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Phase relationship between the relative sunspot number and solar 10.7 cm flux

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A range of analysis approaches, namely continuous wavelet, cross wavelet, and wavelet coherence analyses, are employed to clarify the phase relationship between the smoothed monthly mean sunspot number and solar 10.7 cm flux (F10.7). Analysis shows that there is a region of high spectral power sitting across the Schwabe cycle belt, where the two time series are in phase. However, analysis of the cross-wavelet transform and wavelet coherence unveils asynchronous behavior featured with phase mixing in the high-frequency components of sunspot activity and solar F10.7, which may explain the different activity properties of the photosphere and corona on a short time scale.

sunspot number, solar 10.7 cm flux, cross wavelet, wavelet coherence

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Relative sunspot numbers have been recorded for several hundred years, with monthly mean sunspot data being available from the year 1749. Relative sunspot numbers have frequently been used to describe the level of solar activity. Additionally, the wavelet transform is widely employed to determine the time-frequency of localized and quasi-periodic fluctuations in a time series. Analysis of sunspot number wavelets has led to a belief that solar activity has cycles at different time scales [1,2]. For example, solar activity has a series of cycles with periods of, for example, 27 days, 1.3, 1.7, 2, 5, 11 and 22 years, and even longer [3–5]. The Schwabe and Gleissberg cycles in solar activity have been obtained by wavelet analysis of the relative sunspot number [6,7]. The scope of the Schwabe cycle of the relative sunspot number and the phase relationship between the relative sunspot number and the sunspot group number in the 11-year cycle have also been studied [8]. The Schwabe cycle stands out among the solar cycles.

A sunspot is a phenomenon observed in the solar photosphere. The chromosphere is a layer above the photosphere. The corona is the outermost atmosphere of the Sun. The electromagnetic radiation of the Sun at a frequency of 2800 MHz having wavelength 10.7 cm is an important parameter for measuring the level of solar activity. The intensity of solar F10.7 is expressed by solar radiation flux at a wavelength of 10.7 cm. Observations of F10.7 began in 1947 and are available for more than 60 years. The value of solar F10.7 is low when solar activity is low and high when solar activity is high, with the correlation between the solar F10.7 and relative sunspot number exceeding 0.98 [1,9,10]. The solar F10.7 reflects the activity of the corona, while the relative sunspot number mirrors the activity of the photosphere. It has been questioned whether the levels of solar atmospheric activity in two different layers agree with one another. Considering that relative sunspot numbers and F10.7 are two different series, one has to analyze the phase relation-

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ship between the two using different methodologies, including the cross-recurrence plot and line of synchronization techniques [11,12] and scale-resolved phase coherence analysis [13]. The cross-wavelet transform (XWT) and wavelet coherence (WTC) have been widely employed to study the phase relationship between two time series [12, 14–17]. In this study, wavelet analysis was first carried out to clarify the periodic features of F10.7 and the smoothed monthly mean relative sunspot number. Afterward, the XWT and WTC techniques were employed to depict the phase relationship between the F10.7 and sunspot number at different time scales, in an attempt to clarify synchronization and asynchronization between photospheric activity and coronal activity.

The data of F10.7 used in this paper are adjusted solar radio data recorded at Penticton, B. C., Canada. The unit of F10.7 is the solar flux unit (sfu), where 1 sfu = 10^{-22} W m^{-2} Hz^{-1}.

1 Data analysis

1.1 Continuous wavelet analysis

The smoothed monthly mean F10.7 and relative sunspot number during the period 1947–2010 are shown in Figure 1. The blue and red lines represent the monthly mean F10.7 and relative sunspot number respectively. The figure shows that the smoothed monthly mean F10.7 and relative sunspot number do not always have coinciding maxima and minima. For example, the sunspot number in solar cycle 20 peaked in November 1968 while F10.7 peaked in July 1970; i.e., there was a 20-month gap between the two peaks. Table 1 gives the start and peak times of the smoothed monthly mean sunspot numbers and F10.7 during solar cycles 18–23, and Table 2 gives the gap between times.

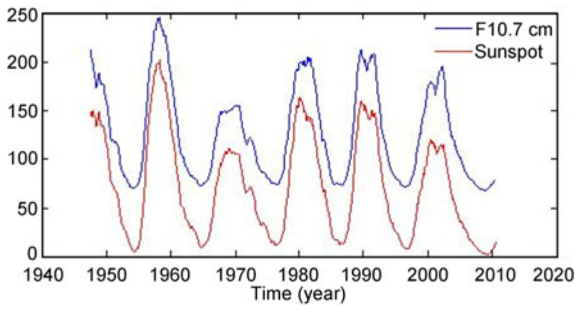

Figure 1 Smoothed monthly mean relative sunspot number and solar F10.7.

The CWT has been widely applied to understand the periodic features of different time series [6,18–27]. The CWT has edge artifacts because the wavelet is not completely localized in time. Thus, it is useful to introduce a cone of influence (COI) in which the transform suffers from these edges effects. The COI is defined such that the wavelet power for a discontinuity at the edges decreases by a factor e^{-2} [28]. Here, the CWT is employed to analyze the time series of the smoothed monthly mean relative sunspot number and solar F10.7, in an attempt to better understand their periodic features. The Morlet wavelet, applied in the study, is expressed as

$$\psi(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2}, \quad (1)$$

where ω_0 is the wave number, which is taken to be 6 [20,28].

Figures 2 and 3 depict the results of continuous wavelet analysis of F10.7 and the relative sunspot number during 1947–2011. The figures present the continuous wavelet power spectra of the smoothed monthly mean relative sunspot number and F10.7. Apparently, the two time series share some features of wavelet power. For example, both time series have large-scale periodicity (the 11-year Schwabe cycle) of high power above the 95% confidence level.

1.2 Cross-wavelet analysis

Cross-wavelet analysis has been frequently applied to determine the phase relationship between two time series. For example, Li et al. [15] analyzed the phase relationship of solar activity between northern and southern hemispheres using the cross-wavelet technique. In this study, the codes provided by Grinsted et al. [28] were employed to give the XWT of the time series of the smoothed monthly mean F10.7 and relative sunspot number in an attempt to clarify the phase relationship between the two (Figure 4). The XWT shows that the relative sunspot number and F10.7 are in phase across an 11-year periodic belt, sharing significant spectral power. Additionally, Figure 4 shows that the two

Table 1 Solar cycles

Solar cycle	Sunspot			F10.7		
	Start month (year)	Maximum month (year)	Endmonth (year)	Start month (year)	Maximum month (year)	End month (year)
18	02/1944	05/1947	04/1954	–	–	04/1954
19	04/1954	03/1958	10/1964	04/1954	03/1958	10/1964
20	10/1964	11/1968	06/1976	10/1964	07/1970	06/1976
21	06/1976	12/1979	09/1986	06/1976	05/1981	09/1986
22	09/1986	07/1989	05/1996	09/1986	07/1989	05/1996
23	05/1996	04/2000	12/2008	05/1996	03/2002	10/2008

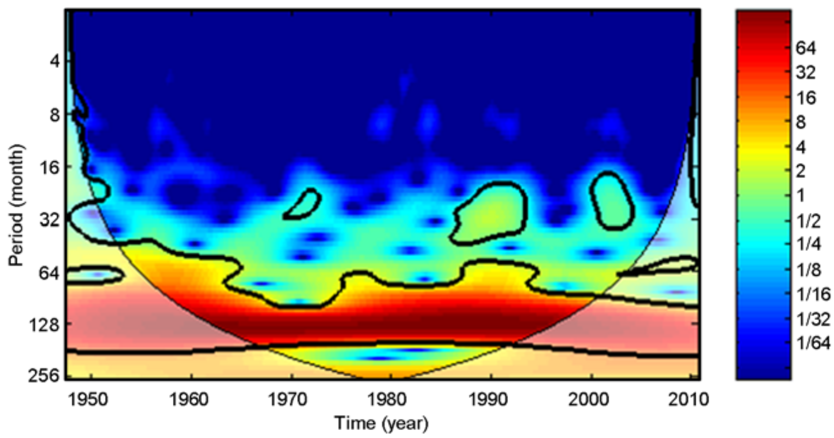

Figure 2 Continuous wavelet power spectra of the monthly mean sunspot number. The thick black contour represents the 95% confidence level, and the region under the thin solid line is the COI.

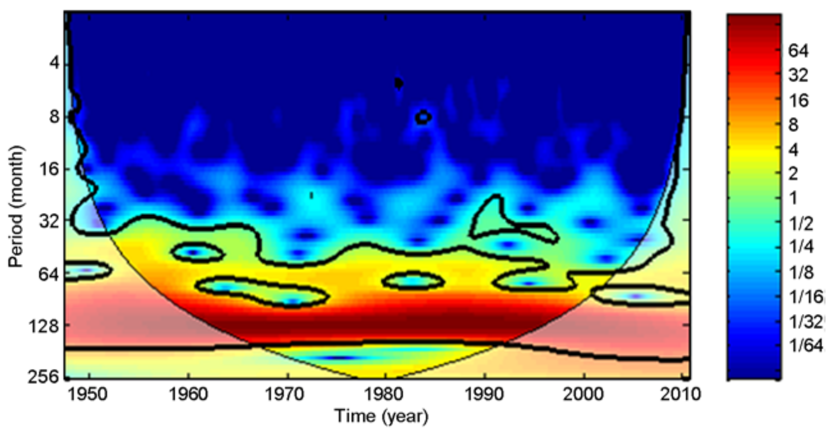

Figure 3 Continuous wavelet power spectra of monthly mean F10.7. The thick black contour represents the 95% confidence level, and the region under the thin solid line is the COI.

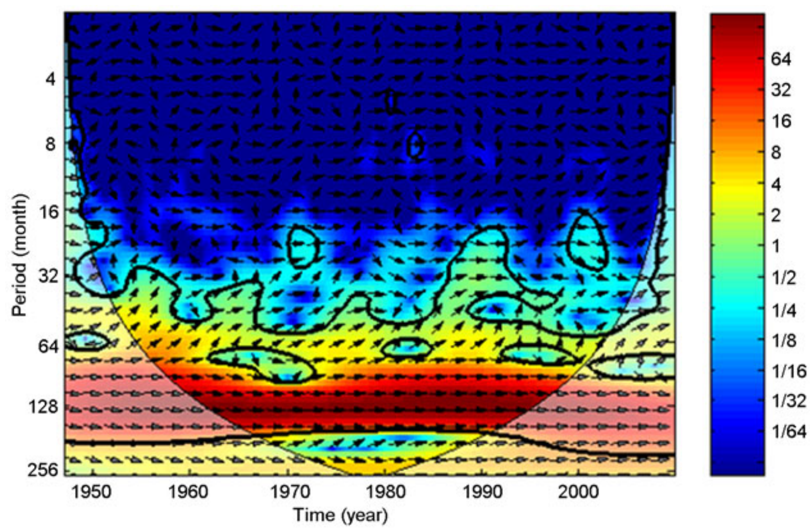

Figure 4 Cross-wavelet spectra of the relative sunspot number and F10.7. The thick black contours indicate the 95% confidence level, and the region below the thin line is the COI. The relative phase relationship is shown by arrows, with arrows pointing right for the in-phase relationship, left for the anti-phase relationship, and straight up for the sunspot activity leading the F10.7 by 90°.

Table 2 Time differences between F10.7 and sunspot number variations (months)[a)]

Solar cycle	Start month	Maximum month	End month
18	–	–	−1
19	−1	0	0
20	0	−20	0
21	0	−17	0
22	0	0	0
23	0	1	2

a) A negative value indicates lagging F10.7 and a positive value an advanced F10.7.

time series become asynchronous for the high-frequency components.

1.3 Wavelet coherence analysis

The WTC between two time series (X and Y) is defined as [28]

$$R_n^2(s)=\frac{|S(s^{-1}W_n^{XY}(s))|^2}{S(s^{-1}|W_n^X(s)|^2)*S(s^{-1}|W_n^Y(s)|^2)}, \quad (2)$$

where S is a smoothing operator. One can take the wavelet coherence as a correlation coefficient to show the localization of time frequency, as the definition and the traditional correlation coefficient are very similar. In this context, the smoothing operator S is

$$S(W)=S_{\text{scale}}(S_{\text{time}}(W_{\text{n}}(S))), \quad (3)$$

where S_{scale} denotes smoothing along the wavelet scale axis, and S_{time} smoothing along the wavelet time axis. In the context of the Morlet wavelet, the smoothing operator is given as [28]

$$S_{\text{time}}(W)|_S=\left(W_n(S)*c_1^{\frac{-t^2}{2S^2}}\right)\Big|_S, \quad (4)$$

$$S_{\text{scale}}(W)|_S=(W_n(S)*c_2\prod(0.6s))|_n, \quad (5)$$

where c_1 and c_2 are normalization constants, and Π a rectangle function. The factor 0.6 is a scale decorrelation length determined empirically for the Morlet wavelet [20]. Both convolutions are discrete. As a result, the normalization coefficients have to be determined numerically [28]. Figure 5 shows the wavelet coherence between the monthly mean sunspot number and F10.7. The WTC suggests strong phase synchronization at period scales around the Schwabe cycle. However, the WTC also indicates noisy behavior with strong phase mixing in the high-frequency components of both the F10.7 and sunspot number.

2 Conclusions and discussion

In the study, wavelet methods were employed to analyze the smoothed monthly mean sunspot number and solar F10.7, and it was found that the two series had notable features of the Schwabe cycle. Both XWT and WTC revealed a prominent periodic belt that passed the 95% confidence level and shared high spectral power. The belt sits around an 11-year cycle (i.e., the Schwabe periodicity). The Schwabe periodicity is in phase both outside and within the COI between the sunspot number and F10.7.

Both XWT and WTC display noisy behavior with strong phase mixing in the high-frequency components of the solar F10.7 and relative sunspot number, which may explain the

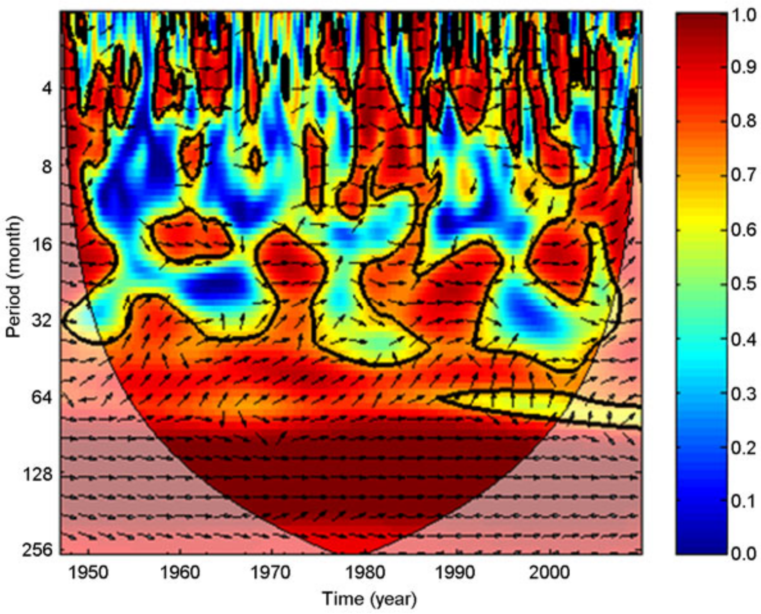

Figure 5 Wavelet coherence of the smoothed monthly mean sunspot number and F10.7. The thick black contours indicate the 95% confidence level, and the region below the thin line is the COI. The relative phase relationship is shown by arrows, with arrows pointing right for the in-phase relationship, left for the anti-phase relationship, and straight up for the sunspot activity leading the F10.7 by 90°.

different photospheric and coronal activities at a time scale less than that of the Schwabe cycle. Sunspot activity is affected by the convection zone and the low layer of the chromosphere. However, the flux of solar radio waves at 10.7 cm is affected by the corona and the high layer of the chromosphere. Physical properties of the convection zone, such as the temperature and density, differ from those of the corona. The physical properties of the low layer of the chromosphere differ from those of the high layer of the chromosphere. As the magnetic flux tube diffuses, the tube in the photosphere differs from that in the corona. All these phenomena may explain the different phases between the sunspot number and solar F10.7 in the high-frequency compoents.

The relative sunspot number and sunspot number/F10.7 data used in the study were obtained from ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_RADIO/FLUX/Penticton_Adjusted/. The wavelet method and code were taken from http://www.pol.ac.uk/home/research/waveletcoherence/. This work was supported by the National Natural Science Foundation of China (41074132, 40931056) and the Foundation for Standout Youth of Anhui Province (2010SQRL039).

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