Constraints on rupture speed of the 2001 M_S 8.1 West Kunlun Mountain Pass earthquake by co-seismic surface rupture slip displacements based on the slip-weakening mechanism with frictional undershoot involved*

SHI Bao-ping^{1),*} (史保平) YANG Yong²⁾ (杨 勇)

1) College of Earth Science, Graduate University of Chinese Academy of Sciences, Beijing 100049, China

2) Institute of Geophysics, China Earthquake Administration, Beijing 100081, China

Abstract

With co-seismic surface rupture slip displacements provided by the field observation for the 2001 M_S 8.1 West Kunlun Mountain Pass earthquake, this paper estimates the rupture speed on the main faulting segment with a long straight fault trace on the surface based on a simple slip-weakening rupture model, in which the frictional overshoot or undershoot are involved in consideration of energy partition during the earthquake faulting. In contrast to the study of Bouchon and Vallée, in which the rupture propagation along the main fault could exceed the local shear-wave speed, perhaps reach the P-wave speed on a certain section of fault, our results show that, under a slip-weakening assumption combined with a frictional undershoot (partial stress drop model), average rupture speed should be equal to or less than the Rayleigh wave speed with a high seismic radiation efficiency, which is consistent with the result derived by waveform inversion and the result estimated from source stress field. Associated with the surface rupture mechanism, such as partial stress drop (frictional undershoot) associated with the apparent stress, an alternative rupture mechanism based on the slip-weakening model has also been discussed.

Key words: slip-weakening model; partial stress drop; frictional undershoot; apparent stress; rupture speed; radiation efficiency

CLC number: P315.3⁺1

Document code: A

Introduction

The 2001 M_S 8.1 West Kunlun Mountain Pass earthquake is one of the largest earthquakes occurred in Chinese mainland within the last fifty years. The earthquake occurred in the east Kunlun fault with an extending in the east-west direction. The observed surface rupture zone extends about 400 km or even more in total length and several hundred meters in width, ranking as the observed longest surface rupture for the strike-slip event on earth. Field investigation also indi-

* Received 2007-10-17; accepted in revised form 2008-04-20.

Foundation item: National Natural Science Foundation of China (40574022), the Central Public Affair Program Award for the Specific Academic Research (ZDJ2007-1) and One-Hundred Individual Program of Chinese Academy of Sciences (99T3004WAZ).

✦ **Author for correspondence:** bshi@gucas.ac.cn

cated that an average of about 2 m horizontal slip displacement was observed and the maximum left-lateral slip displacements on the main fault could reach about 6~7 m horizontally along the fault strike direction (Xu *et al*, 2002). The earthquake occurred in a remote region of northern Tibet on the Kunlun fault, and one of the major strike-slip faults accommodates eastward movement of Tibet relative to Eurasia in response to the northward penetration of India along the Himalayas. The earthquake was not recorded at near-fault distances and only by a handful of digital recording stations at far-field distances. Since the earthquake occurred, many studies have been made, including geological field investigation (Xu *et al*, 2002) and source parameters estimations based on the far-field waveform inversion (Bouchon and Vallée, 2003; XU and CHEN, 2004; CHEN, 2005; Antolik *et al*, 2004; Ozacar and Beck, 2004). Due to the complex rupture process of the fault, there is a significant discrepancy among published researches for this event, which include focal mechanism solutions, epicentral locations, and rupture speeds derived from the waveform inversion. From XU and CHEN (2004), the rupture process was divided into three major subevents, in which the rupture speed is less than the shear wave speed; while Bouchon and Vallée (2003) reported a supershear rupture, in which the rupture speed exceeds the shear wave speed, perhaps reaching P-wave speed, based on the long period surface wave inversion. Recently, Bhat *et al* (2007) reported field observations and showed that this event ruptured faster than the shear wave speed based on the off-fault open cracks which are believed to be caused by the shock wave passing through the fault around from a fracture mechanics point of view.

There have been increased recent reports on supershear earthquake rupture, namely, the propagation speed lies between the shear and the dilatational wave speed of the surrounding medium. The earliest inference of supershear was during the 1979 Imperial Valley earthquake for which Archuleta (1984) noticed that for a better fit of near fault strong motion records, the rupture speed should exceed the shear wave speed. More recent inferences were made during the 1999 Izmit and Duzce events (Bouchon *et al*, 2000, 2001), the 2001 West Kunlun Mountain Pass event (Bouchon and Vallée, 2003), and the 2002 Denali event (Ellsworth *et al*, 2004). Laboratory verification of supershear rupture was provided for the first time by Rosakis *et al* (1999). There remains still, however, much uncertainty about the observation of supershear ruptures in large crustal earthquakes due to lack of sufficient strong motion records. For example, the 2002 Denali event is hypothesized to have gone partly (for about 40 km) (Ellsworth *et al*, 2004) supershear from a single ground motion record. Other claims of supershear rupture propagation come mainly from trying to fit a rupture speed for inversion of ground motion data.

On the other hand, CHEN (2005) reported that the 2001 $M_s 8.1$ West Kunlun Mountain Pass earthquake has a higher apparent stress of 5.3 MPa and a lower average static stress drop of 1.15~1.5 MPa based on the rectangular dislocation model. The static stress drop given by CHEN (2005) seems to be too low due to inappropriate use of the average surface slip of 1.2 m, and also of the focal depth determination. But the conclusion remarks in the paper latterly are still correct in which a partial stress process (frictional undershoot) with an associated abrupt fault locking (Brune, 1970, 1976; Heaton, 1990) could happen during the rupture propagation, which implies that the dynamic frictional strength varies along the fault plane. One of the implications of the partial stress drop model is that higher seismic energy is radiated at the high frequency end of the source spectrum than that would be expected for Orowan model (Orowan, 1960; Brune, 1976), with its obvious repercussions for strong motion modeling, and the earthquake faulting in this case undergoes a higher radiation efficiency and lower fracture energy required in order to advance the rupture propagation. Compared with other large earthquakes, for example, the 2004 Suma-

tra-Andaman earthquake occurred recently, the higher apparent stress and lower static stress drop (CHEN, 2005) also imply intuitively a higher rupture speed during the rupture propagation. For the 2004 Sumatra-Andaman earthquake, Kanamori (2006) showed that the apparent stress, static stress drop, radiation efficiency, and rupture speed are 1.38 MPa, 3.8 MPa, 0.16, and 0.25β , respectively, where β is the shear wave velocity. The apparent stress is much smaller than the static stress drop, which also indicates an overshoot happened during the earthquake faulting (Savage and Wood, 1971).

Comparison of the physical scaling relation of $M_0 = C_1(\Delta\sigma_s V^3) f_r^{-3}$ with the empirical relation of $M_0 \propto f_r^{-3}$ (Brune, 1970), where M_0 , $\Delta\sigma_s$, V , and f_r are the seismic moment, static stress drop, rupture speed, and corner frequency, respectively, and C_1 is a model-dependent constant, suggests that the term of $\Delta\sigma_s V^3$ is a constant, or scale-independent. Therefore, in order to keep the moment conservation in the source parameter inversion, a lower static stress drop means a higher rupture speed which is required so as to keep the term of $\Delta\sigma_s V^3$ to be a constant.

Fault slip weakening model has been commonly used in earthquake source parameter estimation and strong ground motion simulation (Pulido and Irikura, 2000; Favreau and Archuleta, 2002; Day *et al.*, 2005). For earthquake source parameter estimation, a simplified slip-weakening model is commonly used in most studies without consideration of earthquake frictional undershoot or overshoot. Therefore, in order to fit the far-field spectrum, the rupture speed becomes one of the most important parameters to control the waveform shape or the source spectrum decay at high frequency end because we know that the rupture speed is directly related to the seismic energy radiation based on the modern fracture mechanics. Moreover, if frictional undershoot or overshoot occurs during the earthquake faulting, a simplified slip-weakening model without consideration of undershoot/overshoot could result in underestimate or overestimate of rupture velocities. Data analysis by Zuniga (1993) indicates that large earthquakes usually undergo a frictional undershoot. Recently Kanamori (2006) also indicates that the dynamic overshoot or undershoot should be involved in the rupture process, otherwise, the fracture energy derived from the slip weakening model without consideration of undershoot or overshoot process could be less than zero if the frictional undershoot or overshoot occurs during the earthquake faulting.

In this paper, we use the field surface rupture displacements to constrain the dynamic rupture speed of the 2001 $M_s 8.1$ West Kunlun Mountain Pass earthquake based on the slip-weakening model with consideration of dynamic frictional undershoot, combining with a self-similar crack model. In the first part of this paper, we briefly discuss the earthquake energy balance and derive the formulae which will be used in the future calculation of radiation efficiency and rupture speed based on the slip weakening process. In the second part of this paper, we obtained the rupture speed by using surface rupture displacement data based on the slip weakening model. Finally, a brief discussion and conclusions are given.

1 Energy balance of earthquake faulting

1.1 Energy partition

As we know, the earthquake stress drop and the absolute ambient pre-earthquake stress, along with the frictional resisting forces active during sliding, determine the partitioning of stored elastic energy (E) released during faulting into seismic wave energy (E_s) and work done against resisting stresses (E_r) and fracture energy (E_G) requested in order to advance fault rupture propa-

gation. In what follows we outline how this partitioning provides a framework for constraints on the magnitudes of stresses acting before, during and immediately after earthquake fault slip. This will in turn provide a bounding frame work for evaluating various models and inference of stress state.

Define U as the total slip averaged over the fault area A . σ_1 and σ_2 are the initial and final shear stresses on the fault, respectively, and the average resisting shearing stress during slip is σ_r . Following Brune (1976) and Lachenbruch and Sass (1980), Figure 1 shows three possibilities for the relative magnitudes of the final stress and average resisting stress, namely, $\sigma_r > \sigma_2$ overshoot, $\sigma_r = \sigma_2$, total stress drop (Orowan, 1960), and $\sigma_r < \sigma_2$, undershoot or abrupt-locking or partial stress drop.

Figure 1 A schematic view of the average stress levels during fault slip (Brune and Thatcher, 2003)

Dashed line shows case of variable dynamic friction during slip. Three possibilities for the relative magnitudes of the final stress and average resisting stress are shown, namely, $\sigma_r > \sigma_2$ means overshoot, $\sigma_r = \sigma_2$ means total stress drop, and $\sigma_r < \sigma_2$ means undershoot or partial stress drop

The total elastic energy release is expressed by the area under the straight line joining σ_1 and σ_2 in Figure 1 (CHEN, 2003; Kanamori and Brodsky, 2004; Brune and Thatcher, 2003)^①:

$$E = \frac{1}{2}(\sigma_1 + \sigma_2)UA \quad (1)$$

The work done against averaged resisting stresses in the area under the line $\sigma = \sigma_r$ in Figure 1 is

$$E_r = \sigma_r UA \quad (2)$$

More precisely, E_r could be partitioned as two parts, *i.e.*, frictional energy corresponding to the heat generation due to frictional work of E_H during sliding and the fracture energy, E_G , required advancing the rupture propagation. The seismically radiated energy is given by the difference between equation (1) and equation (2), the area between the elastic energy release line and the resisting stress line σ_r :

$$E_s = E - E_r = \left[\frac{1}{2}(\sigma_1 + \sigma_2) - \sigma_r \right] UA \quad (3)$$

Equation (3) also can be rewritten as:

$$\sigma_a = \mu \frac{E_s}{M_0} = \left[\frac{1}{2}(\sigma_1 + \sigma_2) - \sigma_r \right] \quad (4)$$

where $\sigma_a = \mu E_s / M_0$ is known as apparent stress (where μ is shear modulus and $M_0 = \mu UA$ is the scalar seismic moment). The apparent stress has long been used in seismology (Aki, 1967; Wyss and Brune, 1968) and gives us the measurement of radiated seismic energy (E_s) per unit area, per unit slip. This parameter can be determined solely from the two macroscopic parameters E_s and M_0 without making further assumptions on the source dimension if μ , the shear modulus, is known.

Figure 1 graphically shows how the energy partitioning depends on the relative magnitudes

^① CHEN Yun-tai and GU Hao-ding. 2008. Fundamentals of Earthquake Source Theory (Vol.1). Lecture notes of the Graduate School of Chinese Academy of Sciences. Beijing: 169-185 (in Chinese).

of the initial, final, and resisting stresses on the fault. The detail dynamics of earthquake faulting certainly depends on the dynamic shear resistance as shown by the dashed lines in Figure 1, and equation (3) should have a more generalized form with consideration of the complex variations of resisting shear stresses during slip (Rivera and Kanamori, 2005):

$$E_s = E - E_r = \frac{1}{2}(\sigma_1 + \sigma_2)UA - A \int_0^U \sigma_r(u) du \quad (5)$$

Equation (5) describes the rupture process on a fault with more detail, including the effects of energy required to advance the rupture process, which is found in various studies (Madariaga, 1977). But, as we know, the average value remains useful for intuitive understanding. More detail descriptions of how to derive equations of (1) to (5) could be found in the lecture note of CHEN and GU^①.

1.2 Overshoot and undershoot

Equation (4) can be rewritten as (Savage and Wood, 1971):

$$\sigma_a = \frac{1}{2}(\sigma_1 - \sigma_2) - (\sigma_r - \sigma_2) = \frac{1}{2}\Delta\sigma_s - (\sigma_r - \sigma_2) \quad (6)$$

Here, $\Delta\sigma_s = \sigma_1 - \sigma_2$ gives static stress drop. If the difference of $\sigma_r - \sigma_2$ is positive, we obtain the overshoot case (Savage and Wood, 1971); if negative, we obtain undershoot or partial stress drop model (abrupt locking model) (Brune, 1970, 1976), and if the difference is equal to zero, we obtain a total stress drop model or Orowan model (Orowan, 1960). Savage and Wood (1971) exploited the above relation in the form of $\sigma_a < \Delta\sigma_s/2$, which is known as Savage and Wood inequality, implying a frictional overshoot mechanism. From equation (6) we can see that partial stress drop would imply a change in the inequality of $\sigma_a < \Delta\sigma_s/2$. In CHEN's calculation (2005) for the 2001 $M_s 8.1$ West Kunlun Mountain Pass earthquake, the apparent stress is about 5.3 MPa based on the USGS (NEIC) data and static stress drop ranges 1.1~1.5 MPa based on the rectangular dislocation model, which gives a partial stress drop, corresponding to an abrupt locking behavior or self-healing rupture mechanism.

1.3 Simple slip weakening model

Equation (5) can also be rewritten as (Rivera and Kanamori, 2005; Kanamori and Rivera, 2006):

$$E_s = \frac{1}{2}(\sigma_1 - \sigma_2)UA - \left[A \int_0^U \sigma_r(u) du - \sigma_2 UA \right] \quad (7)$$

The first term on the right-hand side is $E_0 = (\sigma_1 - \sigma_2)UA/2 = \Delta\sigma_s UA/2$. The term in the square bracket on the right-hand side gives the difference between the areas labeled by “+” and “-” (Figure 2), which Kostrov (1974) called the “radiation friction”. In practice, it is always difficult to accurately include the contributions for high frequency seismic waves, because high frequency energy is strongly attenuated and scattered during the propagation.

Figure 2 Graphic representation of the energy budget in general case (Rivera and Kanamori, 2005; Kanamori and Rivera, 2006)

^① CHEN Yun-tai and GU Hao-ding. 2008. Fundamentals of Earthquake Source Theory (Vol.1). Lecture notes of the Graduate School of Chinese Academy of Sciences. Beijing: 169-185 (in Chinese).

For simplification, if the frictional process is described by a simple linear function during the sliding, as illustrated in Figure 3, then equation (7) could be rewritten as:

$$\begin{cases} E_s = \frac{1}{2}(\sigma_1 + \sigma_2)UA - (E_G + E_H) \\ E_G + E_H = A \int_0^U \sigma(u) du \\ E_H = \sigma_f UA \end{cases} \quad (8)$$

or

$$\begin{cases} E_s = \frac{1}{2}(\sigma_1 + \sigma_2)UA - (\sigma_r - \sigma_f)UA - \sigma_f UA = \frac{1}{2}(\sigma_1 + \sigma_2)UA - \Delta\sigma_c UA - \sigma_f UA \\ E_G = \Delta\sigma_c UA \end{cases} \quad (9)$$

Figure 3 Simple slip weakening model involving frictional undershoot

The radiation energy E_s , fracture energy E_G , and frictional work (heat) E_H are represented by the quadrangular area of $ACDE$, the triangular area of ABC , and the rectangular area of $BFGD$, respectively. The dynamic stress drop, $\Delta\sigma_d$, is given by $\sigma_1 - \sigma_f$

In equations (8) and (9), σ_f is dynamic frictional stress defined in Figure 3, $\sigma_{US} = \sigma_2 - \sigma_f$, and $\Delta\sigma_c = \sigma_r - \sigma_f$. Therefore, we always have $E_G = \Delta\sigma_c UA$ which is represented by the triangular area of ABC illustrated in Figure 3. E_G is a basic quantity that controls the rupture propagation, especially, the rupture speed, and D_c is the critical slip weakening distance which is related to the rupture initiation. The theoretical model expressed in equations (8) and (9) is a conceptually simple model. On the right-hand side of equation (8) the term of E_G is the fracture energy spent at the tip in the classical sense, and the term of $\sigma_f UA$ is the frictional work done on the fault plane. All the energy dissipation occurs on the fault plane. In contrast, real fault involves energy dissipation in a volume surrounding the fault through

(gain crushing) forming off-fault cracks (Kanamori and Rivera, 2006).

We can rewrite equation (9) as:

$$E_G = \frac{1}{2}(\Delta\sigma_s + 2\sigma_{US} - 2\sigma_a)UA \quad (10)$$

Where $\sigma_{US} = \sigma_2 - \sigma_f \geq 0$ is called dynamic undershoot stress. Similarly, for the overshoot case, we also have

$$E_G = \frac{1}{2}(\Delta\sigma_s - 2\sigma_{OS} - 2\sigma_a)UA \quad (11)$$

Where $\sigma_{OS} = \sigma_r - \sigma_2 \geq 0$ gives dynamic overshoot stress.

In general, the available energy on the left-hand side of equation (1) is partitioned between

radiated energy (E_s) and other types of dissipation such as fracture energy (E_G). The radiated energy E_s can be measured by standard seismological techniques. Everything else is usually combined in a term often associated with fracture energy. This composite term can be called seismological observable fracture energy E_G . In Orowan model (Orowan, 1960) of $\sigma_2 = \sigma_f$, we have

$$\begin{cases} E_s + E_G = \frac{1}{2}(\sigma_1 - \sigma_2)UA \\ E_G = \frac{1}{2}(\Delta\sigma_s - 2\sigma_a)UA \end{cases} \quad (12)$$

1.4 Radiation efficiency

The radiation efficiency is defined by (Husseini and Randall, 1976; Husseini, 1977; Kanamori and Brodsky, 2004):

$$\eta = \frac{E_s}{E_s + E_G} \quad (13)$$

If $\sigma_2 = \sigma_f$, then

$$\eta = 2 \frac{\sigma_a}{\Delta\sigma_s} \quad (14)$$

Equation (14) has been commonly used in the estimation of radiation efficiency related to the earthquake rupture speed (Kanamori and Brodsky, 2004). As we discussed earlier, the commonly used slip weakening model has a simple stress-slip-function, but the stress change during actual faulting could be more complex. As the next level of complexity to the simple slip weakening model, two simple models are overshoot and undershoot models. These models are often used in seismology. An overshoot occurs if the fault moves past the equilibrium point corresponding to the final stress because of inertia and is locked there due to friction. The other case is undershoot (partial stress drop). Partial stress drop could be caused by some obstacles (*e.g.* interlocking asperities) prematurely stopping fault motion (Brune, 1970, 1976). Another possibility is rapid healing of the rupture surface during slippage, which slows down the particle motion of the fault (Heaton, 1990). If an overshoot or undershoot occurs during a rupture process, equation (12) should be modified as

$$\begin{cases} \Delta\sigma_c = \sigma_{US} - \sigma_a + \frac{1}{2}\Delta\sigma_s & \text{Undershoot} \\ \Delta\sigma_c = \frac{1}{2}\Delta\sigma_s - \sigma_a - \sigma_{OS} & \text{Overshoot} \end{cases} \quad (15)$$

and the corresponding radiation efficiencies are

$$\begin{cases} \eta_{US} = \frac{\eta}{1 + 2\sigma_{US}/\Delta\sigma_s} & \text{Undershoot} \\ \eta_{OS} = \frac{\eta}{1 - 2\sigma_{OS}/\Delta\sigma_s} & \text{Overshoot} \end{cases} \quad (16)$$

Equation (16) was first given by Kanamori and Rivera (2006) in which $\eta = 2\sigma_a/\Delta\sigma_s$ comes from equation (14). In general, we could obtain a uniform representation of the radiation efficiency by solving equations of (15) and (16) together or we can derive it directly based on the energy conservation principle in Figure 3. After a little algebra we obtain:

$$\eta = \frac{\sigma_a}{\sigma_a + \Delta\sigma_c} \quad (17)$$

1.5 Radiation efficiency versus rupture speed

As discussed by Kanamori and Brodsky (2004), the energy release rate for a dynamic crack, G , is given as a function of the rupture speed V . Since the fracture energy E_G can be interpreted as the integral of G over the entire fault surface A , we can relate the radiation efficiency η to rupture speed V as follows. In the simplest model discussed by Kanamori and Brodsky (2004), we have

$$E_G = \int G dA = g(V) \int G^* dA = g(V) \Delta W_0 \quad (18)$$

where G^* is called the static energy release rate or crack extension force, $g(V)$ is a universal function of V , and ΔW_0 is the strain energy released (mechanical work) for crack extension. The ratio of the dynamic energy release rate, G , to the static energy release rate, G^* , is given by a function of rupture speed V (Kostrov, 1974; Eshelby, 1949). Freund (1979) showed that the energy release rate, G , for a crack growing at a rupture speed V is given approximately by $G = G^* g(V)$. In this case, the radiation efficiency can be defined as:

$$\eta = \frac{E_s}{E_s + E_G} = \frac{E_s}{\Delta W_0} \quad (19)$$

and

$$\eta = 1 - g(V) \quad (20)$$

For a propagating crack, there is a relationship between the rupture velocity and the real, physical fracture energy. The fraction of the energy that goes into fracture energy controls the speed of crack propagation. At the limiting speed of a crack propagating with the Rayleigh wave speed, the fracture energy is zero. The exact form of the relationship depends on the mode of crack propagation and the loading conditions. For instance, with the assumption that the rest value of the energy release rate at the crack tip is the same as the quasi-static one, for semi-infinite mode-II and mode-III crack extensions, we have:

$$g(V) = \begin{cases} \frac{1 - V/C_R}{\sqrt{1 + V/\beta}} & \text{Mode - II} \\ \frac{1 - V/\beta}{\sqrt{1 + V/\beta}} & \text{Mode - III} \end{cases} \quad (21)$$

where C_R , β , V are the Rayleigh wave, shear wave, and rupture speed, respectively. For fuller discussion on the origin of the fracture energy interpretation see reviews by Kanamori and Brodsky (2004) or Kanamori and Rivera (2006).

2 Rupture speed estimating

The earthquake rupture zone can be divided into three segments as described by XU and CHEN (2004) or Bouchon and Vallée (2003). The 2001 $M_s 8.1$ West Kunlun Mountain Pass earthquake generated more than 400 km surface rupture on the main segment of the Kunlun fault (Figure 4). The measured surface off-set across the fault is about 4.9 m in average and 6~7 m in maximum along the strike direction (Haeussler *et al*, 2004). Correspondingly, this segment from

91°E to 95°E in longitude has been considered to have a super-shear rupture propagation during the earthquake faulting with a pure strike-slip motion, as shown in Figure 4 (Antolik *et al.*, 2004).

For this earthquake, seismic moment and the radiated seismic energy determined by the USGS (NEIC) are $E_s = 3.20 \times 10^{16}$ N·m, $M_0 = 1.80 \times 10^{20}$ N·m, respectively. If we choose $\mu = 3.0 \times 10^4$ MPa, the resultant apparent stress of σ_a is about 5.3 MPa.

As we know, in addition to the seismological method, both geological and geodetic methods can also be used to infer the spatial distribution of seismic slip and hence map static stress drop variation on the earthquake fault plane. For the 2001 $M_{S8.1}$ West Kunlun Mountain Pass earthquake, the extensive surface faulting and measurements of surface fault offset on the main segment of the Kunlun fault along the west-east direction are plotted in Figure 5. The data is originally from the field measurement and high resolution satellite imagery mapping after the earthquake and was published later for research purpose (Xu *et al.*, 2002; Klinger *et al.*, 2005). Although the data scarcity precludes the estimation of the largest static stress change on the fault plane, these results as well as seismological determination of slip distribution indicate considerable heterogeneity in stress drop on earthquake rupture plane. Therefore, from the spatial variation of the relative slip distribution shown in Figure 5, we can estimate the static stress drop with the assumption that the slip variation along the fault reflects the variation of the stress drop on the fault plane to some level. Although the apparent stress of σ_a is a useful dynamic parameter, σ_a itself does not completely reflect the difference in the rupture physics in term of the energy budget. The radiation efficiency η is more representative of the rupture physics.

Figure 4 Schematic illustration of the $M_{S8.1}$ West Kunlun Mountain Pass earthquake faulting

The solid line segments of “3” and “4” represent the orientation of the fault segments that we have used for the rupture speed estimation with the surface-slip constraint. The earthquake rupture is divided into three or four subevents. The arrows indicate rupture direction along the fault (Antolik *et al.*, 2004), and the “X” indicates the location of transition to super shear rupture, which is shown at the beginning of fault segment “3”

Figure 5 Slip distributions on the fault along the strike direction (Xu *et al.*, 2002)

For this earthquake, first, we use mode-II rectangular dislocation model in which $\Delta\sigma_s = C\mu U/W$ for a long strike-slip earthquake, where U and W are the average slip displacement and width on the fault, respectively, and C is geometrical factor given by $C = 4/\pi$ or $C = 2/\pi$, depending on wheth-

er the slip breaks the surface or not (Boore and Dunbar, 1977). For the West Kunlun Mountain Pass earthquake, a large surface rupture is clearly observed on the main segment. Thus, we use the case for a surface break fault model. For estimate of W , following CHEN (2005), we used the values of W as 15 km, 20 km, 30 km, and 40 km, respectively, which represent recent different results derived from source inversions (XU and CHEN, 2004; Antolik *et al*, 2004; Ozacar and Beck, 2004). The interpretation of η is model-dependent, so, for a strike-slip earthquake, we use the widely used mode-II crack in combination with the frictional undershoot (Figure 3), in which $E_G = \Delta\sigma_c UA$. For an earthquake, usually the contribution of fracture energy to the earthquake energy budget is assumed to be very small, and quite often fracture energy is ignored entirely (Kanamori and Heaton, 2000). Still, the relative size of fracture energy for the earthquake could be large; recent estimates from dynamic modeling produce fracture energy that is 60% of the energy associated with the static stress drop (Favreau and Archuleta, 2002). Therefore, we have $E_G = \Delta\sigma_c UA = \Delta\sigma_s \times UA/2 \times 60\%$, or $\Delta\sigma_c = 0.3\Delta\sigma_s$ (Beeler *et al*, 2003). If we choose $W=30$ km, then the static stress drop, $\Delta\sigma_s$, the radiation efficiency, η , and the ratio of the rupture speed to shear wave speed, V/β , are 1.87 MPa, 0.85, and 0.86, respectively. Table 1 summarized the results of the average values of static stress drop, dynamic stress drop, frictional undershoot stress, radiation efficiency, and the ratio of rupture speed to shear wave speed if different fault widths of W are used in this study.

Table 1 Source parameters derived from the slip weakening model with different fault widths (rectangular fault model)

W/km	$\Delta\sigma_s/\text{MPa}$	$\Delta\sigma_d/\text{MPa}$	$\Delta\sigma_u/\text{MPa}$	η	V/β
15	3.76	7.18	3.42	0.74	0.82
20	2.82	6.70	3.89	0.79	0.84
30	1.87	6.20	4.36	0.85	0.86
40	1.41	6.00	4.60	0.89	0.88

Note: W , $\Delta\sigma_s$, $\Delta\sigma_d$, $\Delta\sigma_u$, η , V/β denote fault width, static stress drop, dynamic stress drop, frictional undershoot stress, radiation efficiency and the ratio of rupture speed to shear wave speed, respectively.

In general, the mode-II rectangular dislocation model is the first-order approximation used in the description of earthquake faulting. Real earthquake faulting is more complex and usually is behaved as a combination of mode-II and mode-III of dislocation or crack-like motions. Recent body waveform inversions (Antolik *et al*, 2004; Ozacar and Beck, 2004), based on the finite fault model and constrained by the surface slip displacement distributions, have shown that the slip and stress distributions are heterogeneous on the fault, and in general, the maximum slips occur near the free surface. Based on the mode-III dislocation theory developed by Barnett and Freund (1975) for the strike-slip fault, we have investigated the rupture properties inside the fault, including the spatial distributions of slip, stress, and rupture width on the fault (manuscript in preparing). The resultant slip pattern, average stress drop, and rupture width on the fault are very similar to the body wave inversion results given by Antolik *et al* (2004) and Ozacar and Beck (2004). In Barnett and Freund (1975), the variations of the static stress and slip along the fault depth, X , are:

$$\begin{cases} \Delta\sigma_s(X) = -\mu W'_0 \left[1 - 2 \left(\frac{X}{R} \right)^2 \right] \\ D_s(X) = -\frac{2W'_0}{3R^2} (R^2 - X^2)^{3/2} \\ R = -\frac{3W_0}{2W'_0} \end{cases} \quad (22)$$

where W_0 is the surface slip displacement along the fault strike direction, and W'_0 is the slope of slip variation perpendicular to the fault trace on the fault. For convenience, we choose $W'_0 = -0.55$ m/km used in the study of the 1906 San Francisco earthquake (Weertman, 1965; Barnett and Freund, 1975) in which the rupture type (strike-slip), moment magnitude and rupture length are the same as or similar to the 2001 West Kunlun Mountain Pass earthquake (Haeussler *et al.*, 2004). Using equation (22) and surface rupture data illustrated in Figure 5, the resultant average stress drop and average rupture width in depth are 5.5 MPa and 8.4 km, respectively. The radiation efficiency and the ratio of the rupture speed to the shear wave speed are 0.76 and 0.89, respectively. A brief comparison with previous studies is summarized in Table 2.

Table 2 Source parameters derived from different source models

Rupture models	$\Delta\sigma_s$ /MPa	η from equation (17)	V/β from equation (21)
XU and CHEN (2004)	4.0	0.81	0.85
CHEN (2005)	1.5	0.92	0.89
Antolik <i>et al.</i> (2004)	3.75	0.82	0.86
Mode-III of Barnett and Freund (1975)	5.5	0.76	0.89

3 Discussion

Our study shows that there is a certain range of static stress drop along the fault strike direction for the 2001 $M_s 8.1$ West Kunlun Mountain Pass earthquake. Some of this scatter may be caused by incorrect mapping of surface rupture displacements because of uncertainties such as measured error and data scarcity. Despite of these uncertainties, compared with the apparent stress derived from seismological observation, the frictional undershoot for this earthquake is obvious.

The dynamics of earthquake faulting are the least well-understood aspect of fault mechanics. However, seismological observations indicate that, on average, the apparent stress, which is a measure of the dynamic stress during slip, is relatively small, average ~ 3 MPa, about the same magnitude as the average static stress; this estimate is very sensitive to the accuracy of seismic estimations of radiated energy. In this study, the apparent stress (average) is generally the same order of magnitude as our estimates of the dynamic and static stress drops. There is still uncertainty about this observation, and the conclusions depend greatly on the data sets that are included. In general, the apparent stress could locally be much higher than its mean value because the moment release rate is very non-uniform and much of the energy radiated to teleseismic distance originates from restricted portions of the earthquake fault, which also indicates that the apparent stress is also heterogeneous. Therefore, from the seismological point of view, a sophisticated method to calculate temporal and spatial evolutions of the apparent stress, which combines with the fault friction law such as slip-weakening friction law, is required so as to precisely estimate the rupture parameters.

Rupture velocities inferred from the seismic radiation efficiency criteria are commonly in the range of 0.81β to 0.89β . As rupture speed increases in this range, larger ground motions are produced near the fault. For a rupture propagating at Rayleigh-like speed, this amplification becomes the maximum, as the Rayleigh velocity is the classical upper limit for mode-II ruptures like the 2001 West Kunlun Mountain Pass earthquake. Theoretical research in fracture dynamics (Richards, 1973) predicts that a rupture propagating at near Rayleigh velocity is associated with high slip velocity, thus further increasing near-fault motion. The Rayleigh-like value holds special significance and may help explain some of the other unusual characteristics of this event. During an

earthquake, part of the strain energy released by the rocks is supplied to the crack tip where the advanced fracturing could occur and be maintained. The theory (Freund, 1998; Broberg, 1999; Rosakis, 2002) shows that this energy supply decreases to zero as rupture speed increases to the Rayleigh velocity limit. For a crack propagating at near Rayleigh velocity little energy is available for fracture, which implies that the bond between the crack faces must be very weak. A mode-II or mode-III ruptures cannot propagate at a speed between Rayleigh and shear velocity because in this range the energy supplied to the crack tip would be negative. Beyond the shear wave speed, energy can be supplied again and a few observations of supershear ruptures have been made. Thus, the Rayleigh-like rupture speed of earthquake faulting implies that the fault in this case was weak and broke easily.

Physical arguments in favor of frictional overshoot and frictional undershoot have been unsolved in earthquake source physics. Some studies suggest that only one possibility is allowed when the final stress departs from Orowan's case (Orowan, 1960), implying that overshoot should take place if Orowan's condition is not met. Others have assumed that either frictional overshoot or frictional undershoot is valid in attempt to advance theoretical source studies. Smith *et al* (1991) stated that there is no strong evidence that frictional overshoot ever occurs. Comparing with other large earthquake, such as 2004 Sumatra-Andaman earthquake in which the apparent stress and static stress drop are 1.4 MPa and 3.8 MPa, respectively, for the 2001 West Kunlun Mountain Pass earthquake, the apparent stress and static stress drops in average are 5.3 MPa and 3~5.5 MPa, respectively, indicating a strong frictional undershoot occurred during the earthquake rupture. In this case, the simple slip weakening model and radiation efficiency criteria without consideration of frictional undershoot behavior used to constrain radiation energy and fracture energy, which usually are related to the rupture speed, have to be modified in order to avoid unphysical results of $\eta > 1$ and $E_G < 0$. Abrupt fault locking related to the frictional undershoot or partial stress drop is commonly used to describe earthquake fault frictional motion. In this case, one explanation is that the dynamic frictional stress on the fault plane is strongly heterogeneous, and the rupture can be arrested locally due to the change of fault strength. An alternative explanation is that the ambient stress distribution is strongly heterogeneous for larger earthquakes due to the fault surface roughness (asperity), and the earthquake faulting undergoes a self sustained pulse-shaped rupture (the rupture length is proportional to the asperity size), which is usually observed in recent large earthquakes, including Denali earthquake with a localized rupture of about 18 km (Dunham and Archuleta, 2004).

4 Conclusions

With a simple slip-weakening mechanism, we have modified radiation efficiency criteria involving in consideration of frictional overshoot and undershoot during the earthquake dynamic faulting. Using this criteria, we have estimated the rupture speed for the 2001 $M_s 8.1$ West Kunlun Mountain Pass earthquake, and the results indicate that, for the fault widths of 15 km, 20 km, 30 km, and 40 km, the corresponding rupture speeds of V in average are about 0.82β , 0.84β , 0.86β , and 0.88β , respectively, which are consistent with the source inversion results given by XU and CHEN (2004). The higher apparent stress of 5.3 MPa and the lower static stress drops of 3~5.5 MPa during the earthquake dynamic faulting also indicate a strong frictional undershoot, which corresponds to the partial stress drop model or abrupt fault locking model. Therefore, an anomalous seismic energy is radiated at the high frequency, which could cause a near-fault strong ground

motion related to the large surface damages, such as shear and tensional cracks observed in the rupture zone. The tensional cracks observed in the field has been used as a strong evidence of the earthquake rupturing at supershear speed ($V \gg \beta$) according to the numerical modeling of rupture propagation (Bhat *et al*, 2007). Based on our current study, an alternative physical explanation for this phenomenon could be directly related to the large high frequency seismic energy radiation surrounding fault if a frictional undershoot occurs during earthquake faulting, and the surface damage (fracture zone) could be caused by the strong ground motion under an anomalous high frequency seismic radiation. Related to the supershear rupture of earthquake faulting, more theoretical works and near-fault strong motion observations are required in the future study of earthquake dynamics.

References

- Aki K. 1967. Scaling law of seismic spectrum [J]. *J Geophys Res*, **72**: 1 217-1 231.
- Antolik M, Abercrombie R E, Ekstrom E. 2004. The 14 November 2001 Kokoxili (Kunlunshan), Tibet, earthquake: Rupture transfer through a large extensional step-over [J]. *Bull Seism Soc Amer*, **94**: 1 173-1 194.
- Archuleta R J. 1984. A faulting model for the 1979 Imperial Valley earthquake [J]. *J Geophys Res*, **89**: 4 559-4 585.
- Barnett D M and Freund L B. 1975. An estimate of strike-slip fault friction stress and fault depth from surface displacement data [J]. *Bull Seism Soc Amer*, **65**: 1 259-1 266.
- Beeler N M, Wong T F, Hickman S H. 2003. On the expected relationships among apparent stress, static stress drop, effective shear fracture energy, and efficiency [J]. *Bull Seism Soc Amer*, **93**: 1 381-1 389.
- Bhat H S, Dmowska R, King C P. 2007. Off-fault damage patterns due to supershear ruptures with application to the 2001 M_s 8.1 Kokoxili (Kunlun) Tibet earthquake [J]. *J Geophys Res*, **112**: B06301.
- Boore D M and Dunbar W S. 1977. Effect of the free surface on calculated stress drops [J]. *Bull Seism Soc Amer*, **67**: 1 661-1 664.
- Bouchon M, Bouin M P, Karabulut H, *et al*. 2001. How fast is rupture during an earthquake? New insights from the 1999 Turkey earthquakes [J]. *Geophys Res Lett*, **28**: 2 723-2 726.
- Bouchon M, Toksoz N, Karabulut H. 2000. Seismic imaging of the 1999 Izmit (Turkey) rupture inferred from the near-fault recordings [J]. *Geophys Res Lett*, **27**: 3 013-3 016.
- Bouchon M and Vallée M. 2003. Observation of long supershear rupture during the magnitude 8.1 Kunlunshan earthquake [J]. *Science*, **301**: 824-826.
- Broberg K B. 1999. *Cracks and Fracture* [M]. New York: Elsevier: 328-508.
- Brune J N. 1970. Tectonic stress and the spectra of seismic shear waves from earthquakes [J]. *J Geophys Res*, **75**: 4 997-5 009.
- Brune J N. 1976. The physics of earthquake strong motion [G]/Lomnitz C and Rosenblueth E eds. *Seismic Risk and Engineering Decisions*. New York: Elsevier: 141-177.
- Brune J N and Thatcher W. 2003. Strength and energetics of active fault zones [G]/*International Handbook of Earthquake and Engineering Seismology, Part A*. San Diego: Academic Press: 569-588.
- CHEN Xue-zhong. 2005. Estimation of the stress levels in the focal region before and after the 2001 M_s 8.1 Western Kunlun Mountain Pass earthquake [J]. *Acta Seismologica Sinica*, **18**(6): 651-655.
- CHEN Yun-tai. 2003. Earthquake energy and seismic efficiency [M]/*Source Parameters: The Application of Digital Seismology in the Earthquake Prediction*. Beijing: Seismological Press: 51-72 (in Chinese).
- Day S M, Dalgner L A, Lapusta N. 2005. Comparison of finite difference and boundary integral solutions to three-dimensional spontaneous rupture [J]. *J Geophys Res*, **110**: B12307, doi:10.1029/2005JB003813.
- Dunham E M and Archuleta R J. 2004. Evidence for a supershear transient during the 2002 Denali fault earthquake [J]. *Bull Seism Soc Amer*, **94**: 256-268.
- Ellsworth W L, Celebi M, Evans J R. 2004. Processing and modeling of the Pump Station 10 record from the November 3, 2002, Denali fault, Alaska earthquake [C]/*Proceedings 11th Intern Conf Soil Dynam Earthq Eng* 1. Berkeley, California: 471-477
- Eshelby J D. 1949. Uniformly moving dislocations [J]. *Proc Phys Soc (London)*, **A62**: 307-314.
- Favreau P and Archuleta R. 2002. Direct seismic energy modeling and application to the 1979 Imperial Valley earthquake [J]. *Geophys Res Lett*, **30**: 1198, doi:10.29/2002GL015968.
- Freund L B. 1979. The mechanics of dynamic shear crack propagation [J]. *J Geophys Res*, **84**: 2 199-2 209.
- Freund L B. 1998. *Dynamic Fracture Mechanics* [M]. New York: Cambridge University Press: 231-242.
- Haeussler P J, Schwartz D P, Dawson T E. 2004. Surface rupture of the 2002 Denali fault, Alaska, earthquake and comparison with other strike-slip ruptures [J]. *Earthquake Spectra*, **20**: 565-578.
- Heaton T H. 1990. Evidence for and implications of self-healing pulses of slip in earthquake rupture [J]. *Phys Earth Planet Int*, **64**: 1-20.
- Husseini M I. 1977. Energy balance for formation along a fault [J]. *Geophys J R astr Soc*, **49**: 699-714.
- Husseini M I and Randall M. 1976. Rupture velocity and radiation efficiency [J]. *Bull Seism Soc Amer*, **66**: 1 173-1 187.
- Kanamori H. 2006. *The Radiated Energy of the 2004 Sumatra-Andaman Earthquake* [M]. Washington DC: American Geophysical Union, **170**: 59-68.
- Kanamori H and Brodsky E E. 2004. The physics of earthquakes [J]. *Rep Prog Phys*, **67**: 1 429-1 496.

- Kanamori H and Heaton T H. 2000. *Microscopic and Macroscopic Physics of Earthquakes* [M]. Washington DC: American Geophysical Union, **120**: 147-155.
- Kanamori H and Rivera L. 2006. *Energy Partition During an Earthquake* [M]. Washington DC: American Geophysical Union, **170**: 3-14.
- Klinger Y, Xu X, Tapponnier P. 2005. High-resolution satellite imagery mapping of the surface rupture and slip distribution of the $M_w \sim 7.8$, 14 November 2001 Kokoxili earthquake, Kunlun fault, northern Tibet, China [J]. *Bull Seism Soc Amer*, **95**: 1 970-1 987.
- Kostrov B. 1974. Seismic moment and energy of earthquakes, and seismic flow of rock [J]. *Izv Earth Physics*, **1**: 23-40.
- Lachenbruch A H and Sass J H. 1980. Heat flow and energetics of the San Andreas fault zone [J]. *J Geophys Res*, **85**: 6 185-6 222.
- Madariaga R. 1977. High frequency radiation from crack (stress drop) models of earthquake faulting [J]. *Geophys J R astr Soc*, **51**: 625-651.
- Orowan E. 1960. Mechanism of seismic faulting [J]. *Geol Soc Am Mem*, **79**: 323-345.
- Ozacar A A and Beck S L. 2004. The 2002 Denali fault and 2001 Kunlun fault earthquakes: Complex rupture processes of two large strike-slip events [J]. *Bull Seis Soc Amer*, **94**: S278-S292.
- Pulido N and Irikura K. 2000. Estimation of dynamic rupture parameters from the radiated seismic energy and apparent stress [J]. *Geophys Res Lett*, **27**: 3 945-3 948.
- Richards P G. 1973. The dynamic field of a growing plane elliptical shear crack [J]. *Int J Sol Struct*, **9**: 843-861.
- Rivera L and Kanamori H. 2005. Representations of the radiated energy in earthquakes [J]. *Geophys J Int*, **162**: 148-155.
- Rosakis A J. 2002. Intersonic shear cracks and fault ruptures [J]. *Adv Phys*, **51**: 1 189-1 257.
- Rosakis A J, Samudrala O, Coker D. 1999. Cracks faster than the shear wave speed [J]. *Science*, **284**: 1 337-1 340.
- Savage J C and Wood M D. 1971. The relation between apparent stress and stress drop [J]. *Bull Seism Soc Amer*, **61**: 1 381-1 388.
- Smith K D, Brune J N, Priestly K F. 1991. The seismic spectrum, radiated energy, and the Savage and Wood inequality for complex earthquakes [J]. *Tectonophysics*, **188**: 303-320.
- Weertman J. 1965. Relationship between displacements on a free surface and the stress on a fault [J]. *Bull Seism Soc Amer*, **55**: 945-953.
- Wyss M and Brune J N. 1968. Seismic moment stress and source dimension for earthquake in California-Nevada region [J]. *J Geophys Res*, **73**: 4 681-4 694.
- XU Li-sheng and CHEN Yun-tai. 2004. Temporal and spatial rupture process of the great Kunlun Mountain Pass earthquake of Nov. 14, 2001, from GDSN long period waveform data [J]. *Science in China (Ser D)*, **34**(3): 256-264.
- Xu X, Chen W, Ma W, *et al.* 2002. Surface rupture of the Kunlunshan earthquake ($M_s 8.1$), northern Tibetan plateau, China [J]. *Seism Res Lett*, **73**: 884-892.
- Zuniga F R. 1993. Frictional overshoot and partial stress drop. Which one [J]? *Bull Seism Soc Amer*, **83**: 939-944.