Effect of initial perturbations on the simulation capability of numerical models

LIN WanTao^{1*}, WU LiFei², LIN YiHua¹ & XUE Feng¹

¹ State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China;

² Department of Mathematics and Physics, North China Electric Power University, Beijing 102206, China

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The effect of an initial perturbation on simulation is studied in a series of numerical experiments using a one-dimensional nonlinear advection equation model. It is shown that the capability of the model tends to decline with an increase in initial perturbation. The error in the initial perturbation grows in a nonlinear manner, indicating that the simulating results can be improved only to a limited degree by increasing the accuracy of the initial data.

initial perturbation, capability of model, model error, DTE

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The equation of motion of the atmosphere and ocean is a complex nonlinear evolution equation. Because of the complexity of the equation, it is generally difficult to obtain an analytical solution. It is therefore necessary to obtain a numerical solution from the discrete form of the equation. The numerical method is also referred to as using an atmospheric or ocean model. Since the 1980s, the global climate system model has been an important tool in climate simulation and future climate projection [1–3].

Results obtained using an atmospheric or oceanic model depend on the simulating capability of the model, which is closely related to the initial values used in the model. Because the movement of the atmosphere or ocean is essentially nonlinear, its motion state depends on the initial field (i.e. the initial values) [4]. The movement state in the model relies on the initial field in a similar way [5]. In fact, no matter how accurate the observation data are, there are unavoidable errors in the initial field. These errors may affect the prediction of the movement state by the model [6]. In an effort to account for the effect of initial perturbation on

simulation results, data assimilation technology has been advanced rapidly since the 1980s. The quality of the initial field can be improved greatly through data assimilation. As a result, the simulating capability of the model is enhanced by a more realistic description of the actual state obtained with the assimilated data [7–9]. A question that remains unanswered is to what degree data assimilation can improve the capability of the model. This paper studies this question for the typical example of a simple model based on a one-dimensional nonlinear advection equation in a series of numerical experiments.

1 Model and method

The most significant movement in the atmosphere and ocean is horizontal movement, which can be characterized by a one-dimensional nonlinear advection equation with a nonlinear advection term. Therefore, the simulation capability of this simple model can represent the characteristics of a complex model.

The one-dimensional nonlinear advection equation is

*Corresponding author (email: linwt@lasg.iap.ac.cn)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \quad a \leq x \leq b, \quad 0 \leq t \leq T, \quad (1)$$

$$u(x, 0) = \phi(x),$$

where u is a function of space x and time t , and a , b , and T are constants.

For equation (1), we use the leapfrog scheme to obtain the discrete form of the model:

$$\frac{u_j^{n+1} - u_j^{n-1}}{2\Delta t} + u_j^n \frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} = 0. \quad (2)$$

In the following experiments, the solution area is $\Sigma = [0, 1] \times [0, 10]$, the spatial step is 0.01, the temporal step is 0.001, the initial value is $u(x, 0) = x + 5.21078$, the left boundary is $u(0, t) = 5.21078/(1+t)$, and the right boundary is $u_j^n = 3u_{j-1}^n - 3u_{j-2}^n + u_{j-3}^n$. Under these conditions, Model (2) has good simulation capability [10,11]. We refer to the experiment run under these conditions as the control experiment (CNTL); the experiment is described in Figure 1.

The effects of initial perturbation on the simulation capability are determined in the following five experiments (Table 1).

2 Results

Following the literature [12], we define the perturbation energy of the model (the domain-integrated difference total energy (DTE)) as

$$\text{DTE} = \frac{1}{2} \sum_{i=1}^N \delta u_i^2, \quad (3)$$

where δu is the spatial lattice point difference between CNTL and the perturbation experiments. DTE denotes the growth of error with time in the perturbation experiment relative to the control experiment. The results of the five perturbation experiments are shown in Figure 2.

Figure 2 shows that the growth trend of the error in the perturbation experiment is clearly nonlinear. After a certain time, similar to the occurrence of a shock wave, the model error tends to increase suddenly. At this time, the model loses its simulation capability completely. On the basis of the results in Figure 2, we further analyzed the relationship between the simulation capability and error growth. Figure 1 shows that the model has a perfect simulation capability in CNTL. If we take the length of the integral time as a standard,

Figure 1 Initial condition $u(x, 0) = x + 5.21078$.

Table 1 Perturbation experiments

Experiment	Purpose (To test the effect of initial perturbation with a different value)	Initial value
1	a large value	$u(x, 0) = x + 5.21078 + 10^{-1}$
2	a relatively large value	$u(x, 0) = x + 5.21078 + 10^{-2}$
3	a small value	$u(x, 0) = x + 5.21078 + 10^{-3}$
4	a relatively tiny value	$u(x, 0) = x + 5.21078 + 10^{-4}$
5	a tiny value	$u(x, 0) = x + 5.21078 + 10^{-5}$

Exp.1

Exp.2

Exp.3

Figure 2 Left: Results of CNTL and perturbation experiments; right: the time evolution of DTE.

Table 2 Score of the simulation capacity of the model in CNTL and perturbation experiments

CNTL	Exp.1	Exp.2	Exp.3	Exp.4	Exp.5
1	0.3205	0.842	0.993	0.935	0.993

the score of the model in CNTL is 1. The score of each perturbation experiment is given in Table 2.

Table 2 shows that, when the initial perturbation reduces to a certain level (10^{-3}), the simulation capability of the model cannot be enhanced further. In other words, the simulation capability is limited to a certain degree owing to the rapidly nonlinear increase in the error with time.

3 Conclusion

Using a one-dimensional nonlinear advection equation model,

we conducted a series of numerical experiments to study the effect of initial perturbations on the simulation capability. The major conclusions are summarized as follows.

(1) The effect of the initial perturbation on the simulation capability of the model is significant. The simulation capability tends to decrease gradually with an increase in error. When the error reaches a certain level, the model may lose its simulation capability completely.

(2) The initial perturbation error grows in a nonlinear manner. This indicates that even if the initial data are accurate or there is only a small error in the observational data, the simulation capability cannot be enhanced further with

more accurate initial values owing to a nonlinear increase in the error in the model.

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