corresponding grid. In other words, the absolute locations of the three selected “observation points” remain exactly the same for all meshing schemes, thus the results obtained in different meshing schemes could be fairly compared.

To quantitatively evaluate the accuracy of the simulation results, we define the error function for the simulation result with scheme n as follows

$$E_{\text{error}}(n) = \frac{t_n - t_{n-1}}{t_{n-1}} \quad (n=1, 2, \dots, 8) \quad (2)$$

Figure 3 Stress as a function of time at observation point C (see Figure 1) in four different meshing schemes with ($D_c=0.348$ m, $\bar{T}_c=0.36$)

The rectangular zone below is zoomed into the subfigure in the right corner, in which t_n ($n=1, 2, 3, 4$) are the characteristic time values in “scheme- n ” defined in the text, respectively

Figure 2 Sketch for the slip-weakening law
 x and y axis denote slip and stress, respectively.
 σ_u is the peak strength, σ_0 is the absolute initial stress, σ_r is the residual stress, and D_c is the critical slip-weakening distance

Here n represents “scheme n ” and t_n is a characteristic value of a real-time variable measured at the observation points. For convenience we here use point C and designate the time when its stress drops to 0 as t_n . Such definition has clear physical meaning and is easy to implement. Moreover, good correlation between this characteristic time and relative errors of several other physical quantities is also found in our simulations. Hence, it is reasonable to use this definition in error analysis. Figure 3 demonstrates the error analysis.

2 Numerical results with different combinations of D_c and T_c

First, we determine the T_{cc} corresponding to three different D_c through numerical tests. For the dimensionless initial shear stress $\bar{T}_c (=T_c/T_u)$, the accuracy of our tests is chosen as 0.02. As shown in Table 1, the dimensionless characteristic value $\bar{T}_{cc}$ is the average of two adjacent values of $\bar{T}_c$ which generate two distinct types of rupture processes, *i.e.*, the “non-growth rupture” and “rupture”. Figures 4a and 4b show the snapshots of the rupture processes when $\bar{T}_c=0.28$ and $\bar{T}_c=0.30$ respectively (with $D_c=0.348$ m). It is obvious that when $\bar{T}_c=0.28$ the rupture process stops shortly after its nucleation, which is called “non-growth rupture”. For $\bar{T}_c=0.30$ the rupture keeps propagating outward and called “rupture”. Although the difference between the two values of $\bar{T}_c$ is only 0.02, the rupture processes are totally different. Figures 5a and 5b show the evolutions of slip and stress at the observation points A , B and C in each case. Comparing the slip and stress curves of point C in Figures 5a and 5b, we can see that with $\bar{T}_c=0.28$, rupture never occurs.

Figure 4a Snapshot of slip rate (non-dimensional) in the rupture propagation with $D_c=0.348$ m and $\bar{T}_c=0.28$
Numbers on subfigures indicate time steps

Figure 4b Snapshot of slip rate (non-dimensional) in the rupture propagation process with $D_c=0.348$ m and $\bar{T}_c=0.30$
Numbers on subfigures indicate time steps

Figure 5a Slip and stress as a function of time at points *A*, *B* and *C* with $D_c=0.348$ m and $\bar{T}_c=0.28$

Next, choosing 2% as the allowed error, we run a number of simulations for each combination of D_c and T_c by gradually refining the computing grids with the special meshing scheme defined above, and then determine the grid size of the first “scheme *n*” whose error is below 2% as the maximum grid size for the corresponding D_c and T_c . Figure 6 shows the maximum grid sizes for the twelve combinations of D_c and T_c according to our simulations.

Because the values of $\bar{T}_{ec}$ for different D_c are different as shown in Table 1, the selected four values of T_c for each D_c are chosen in following way

$$\bar{T}_{e1} = \bar{T}_{ec}(D_c) + 0.01, \bar{T}_{e2} = \bar{T}_{ec}(D_c) + 0.06, \bar{T}_{e3} = \bar{T}_{ec}(D_c) + 0.11, \bar{T}_{e4} = \bar{T}_{ec}(D_c) + 0.21$$

that is, they start from $\bar{T}_{ec}(D_c)$.

Figure 5b Slip and stress as a function of time at points A , B and C with $D_c=0.348$ m and $\bar{T}_e=0.30$

Table 1 Decision of characteristic value of $\bar{T}_{ec}(D_c)$

D_c/m	Non-growth rupture $\bar{T}_e$	Rupture $\bar{T}_e$	Characteristic value $\bar{T}_{ec}$
0.070	0.16	0.18	0.17
0.348	0.28	0.30	0.29
0.695	0.47	0.49	0.48

Figure 6 The max Δs for different (D_c , T_e) under a relative error bound (2%)
x and y axis represent effective stress and Δs , respectively

3 Discussion and conclusions

From Figure 6 we can see that for a fixed D_c , the maximum grid size is proportional to $(\bar{T}_e - \bar{T}_{ec})$. Comparisons between different curves in this figure clearly indicate that as D_c decreases all the computational grids should be condensed to ensure sufficient accuracy. These conclusions provide very important and useful guidelines for efficiently and accurately simulating the seismic source rupture processes with BIEM.

Because our meshing scheme is not continuous and only four values of T_e are selected for each D_c , the curves in Figure 6 are not smooth enough. Furthermore, since the error of “scheme 0” could not be computed with equation (2), our minimum grid division is chosen as 40×40 (*i.e.*, “scheme 1”), which results in a “saturation” in the simulation meshing scheme shown in Figure 6 in which the curve for $D_c=0.695$ m

tion (2), our minimum grid division is chosen as 40×40 (*i.e.*, “scheme 1”), which results in a “saturation” in the simulation meshing scheme shown in Figure 6 in which the curve for $D_c=0.695$ m

becomes flat.

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