Existence and construction of a quantum channel with given inputs and outputs

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We derive that sufficient and necessary conditions for existence of a quantum channel ϕ and a generalized unitary operation $\mathcal{E}$ sending A_i to B_i ($1 \leq i \leq k$) for two given families $\{A_i\}_{i=1}^k, \{B_i\}_{i=1}^k$ of matrices, respectively. As an application, a sufficient and necessary condition for existence of a unitary duality quantum computer with given input-output states is obtained.

existence, quantum channel, generalized unitary operation, duality quantum computer

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1 Introduction

The theory of quantum information and quantum computation [1] is a result of the effort to generalize classical information theory to the quantum world. According to quantum mechanics, a quantum system is described by a finite dimensional Hilbert space $\mathcal{H}$, a state of the system is described by a unit vector in $\mathcal{H}$, or a density operator (a positive operator of trace 1). An evolution of the system is called a quantum operation and denoted by a unitary operator (called a quantum gate) or a completely positive trace-preserving (CPTP) mapping (called a quantum channel). A convex combination of unitary operators is called a generalized quantum gate [2–4], which came from a duality quantum computer [5–14].

In quantum information science, it is necessary and useful to discuss the existence and construction of a quantum channel that maps a given family of density operators to another.

We consider a composite quantum system $\mathcal{H}_A \otimes \mathcal{H}_B$ composed of two parts, finite dimensional Hilbert spaces $\mathcal{H}_A$ and $\mathcal{H}_B$. The first example is the following.

Quantum cloning question (QCQ). Given a set $\{\rho_1, \rho_2, \dots, \rho_k\}$ of states on $\mathcal{H}_A$ and a state Σ on $\mathcal{H}_B$, to find a quantum channel $\phi: B(\mathcal{H}_A) \rightarrow B(\mathcal{H}_B)$ such that

$$\phi(\rho_s \otimes \Sigma) = \rho_s \otimes \rho_s (s = 1, \dots, k),$$

where $B(\mathcal{H})$ denotes the algebra of all bounded linear operators on $\mathcal{H}$. If such a ϕ exists, we call $\rho_1, \rho_2, \dots, \rho_k$ can be cloned simultaneously [15,16].

The second example is the so-called classical correlation question.

Classical correlation question (CCQ). Given a state ρ on $\mathcal{H}_A \otimes \mathcal{H}_B$, to find a von Neumann measurement $\Pi = \{\Pi_s^A \otimes \Pi_t^B : 1 \leq s \leq m, 1 \leq t \leq n\}$ such that

$$\Pi(\rho) := \sum_{s=1}^m \sum_{t=1}^n (\Pi_s^A \otimes \Pi_t^B) \rho (\Pi_s^A \otimes \Pi_t^B) = \rho,$$

where Π_s^A and Π_t^B are one-dimensional orthogonal projections on $\mathcal{H}_A$ and $\mathcal{H}_B$, respectively. If such a Π exists, we call the state ρ is classical correlated [17–22]. Gisin [23] presented a theorem in 1991 that states that any pure entangled two-qubit state violates the CHSH inequality. Specifically, the CHSH inequality is both sufficient and necessary for the separability of two-qubit states and any pure entangled quantum state violates one of Bell-like inequalities [24].

Thus, the following question arises naturally: given families $\{A_i\}_{i=1}^k \subset \mathcal{M}_N$ and $\{B_i\}_{i=1}^k \subset \mathcal{M}_M$ of matrices, to discuss the existence and construction of a quantum channel

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$\phi : \mathcal{M}_N \rightarrow \mathcal{M}_M$ such that $\phi(A_i) = B_i$ for every $1 \leq i \leq k$, $\mathcal{M}_{S,T}$ denotes the set of all $S \times T$ matrices and when $S = T$, write it as $\mathcal{M}_S$.

In Section 2, some preliminaries will be given. In Section 3, we will derive that sufficient and necessary conditions for existence of a quantum channel ϕ and a generalized unitary operation $\mathcal{E}$ sending A_i to B_i ($1 \leq i \leq k$) for two given families $\{A_i\}_{i=1}^k, \{B_i\}_{i=1}^k$ of matrices, respectively. In Section 4, a sufficient and necessary condition for existence of a unitary duality quantum computer with given input-output states will be obtained as an application.

2 Preliminaries

For a complex matrix A , $A^\dagger$ denotes the transpose of the complex conjugate of A . We say a square matrix A is Hermitian if $A = A^\dagger$, A is positive semi-definite if A is Hermitian and its eigenvalues are nonnegative. $\mathcal{M}_N(\mathcal{M}_M) = \mathcal{M}_N \otimes \mathcal{M}_M$ is the collection of all $N \times N$ block matrices with $M \times M$ matrices as entries. A linear map $\phi : \mathcal{M}_N \rightarrow \mathcal{M}_M$ is positive if $\phi(A)$ is positive semi-definite for any positive semi-definite matrix A in $\mathcal{M}_N$. Define $I_p \otimes \phi : \mathcal{M}_p(\mathcal{M}_N) \rightarrow \mathcal{M}_p(\mathcal{M}_M)$ by $(I_p \otimes \phi)([A_{jk}]) = [\phi(A_{jk})]$. Then we call ϕ is completely positive (CP) if $I_p \otimes \phi$ is positive for all positive integers p .

Consider an $N \times M$ rectangular matrix $A = [a_{ij}]$, we define the reshaping of A as

$$\text{res}(A) = (a_{11}, \dots, a_{1M}, a_{21}, \dots, a_{2M}, \dots, a_{N1}, \dots, a_{NM})^t, \quad (2.1)$$

where X^t denotes the transpose of a matrix X . Clearly, the length of $\text{res}(A)$ is NM .

Conversely, any vector of length NM may be reshaped into an $N \times M$ rectangular matrix. For example,

$$(a_1, a_2, a_3, a_4)^t \leftrightarrow \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}.$$

For arbitrary $MN \times ST$ matrix C , usually we use both indices to denote $C = [c_{ij}]$, but more conveniently, we will use the block-form of C as

$$C = [C_{m,n}] \in \mathcal{M}_{M,N}(\mathcal{M}_{S,T})$$

and then write as

$$C = [C_{m,n}] = [c_{m,n}^{\mu,\nu}], \quad (2.2)$$

where $c_{m,n}^{\mu,\nu}$ denotes the (μ, ν) -entry of (m, n) -block matrix $C_{m,n}$. For example,

$$C = \begin{pmatrix} \begin{matrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{matrix} & \begin{matrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{matrix} \\ \begin{matrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{matrix} & \begin{matrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{matrix} \end{pmatrix} = [c_{m,n}^{\mu,\nu}].$$

Especially, when $C = A \otimes B$, where

$$A = [a_{mn}] \in \mathcal{M}_{M,N}, B = [b_{\mu\nu}] \in \mathcal{M}_{S,T},$$

we have $c_{m,n}^{\mu,\nu} = a_{mn}b_{\mu\nu}$.

For arbitrary $MN \times ST$ matrix C defined by eq. (2.2), we define a block-matrix

$$C^K = [C_{m,\mu}^K] = [c_{m,\mu}^K], \quad (2.3)$$

where $c_{m,\mu}^K$ denotes the (n, ν) -entry of (m, μ) -block matrix $C_{m,\mu}^K$.

Let $\phi : \mathcal{M}_N \rightarrow \mathcal{M}_M$ be a completely positive map. Then from Choi's Theorem [25], we know that ϕ can be represented as

$$\phi(\rho) = \sum_{i=1}^k V_i \rho V_i^\dagger, \quad \forall \rho \in \mathcal{M}_N. \quad (2.4)$$

Now, we reshape V_i into a length MN column vector $\text{res}(V_i)$ following eq. (2.1). Define

$$V_\phi := (\text{res}(V_1), \text{res}(V_2), \dots, \text{res}(V_k)),$$

and put

$$D_\phi := V_\phi V_\phi^\dagger = \sum_{i=1}^k (\text{res}(V_i))(\text{res}(V_i))^\dagger, \quad (2.5)$$

which is an $MN \times MN$ matrix.

The following properties on matrix D_ϕ were essentially derived in [26].

Proposition 2.1 Suppose that $\phi : \mathcal{M}_N \rightarrow \mathcal{M}_M$ is a completely positive map, then

(1) the matrix D_ϕ defined by eq. (2.5) is uniquely determined by ϕ and positive semi-definite; moreover, $D_{a\phi_1+b\phi_2} = aD_{\phi_1} + bD_{\phi_2}$ holds for arbitrary positive numbers a, b and CP maps ϕ_1 and ϕ_2 .

(2) ϕ is unital if and only if $\text{tr}_2 D_\phi := \sum_{\mu=1}^M z_{n,\mu}^{m,\mu} = I_M$; and ϕ is trace-preserving if and only if $\text{tr}_1 D_\phi := \sum_{m=1}^M z_{m,\mu}^{m,\nu} = I_N$, where $\text{tr}_s X$ denotes that the partial trace of X for the s th system and I_M the $M \times M$ identity matrix.

(3) $\text{res}(\phi(\rho)) = D_\phi^\times \text{res}(\rho)$ for each $\rho \in \mathcal{M}_N$.

3 Main results

Theorem 3.1 Let $\{A_i\}_{i=1}^k \subset \mathcal{M}_N$ and $\{B_i\}_{i=1}^k \subset \mathcal{M}_M$. Then the following conditions are equivalent.

(a) There exists a quantum channel $\phi : \mathcal{M}_N \rightarrow \mathcal{M}_M$ such that $\phi(A_i) = B_i$ for every $1 \leq i \leq k$.

(b) There exists an $MN \times MN$ positive semi-definite matrix E such that $\text{tr}_1 E = I_N$ and

$$\text{res}(B_i) = E^K \text{res}(A_i) (1 \leq i \leq k). \quad (3.1)$$

Proof (a) $\Rightarrow$ (b) Suppose that there exists a quantum channel ϕ satisfying $\phi(A_i) = B_i$ for every $1 \leq i \leq k$. Take

$E = D_\phi$. Proposition 2.1 implies that E is an $MN \times MN$ positive semi-definite matrix with $\text{tr}_1 E = I_N$ and satisfies (3.1).

(b) $\Rightarrow$ (a) Suppose there exists an $MN \times MN$ positive semi-definite matrix E satisfying $\text{tr}_1 E = I_N$ and (3.1). First, E has a decomposition $E = FF^\dagger$ for some $MN \times MN$ matrix $F = [F_1, F_2, \dots, F_{MN}]$. Furthermore, we reshape the j th column F_j of F into an $M \times N$ matrix V_j for every $1 \leq j \leq MN$. Thus, $\text{res}(V_j) = F_j$ for all $1 \leq j \leq k$. Define $\phi(\rho) = \sum_{j=1}^{MN} V_j \rho V_j^\dagger$. Then Choi's Theorem reads $\phi: \mathcal{M}_N \rightarrow \mathcal{M}_M$ is a completely positive map. By (3.1) and Proposition 2.1(3), we get

$$\text{res}(B_i) = E^k \text{res}(A_i) = \text{res}(\phi(A_i))$$

for all $1 \leq i \leq k$. Lastly, by Proposition 2.1 (2) implies that ϕ is trace-preserving and then a quantum channel. $\square$

Example 1 Let

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}, A_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, B_2 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, B_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

It is easy to check that there exists only one matrix

$$E = \begin{pmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 1 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \end{pmatrix}$$

such that $\text{res}(B_i) = E \text{res}(A_i)$ ($i = 1, 2, 3, 4$). Since -1 is an eigenvalue of

$$E^k = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & 1 & -\frac{1}{2} & 0 \end{pmatrix},$$

E^k is not positive semi-definite. Theorem 3.1 implies that there does not exist a quantum channel ϕ such that $\phi(A_i) = B_i$ ($i = 1, 2, 3, 4$).

Example 2 Let

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, A_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}, B_2 = \begin{pmatrix} 2 & \frac{1}{2} \\ \frac{1}{2} & 2 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, B_4 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

It is easy to check that the only positive semi-definite matrix E satisfying

$$\text{res}(B_i) = E^k \text{res}(A_i) (i = 1, 2, 3, 4) \text{ and } \text{tr}_1 E = I_2$$

is

$$E = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

From Theorem 3.1, we see that there exists a unique quantum channel ϕ such that $\phi(A_i) = B_i$ ($i = 1, 2, 3, 4$).

A quantum channel $\mathcal{E}$ on $\mathcal{M}_N$ is said to be a generalized unitary operation (GUO) if it can be written as

$$\mathcal{E}(X) = \sum_{j=1}^d |p_j|^2 U_j X U_j^\dagger, \quad \forall X \in \mathcal{M}_N$$

for some $N \times N$ unitary matrices U_j and scalars p_j with $\sum_{j=1}^d |p_j|^2 = 1$.

Clearly, every GUO $\mathcal{E}$ preserves the unit: $\mathcal{E}(I_N) = I_N$, equivalently, $\text{tr}_2(D_{\mathcal{E}}) = I_N$ (Proposition 2.1(2)).

In Example 2, since $\text{tr}_2(E) = \text{tr}_2(D_\phi) \neq I_2$, the unique quantum channel ϕ mapping each A_i onto B_i is not a GUO. This shows that there does not exist a GUO ϕ on $\mathcal{M}_2$ sending the given A_i to B_i for all $i = 1, 2, 3, 4$.

Next, let us discuss the existence of generalized unitary operations that send a given family of matrices to another.

Theorem 3.2 Let $\{A_i\}_{i=1}^k \subset \mathcal{M}_N$ and $\{B_i\}_{i=1}^k \subset \mathcal{M}_N$. Then there exists a generalized unitary operation $\mathcal{E}$ such that $\mathcal{E}(A_i) = B_i$ ($i = 1, 2, \dots, k$) if and only if there exists a group $\{\mathbf{a}_j\}$ of column vectors such that matrices $E_j := \mathbf{a}_j \mathbf{a}_j^\dagger$ ($1 \leq j \leq d$) satisfies $\text{tr}_s E_j = I_N$ ($s = 1, 2$) and there exists a probability distribution $\{q_j : j = 1, 2, \dots, d\}$ such that the matrix $E = \sum_{j=1}^d q_j E_j$ satisfies condition (3.1).

Proof Necessity. Suppose that there exists a generalized unitary operation $\mathcal{E} : X \mapsto \sum_j |p_j|^2 U_j X U_j^\dagger$ such that

$$B_i = \mathcal{E}(A_i) = \sum_{j=1}^d |p_j|^2 U_j A_i U_j^\dagger (i = 1, 2, \dots, k). \quad (3.2)$$

Put $\mathbf{a}_j = \text{res}(U_j)$ and $E_j = \mathbf{a}_j \mathbf{a}_j^\dagger$. Then

$$\text{res}(U_j \rho U_j^\dagger) = E_j^k \text{res}(\rho) (\forall \rho \in \mathcal{M}_N), \text{tr}_s E_j = I_N \quad (3.3)$$

for all j and $s = 1, 2$. Let $E = \sum_{j=1}^d |p_j|^2 E_j$. Then (3.3) implies that (3.1) holds.

Sufficiency. Suppose that there exists a sequence of matrices $E_j = \mathbf{a}_j \mathbf{a}_j^\dagger$ with $\text{tr}_s E_j = I_N (s = 1, 2)$, where $\mathbf{a}_j$ denote some column vectors, such that (3.1) holds for $E = \sum_{j=1}^d q_j E_j$ with $\sum_{j=1}^d q_j = 1$ and $q_j \geq 0$. Then we reshape the $\mathbf{a}_j$ into an $N \times N$ matrix U_j such that $\text{res}(U_j) = \mathbf{a}_j$ for every j . Then it follows from (3.1) that $B_i = \sum_{j=1}^d q_j U_j A_i U_j^\dagger$ for $i = 1, 2, \dots, k$. Define $p_j = \sqrt{q_j}$ and

$$\mathcal{E}(X) = \sum_{j=1}^d p_j^2 U_j X U_j^\dagger, \quad \forall X \in \mathcal{M}_N.$$

Then we get a generalized unitary operation $\mathcal{E}$ satisfying $\mathcal{E}(A_i) = B_i$ for all i . $\square$

Example 3 Let

$$A_1 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 & \frac{2}{3} \\ \frac{2}{3} & 1 \end{pmatrix}, B_2 = \begin{pmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{1}{3} \end{pmatrix}.$$

It is easy to check that the matrix

$$\begin{aligned} E &= \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} (1 \ 0 \ 0 \ 1) + \frac{1}{3} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} (0 \ 1 \ 1 \ 0) \\ &\quad + \frac{1}{6} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} (1 \ 0 \ 0 \ -1) \\ &:= \frac{1}{2} E_1 + \frac{1}{3} E_2 + \frac{1}{6} E_3 \end{aligned}$$

satisfies $\text{res}(B_i) = E^\kappa \text{res}(A_i)$ for $i = 1, 2$ and $\text{tr}_s E_j = I_2$ for $j = 1, 2, 3, s = 1, 2$. Thus, Theorem 3.2 yields that there exists a generalized unitary operation ϕ on $\mathcal{M}_2$ such that $\phi(A_i) = B_i (i = 1, 2)$. From the proof of Theorem 3.2, we see that the GUO ϕ is

$$\phi : X \mapsto \frac{1}{2} U_1 X U_1^\dagger + \frac{1}{3} U_2 X U_2^\dagger + \frac{1}{6} U_3 X U_3^\dagger (i = 1, 2),$$

where

$$U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, U_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, U_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

which are unitary matrices.

4 An application to duality quantum computers

Based on the general principle of quantum interference, Long proposed in [5,6] a new type of computing machines called duality quantum computers. Gudder has given the mathematical theory for the duality quantum computer [9,10]. For the further discussions and some applications of duality quantum computers, please refer to [7,8,12–14]. In [13], based on Long's idea of duality quantum computers, we proposed complex duality quantum computers (CDQCs) acting on vector-states and operator-states. By [13], a CDQC acting on operator-states is a mapping of the form

$$\mathbf{C}_{\overline{p}} \circ \left(\bigoplus_{i=0}^{n-1} \varepsilon_i \right) \circ \mathbf{D}_p : B(\mathcal{H}) \rightarrow B(\mathcal{H}), \quad (4.1)$$

where $p = (p_0, p_1, \dots, p_{n-1}) \in \mathbb{C}^n$ with $\sum_{i=0}^{n-1} |p_i|^2 = 1$, $\{\varepsilon_0, \varepsilon_1, \dots, \varepsilon_{n-1}\}$ a group of CPTP mappings on $B(\mathcal{H})$ and operators

$$\mathbf{D}_p : B(\mathcal{H}) \rightarrow B(\mathcal{H})^n, \mathbf{C}_{\overline{p}} : B(\mathcal{H})^n \rightarrow B(\mathcal{H}) \quad (4.2)$$

are given by

$$\mathbf{D}_p(T) = \bigoplus_{i=0}^{n-1} p_i T, \mathbf{C}_{\overline{p}} \left(\bigoplus_{i=0}^{n-1} T_i \right) = \sum_{i=0}^{n-1} \overline{p_i} T_i. \quad (4.3)$$

Given a CDQC (4.1), we define

$$\mathbf{U}_p = \mathbf{C}_{\overline{p}} \left(\bigoplus_{i=0}^{n-1} \varepsilon_i \right) \mathbf{D}_p : B(\mathcal{H}) \rightarrow B(\mathcal{H}). \quad (4.4)$$

Then $\mathbf{U}_p(T) = \sum_{i=0}^{n-1} |p_i|^2 \varepsilon_i(T)$ for all $T \in B(\mathcal{H})$. That is,

$$\mathbf{U}_p = \sum_{i=0}^{n-1} |p_i|^2 \varepsilon_i. \quad (4.5)$$

Clearly, $\mathbf{U}_p$ is a quantum channel on $B(\mathcal{H})$. Especially, in the case where ε_i is the unitary isomorphism defined by $\varepsilon_i(X) = U_i X U_i^\dagger$ for all i , the $\mathbf{U}_p$ reads

$$\mathbf{U}_p(T) = \sum_{i=0}^{n-1} |p_i|^2 U_i T U_i^\dagger, \quad \forall T \in B(\mathcal{H})$$

and therefore is a generalized unitary operation on $B(\mathcal{H})$. Moreover, we say that a CDQC (4.1) is a unitary duality quantum computer (UDQC) if ε_i 's are unitary isomorphisms.

In the case that $\dim(\mathcal{H}) = N$, $B(\mathcal{H}) = \mathcal{M}_N$. Given input states $\{A_i\}_{i=1}^k \subset \mathcal{M}_N$ and output states $\{B_i\}_{i=1}^k \subset \mathcal{M}_N$, Theorem 3.2 gives indeed a necessary and sufficient condition for a UDQC such that $\mathbf{U}_p(A_i) = B_i (i = 1, 2, \dots, k)$ to exist.

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