Amplifying stationary quantum discord and entanglement between a superconducting qubit and a data bus by time-dependent electromagnetic field

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We study the dynamics of quantum discord and entanglement between a superconducting qubit and a data bus, which is driven by a controllable time-dependent electromagnetic field, in the presence of phase decoherence and find that the quantum discord and entanglement remain at a stationary non-zero value for long time evolution. It is shown that the amount of stationary quantum discord and entanglement can be enhanced by applying the time-dependent electromagnetic field.

stationary quantum discord, stationary entanglement, time-dependent electromagnetic field

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Quantum entanglement plays a crucial role in quantum information and computation. It is a special quantum correlation and has been recognized as an essential resource in quantum information processing [1]. The experimental demonstrations of two-particle entanglement and multi-particle entanglement in the cavity quantum electrodynamics have been reported [2,3] and some applications about the entanglement or the nonclassical correlations have also been realized in recent experiments [4–6]. However, some recent studies show that there are other kinds of quantum correlations that can offer support for the quantum tasks. It has been found both theoretically [7–9] and experimentally [10,11] that other nonclassical correlation, namely, quantum discord [12] can be responsible for computational speedup for certain quantum tasks [13–19] compared to their classical counterparts. Quantum discord, introduced by Ollivier and Zurek [12], is a general measure of quantum correlation and defined as the difference between the quantum mutual information and the classical correlation. The main difference between the quantum discord and entanglement is the fact that the quantum discord is nonzero even for separate mixed states. Therefore,

the quantum discord may be regarded as a more general and fundamental resource for the quantum information processing. Recently, much attention has been focused on the study of the dynamics of quantum discord for some open systems [15,20,21]. It has been shown that the quantum discord can be completely unaffected by the certain decoherence environment during an initial time interval [22] and this phenomenon has been verified by the recent experiment [23].

On the other hand, the superconducting qubits have been considered as possible candidates for quantum information processing and attracted much attention in recent years [24–31]. It has been experimentally demonstrated that the superconducting qubits possess macroscopic quantum coherence and can be used to construct the conditional two-qubit gate. It is necessary to explore possibility for scaling up to many qubits in order to perform the complex quantum information task. Several schemes have been proposed to selectively couple any pair of qubits with a common data bus [26–29]. In [30], Liu et al. proposed a method to use a controllable time-dependent electromagnetic field to couple a superconducting qubit and data bus, where quantum information can be transferred from one qubit to another.

In this paper, we investigate the dynamics of quantum

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discord and entanglement between a data bus and a superconducting qubit, which is driven by a controllable time-dependent electromagnetic field (TDEF), in the presence of phase decoherence and find that the quantum discord and entanglement remain at a stationary non-zero value for long time evolution. This means that the stationary quantum discord and entanglement can arise in the interaction of the superconducting qubit and data bus as the time approach to infinite. We also examine the influence of the TDEF on the dynamics of quantum discord and entanglement between the superconducting qubit and data bus and show that the amount of stationary quantum discord and entanglement can be increased by applying the TDEF.

1 Time evolution of superconducting qubit system driven by an electromagnetic field with phase decoherence

We now consider a superconducting flux qubit interacting with a data bus and a controllable TDEF. The data bus can be realized simply by an LC circuit and the interaction between a single superconducting flux qubit and a LC circuit has been realized experimentally. The Hamiltonian of the system can be described as [30,31] (note that we have set $\hbar = 1$ in this paper)

$$H = H_0 + H_F, \quad (1)$$

$$H_0 = \frac{\omega_q}{2} \sigma_z + \frac{\omega^2 L Q^2}{2} + \frac{\omega^2 C \tilde{\phi}^2}{2} + \left[\chi \left(\sqrt{\frac{\omega C}{2}} \tilde{\phi} - i \sqrt{\frac{\omega L}{2}} Q \right) \sigma_+ + H.c. \right], \quad (2)$$

$$H_F = \lambda (e^{-i\omega_e t} \sigma_+ + e^{i\omega_e t} \sigma_-), \quad (3)$$

where $H.c.$ means Hermitian conjugate, $\omega = 1/\sqrt{LC}$ (with the the capacitor C and the self-inductance L) and $\omega_q = \frac{1}{2} \sqrt{I^2 (\Phi_e - \frac{\Phi_0}{2})^2 + T_{RL}^2}$ are the frequency of data bus and qubit, respectively. Here, Φ_e is the externally applied flux, Φ_0 is the flux quantum, I is the qubit loop-current and the parameter T_{RL} denotes the tunnel coupling between the two potential wells of the qubit. The H_F denotes a controllable TDEF and ω_e is the frequency of the TDEF, λ is the coupling constants of the interactions of qubit with the TDEF. The magnetic flux $\tilde{\phi}$ through the LC circuit and the charge Q operators obey the commutation relation $[Q, \tilde{\phi}] = i$. The operators σ_z and $\sigma_{\pm}$ are defined by $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, where $|e\rangle$ and $|g\rangle$ are the excited state and ground state of the qubit. χ is the coupling constant between the qubit and the data bus and can be written as $\chi = M \sqrt{\frac{\omega}{2L}} \langle e|I|g \rangle$ (in this paper we have set χ to be a real number so that $\chi = \chi^*$).

In the rotating reference frame, through an unitary trans-

formation $U = U_2 U_1$, where

$$U_1 = \exp\left(-\frac{i\omega_e t}{2} \sigma_z\right), \quad (4)$$

$$U_2 = \exp\left[-\frac{i\omega_e t}{2} \left(\frac{\omega_q - \omega_e}{2\delta} \sigma_z + \frac{\lambda}{\delta} \sigma_x\right)\right],$$

the Hamiltonian of eq. (2) is transformed to [30]

$$H' = \frac{\omega L Q^2}{2} + \frac{\omega C \tilde{\phi}^2}{2} + \frac{\delta + \omega_e}{2} \left(\frac{\omega_q - \omega_e}{\delta} \sigma_z + \frac{2\lambda}{\delta} \sigma_x \right) + \left[\frac{\chi(\delta + \omega_q - \omega_e)}{2\delta} \left(\sqrt{\frac{\omega C}{2}} \tilde{\phi} - i \sqrt{\frac{\omega L}{2}} Q \right) \otimes \left(\frac{\omega_q - \omega_e}{2\delta} \sigma_x + \frac{i}{2} \sigma_y - \frac{\lambda}{\delta} \sigma_z \right) + H.c. \right], \quad (5)$$

where $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$, $\sigma_y = -i(|e\rangle\langle g| - |g\rangle\langle e|)$ and $\delta = \sqrt{(\omega_q - \omega_e)^2 + 4\lambda^2}$.

We consider the pure phase decoherence mechanism only. In this situation, the master equation governing the time evolution for the quantum system with phase decoherence is given by [32]

$$\frac{d\rho}{dt} = -i[H', \rho] - \frac{\gamma}{2} [H', [H', \rho]], \quad (6)$$

where γ is the phase decoherence coefficient. Eq. (6) reduces to the ordinary von Neuman equation when $\gamma \rightarrow 0$. The formal solution of the master equation can be expressed as [32]

$$\rho(t) = \sum_{k=0}^{\infty} \frac{(\gamma t)^k}{k!} M^k(t) \rho(0) M^{\dagger k}(t), \quad (7)$$

where $\rho(0)$ is the initial density operator of the system and $M^k(t)$ is defined by

$$M^k(t) = H'^k \exp(-iH't) \exp\left(-\frac{\gamma t}{2} H'^2\right). \quad (8)$$

With the help of the $SU(2)$ dynamical algebraic structure, we can rewrite H' as [33]

$$H' = \Delta S_0 + \omega \left(K - \frac{1}{2} \right) + \frac{\sqrt{K}}{2} \left(1 + \frac{\omega_q - \omega_e}{\delta} \right) \chi (S_+ + S_-), \quad (9)$$

where $\Delta = \delta + \omega_e - \omega$, $K = \frac{1}{2} \left(\frac{\omega_q - \omega_e}{\delta} \sigma_z + \frac{2\lambda}{\delta} \right) + \frac{1}{2} (L\tilde{\phi}^2 + LQ^2)\omega$ is a constant of motion which commutes with the Hamiltonian H' and the operators

$$S_+ = \sqrt{\frac{1}{K}} \left(\sqrt{\frac{\omega C}{2}} \tilde{\phi} - i \sqrt{\frac{\omega L}{2}} Q \right) \left(\frac{\omega_q - \omega_e}{2\delta} \sigma_x + \frac{i}{2} \sigma_y - \frac{\lambda}{\delta} \sigma_z \right),$$

$$S_- = \sqrt{\frac{1}{K}} \left(\sqrt{\frac{\omega C}{2}} \tilde{\phi} + i \sqrt{\frac{\omega L}{2}} Q \right) \left(\frac{\omega_q - \omega_e}{2\delta} \sigma_x - \frac{i}{2} \sigma_y - \frac{\lambda}{\delta} \sigma_z \right),$$

$$S_0 = \frac{1}{2} \left(\frac{\omega_q - \omega_e}{\delta} \sigma_z + \frac{2\lambda}{\delta} \sigma_x \right), \quad (10)$$

satisfy the following commutation relations:

$$[S_0, S_{\pm}] = \pm S_{\pm}, [S_+, S_-] = 2S_0, \quad (11)$$

which means that the operators S_i ($i = 0, \pm$) are the generators of the $SU(2)$ algebra.

By means of the $SU(2)$ dynamical algebraic structure, we can diagonalize the Hamiltonian in eq. (9) and obtain the explicit expression for the operator $M^k(t)$ in eq. (8)

$$\begin{aligned} M^k(t) = & \frac{1}{2} \left[f_+(K)^k \exp(-if_+(K)t) \exp\left(-\frac{\gamma f_+(K)^2 t}{2}\right) \right. \\ & + f_-(K)^k \exp(-if_-(K)t) \exp\left(-\frac{\gamma f_-(K)^2 t}{2}\right) \\ & + \left[f_+(K)^k \exp(-if_+(K)t) \exp\left(-\frac{\gamma f_+(K)^2 t}{2}\right) \right. \\ & \left. - f_-(K)^k \exp(-if_-(K)t) \exp\left(-\frac{\gamma f_-(K)^2 t}{2}\right) \right] \\ & \otimes \left[\frac{\Delta}{4\Omega_K} (2 \cos \theta \sigma_z + \sin \theta \sigma_x) \right. \\ & + \frac{\chi \cos^2 \frac{\theta}{2}}{2\Omega_K} (a(-\sin \theta \sigma_z + \cos \theta \sigma_x + i\sigma_y) \\ & \left. + a^\dagger(-\sin \theta \sigma_z + \cos \theta \sigma_x - i\sigma_y)) \right], \end{aligned} \quad (12)$$

where

$$\begin{aligned} \Omega_K &= \sqrt{\Delta^2 + 4K\left(\chi \cos^2 \frac{\theta}{2}\right)^2}, \\ f_\pm(K) &= \omega\left(K - \frac{1}{2}\right) \pm \frac{1}{2}\Omega_K, \\ \theta &= \arctan\left(\frac{2\lambda}{\omega_q - \omega_e}\right) \end{aligned} \quad (13)$$

and the operators

$$\begin{aligned} a &= \sqrt{\frac{\omega C}{2}} \tilde{\phi} - i \sqrt{\frac{\omega L}{2}} Q, \\ a^\dagger &= \sqrt{\frac{\omega C}{2}} \tilde{\phi} + i \sqrt{\frac{\omega L}{2}} Q, \end{aligned} \quad (14)$$

satisfy the commutation relation $[a, a^\dagger] = 1$.

We assume that the superconducting qubit is initially prepared in the dressed state $|+\rangle = \cos \frac{\theta}{2}|e\rangle + \sin \frac{\theta}{2}|g\rangle$ and the data bus is initially prepared in the fock state $|n\rangle$. Then, the initial density operator of the whole system reads

$$\rho(0) = |+\rangle|n\rangle\langle+|\langle n|. \quad (15)$$

By making use of the eqs. (7), (12), (14), (15), we obtain the density matrix of the system at time t by means of the standard basis $|-\rangle|n\rangle$, $|-\rangle|n+1\rangle$, $|+\rangle|n\rangle$, $|+\rangle|n+1\rangle$ (where $|+\rangle = \cos \frac{\theta}{2}|e\rangle + \sin \frac{\theta}{2}|g\rangle$, $|-\rangle = -\sin \frac{\theta}{2}|e\rangle + \cos \frac{\theta}{2}|g\rangle$), as follows:

$$\rho(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix},$$

where

$$\rho_{11} = \rho_{44} = \rho_{14} = \rho_{41} = 0,$$

$$\begin{aligned} \rho_{22} &= \frac{2(\chi \cos^2 \frac{\theta}{2})^2(n+1)}{\Omega_{n+1}^2} \left[1 - \cos(\Omega_{n+1}t) \exp\left(-\frac{\gamma \Omega_{n+1}^2 t}{2}\right) \right], \\ \rho_{33} &= \frac{1}{2} \left[1 + \frac{\Delta^2}{\Omega_{n+1}^2} + \left(1 - \frac{\Delta^2}{\Omega_{n+1}^2}\right) \cos(\Omega_{n+1}t) \exp\left(-\frac{\gamma \Omega_{n+1}^2 t}{2}\right) \right], \\ \rho_{32} &= (\rho_{23})^* = \frac{\chi \cos^2 \frac{\theta}{2} \sqrt{n+1}}{\Omega_{n+1}} \left[\frac{\Delta}{\Omega_{n+1}} + (i \sin(\Omega_{n+1}t) \right. \\ & \left. - \frac{\Delta}{\Omega_{n+1}} \cos(\Omega_{n+1}t)) \exp\left(-\frac{\gamma \Omega_{n+1}^2 t}{2}\right) \right]. \end{aligned} \quad (16)$$

It is obvious that the density matrix $\rho(t)$ belongs to the class of X structure states. The advantage of X structure states is that, not only it is easy to calculate the quantum discord and entanglement, but it keeps the X structure unchanged during time evolution as well [34].

2 The dynamics of quantum discord and entanglement for the superconducting qubit system driven by an electromagnetic field with phase decoherence

In this section, we study the influence of phase decoherence and TDEF on the dynamics of quantum discord and entanglement of a superconducting qubit interacting with a data bus in the presence of phase decoherence. The definition of quantum discord is based on quantum mutual information which contains both classical and quantum correlations. For a bipartite system ρ^{AB} , its total correlations can be measured by their quantum mutual information [12]:

$$I(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}), \quad (17)$$

where $S(\rho) = -Tr(\rho \log_2 \rho)$ is the von Neumann entropy, and ρ^A and ρ^B denote the reduced density matrices of parts A and B , respectively. The quantum discord is defined as the difference between the quantum mutual information and the classical correlation [12]:

$$\mathcal{D}(\rho^{AB}) = I(\rho^{AB}) - C(\rho^{AB}), \quad (18)$$

where $C(\rho^{AB})$ is the classical correlation which depends on the maximal information obtained with measurement on one of the subsystems and can be expressed as [34]

$$C(\rho^{AB}) = \max_{\{B_k\}} [S(\rho^A) - S(\rho^{AB}|\{B_k\})], \quad (19)$$

where $\{B_k\}$ is a complete set of projectors preformed on subsystem B locally, $S(\rho^{AB}|\{B_k\}) = \sum_k p_k S(\rho_k)$ is the quantum conditional entropy, $\rho_k = 1/p_k (I \otimes B_k) \rho^{AB} (I \otimes B_k)$ is the conditional density operator and $p_k = tr_{(AB)}[(I \otimes B_k) \rho^{AB} (I \otimes B_k)]$ is the probability.

The eigenvalues of the density matrix $\rho(t)$ for the qubit-data bus system with TDEF applied under the phase decoher-

ence can be derived as

$$\begin{aligned}\lambda_{1,2} &= \frac{1}{2} \left[\rho_{11} + \rho_{44} \pm \sqrt{(\rho_{11} - \rho_{44})^2 + 4|\rho_{14}|^2} \right], \\ \lambda_{3,4} &= \frac{1}{2} \left[\rho_{22} + \rho_{33} \pm \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{23}|^2} \right].\end{aligned}\quad (20)$$

Then, the quantum mutual information can be expressed as

$$I(\rho(t)) = S(\rho^{(1)}(t)) + S(\rho^{(2)}(t)) + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i, \quad (21)$$

where

$$\begin{aligned}S(\rho^{(1)}(t)) &= -(\rho_{11} + \rho_{22}) \log_2(\rho_{11} + \rho_{22}) \\ &\quad - (\rho_{33} + \rho_{44}) \log_2(\rho_{33} + \rho_{44}), \\ S(\rho^{(2)}(t)) &= -(\rho_{11} + \rho_{33}) \log_2(\rho_{11} + \rho_{33}) \\ &\quad - (\rho_{22} + \rho_{44}) \log_2(\rho_{22} + \rho_{44}),\end{aligned}\quad (22)$$

where $\rho^{(i)}(t)$ is the reduced density matrix of the i th partite of the qubit-data bus system. By substituting ρ_{ij} ($i, j = 1, 2, 3, 4$) of eq. (16) into eqs. (18)–(22), the explicit expressions of quantum discord $\mathcal{D}(\rho(t))$ and the quantum mutual information $I(\rho(t))$ can be obtained [34]. Here we do not list the concrete expressions of the quantum discord and quantum mutual information because it is rather long and complex.

In order to quantify the entanglement dynamics of the qubit-data bus system and make a comparison with the quantum discord dynamics, we adopt the measure concurrence which is defined by [35]

$$E = \max \{0, \sqrt{\eta_1} - \sqrt{\eta_2} - \sqrt{\eta_3} - \sqrt{\eta_4}\}, \quad (23)$$

where the η_i ($i=1, 2, 3, 4$) are the eigenvalues in decreasing order of the magnitude of the “spin-flipped” density matrix operator $R = \rho(\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$ and σ_y is the Pauli Y matrix, i.e., $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. Combining the definition of

concurrence with the density matrix $\rho(t)$ of eq. (16), we can find the explicit expressions of the concurrence between the superconducting qubit and data bus:

$$E(t) = 2 \max(0, |\rho_{32}|). \quad (24)$$

We plot the quantum discord (a) and entanglement (b) between the superconducting qubit and data bus in Figure 1 as a function of the evolution time t and the decoherence coefficient γ with $\chi = 1, \omega = 1, \omega_q = 2, \omega_e = 0, \lambda = 0, n = 0$. It can be seen from Figure 1 that the quantum discord and entanglement are always larger than zero during the time evolution if $\gamma \neq 0$ and $\omega \neq \omega_q$, which means that the phase decoherence does not completely destroy the quantum discord and entanglement but a stationary quantum discord and entanglement can arise in the interaction of the superconducting qubit and data bus for long-time evolution.

In Figure 2, the quantum discord (a) and entanglement (b) between the superconducting qubit and data bus are plotted as a function of the evolution time t for two different values of ω_q : $\omega_q = 1$ (solid line), $\omega_q = 2$ (dashed line) with $\chi = 1, \omega = 1, \omega_e = \lambda = 0, \gamma = 0.05, n = 0$. We can see from Figure 2 that the quantum discord and entanglement between the superconducting qubit and data bus oscillate and eventually become zero when $\omega = \omega_q$ in the presence of phase decoherence. But the quantum discord and entanglement between the superconducting qubit and data bus are not completely destroyed and remain at a stationary non-zero value in the case of $\omega \neq \omega_q$ when the time approaches infinite.

In order to study the influence of TDEF on the dynamics of quantum discord and entanglement between the superconducting qubit and data bus, we plot the discord (a) and entanglement (b) between the superconducting qubit and data bus in Figure 3 as a function of the evolution time t for different values of ω_e and λ : $\omega_e = \lambda = 0$ (solid line), $\omega_e = 1, \lambda = 0.65$ (dashed line) with $\chi = 1, \omega = 1, \omega_q = 2, \gamma = 0.05, n = 0$. By comparing the solid line and dashed line in Figure 3(a) and (b), we can see clearly that the amount of stationary quantum

Figure 1 The quantum discord (a) and entanglement (b) between the superconducting qubit and data bus are plotted as a function of the evolution time t and the decoherence coefficient γ with $\chi = 1, \omega = 1, \omega_q = 2, \omega_e = 0, \lambda = 0, n = 0$.

Figure 2 (Color online) The quantum discord (a) and entanglement (b) between the superconducting qubit and data bus are plotted as a function of the evolution time t for different values of ω_q : $\omega_q = 1$ (solid line), $\omega_q = 2$ (dashed line) with $\chi = 1, \omega = 1, \omega_e = \lambda = 0, \gamma = 0.05, n = 0$.

Figure 3 (Color online) The quantum discord (a) and entanglement (b) between the superconducting qubit and data bus are plotted as a function of the evolution time t for different values of ω_e and λ : $\omega_e = \lambda = 0$ (solid line), $\omega_e = 1, \lambda = 0.65$ (dashed line) with $\chi = 1, \omega = 1, \omega_q = 2, \gamma = 0.05, n = 0$.

discord and entanglement between the superconducting qubit and data bus can be increased by adjusting the frequency ω_e and strength λ of the TDEF in the presence of phase decoherence.

Finally, the influence of the initial state of data bus on the dynamics of quantum discord and entanglement between the superconducting qubit and data bus are studied as the quantum discord (a) and entanglement (b) are plotted as a function of the evolution time t in Figure 4 for different values of n : $n = 0$ (solid line), $n = 1$ (dashed line) with $\chi = 1, \omega = 1, \omega_q = 2, \omega_e = \lambda = 0, \gamma = 0.05$. It is found that the oscillation of quantum discord and entanglement end earlier and the stationary amount of quantum discord and en-

tanglement become smaller as n increases. Therefore, in order to generate optimal amount of stationary quantum discord and stationary entanglement between the qubit and data bus, it would be best the initial state of data bus is prepared in $|0\rangle$.

3 Conclusions

In this paper, we investigate the dynamics of quantum discord and entanglement between a superconducting qubit and a data bus, which is driven by a controllable TDEF, in the presence of phase decoherence. It is found that the quantum discord and entanglement between the superconducting

Figure 4 (Color online) The quantum discord (a) and entanglement (b) between the superconducting qubit and data bus are plotted as a function of the evolution time t for different values of n : $n = 0$ (solid line), $n = 1$ (dashed line) with $\chi = 1, \omega = 1, \omega_q = 2, \omega_e = \lambda = 0, \gamma = 0.05$.

qubit and data bus rapidly decay with phase decoherence in the case of $\omega = \omega_q$. However, in the case of $\omega \neq \omega_q$, the quantum discord and entanglement between the superconducting qubit and data bus are not completely destroyed by the phase decoherence, which means that a stationary quantum discord and entanglement can arise in the interaction of the superconducting qubit with data bus as the time approach to infinite. We also examine the influence of the TDEF on the dynamics of quantum discord and entanglement between the superconducting qubit and data bus and show that the amount of stationary quantum discord and entanglement can be increased by applying the TDEF. Finally we study the influence of the initial state of data bus on the dynamics of quantum discord and entanglement between the superconducting qubit and data bus and show that the oscillation of quantum discord and entanglement end earlier and the stationary amount of quantum discord and entanglement become smaller as n increases. The approach adopted here may be extended to improve the implementation of quantum tasks which based on the quantum correlations.

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