Study of two-section composite chain of beads under vertical vibration

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We investigated the dynamic properties of a two-section composite chain of beads under vertical vibration. By analyzing the chain's motion, including phase-shift, mean dilation, center-of-mass displacement, and energy, we found that with different bead arrangements, the chain behaved in different ways. We believe that interaction existing at the interface between bead sections provides the underlying cause. This interaction causes different energy in different arrangements, which leads to different dynamic characteristics.

granular materials, two-section chain of beads, vertical vibration

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The dynamic properties of one-dimensional chain of beads have attracted much interest from scientists in recent years. Nesterenko [1] initially discovered that there is a new type of solitary wave. Different from well-known weakly nonlinear KdV solitons, this solitary wave has strong nonlinearity due to unique properties of granular materials. Related research revealed the characteristics of the energy propagation in granular materials. Following Nesterenko, most researchers focused mainly on solitary waves, especially in composite granular chains consisting of more than one type of beads with different sizes, masses, materials, etc. Several valuable results were discovered on how these solitary waves behave at the interfaces between different bead sections [2–6]. For example, if waves propagate from heavy to light beads, most of the energy will be transmitted through the interface; in contrast if waves propagate from light to heavy beads, most of the energy will be reflected. Such single-direction transmission, which can be thought of as an “acoustic-diode” effect, is due to the strong nonlinearity arising from granular materials.

The study on granular chain behavior under continuous

vertical vibration was initially conducted by Luding et al. [7,8]. They pointed out that for a chain of beads with high coefficient of restitution (i.e. “hard” beads), as the vibrational acceleration increases, it will transite from a condensed phase (where all beads stay in contact with each other and move as a whole with the vibration source) to a fluidized phase (where all beads move independently from one another and away from the source). However, for a chain of beads with low coefficient of restitution (i.e. “soft” beads), one sees a transition from a condensed phase to a clustered phase (where all beads move away from the source but still approximately stay in contact with each other) without fluidization, essentially because the dissipation of energy in “soft” bead collisions is greater than that in “hard” beads. Luding's work was based on a chain consisting of only one type of bead (referred to as “single-section chain”). Although still focused on single-section chains, these studies were extended by others [9–11] to include the use of Boltzmann's equation and molecular dynamic simulation [12–14] to analyze the motion of the granular materials; few studies report on the behavior of composite chains.

In the present paper, we describe the dynamics of composite granular chains under continuous vertical vibration.

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We choose beads of two types with different parameters. Dynamic properties of the two-section composite chain are studied under continuous vertical vibrations. We especially investigate the interface effects between the two bead sections on energy propagation as well as properties of the whole system.

1 Materials and methods

For our experiments, we chose cuprum and aluminum beads; the former are considered to be heavy and “hard” with a density of $8.9 \times 10^3 \text{ kg/m}^3$ and a coefficient of restitution of approximately 0.9, whereas the latter are considered light “soft” beads with a density of $2.7 \times 10^3 \text{ kg/m}^3$ and a coefficient of restitution of approximately 0.6. All beads are of the same shape and size (3 mm in diameter) and are stacked one on top of the other in a long square tube. As indicated in Figure 1, the edge length of the cross section is slightly larger than 3 mm, so that all beads can move freely in the tube and constitute a one-dimensional granular chain. The tube is fixed to a vertically vibrating plate with its top left open. We also fixed one of cuprum beads at the bottom of the tube that we refer to as the “source bead”. The displacement of the vibrating plate is sinusoidal $d = A \sin(2\pi ft)$, where A is the driving amplitude, and f the driving frequency. We chose a dimensionless acceleration Γ (defined as $\Gamma = 4\pi^2 f^2 A/g$, where g is the gravity acceleration) and f as two controlled parameters. The frequency range is from 15 to 25 Hz. In this range, Γ can reach higher values (≥ 5), which allows us to observe the most experimental phenomena.

We use a high-speed camera (Motion Scope PCI 2000 SC) to record bead movements. The camera is controlled by a phase-locked circuit, so all photos are taken at fixed phase points with respect to the driving period. Over each period, 20 photos are taken at regular time intervals. For each set of

parameters, when the chain reaches its stable state, a total of 4096 photos (corresponding to 204 driving cycles) are taken at one time. All photos are analyzed digitally in the subsequent analysis.

2 Results and discussion

We began with five cuprum beads in the bottom of the tube, and five aluminum beads on top of cuprum beads; this arrangement is referred to as a “C-A chain”. For different sets of driving parameters, experimental data are recorded and analyzed. Each set of photos are statistically processed by calculating the average values. Figure 2 shows the displacement curves of the granular chain at $f = 15 \text{ Hz}$. For each bead, we calculate the average displacements at all the same phase over all vibration periods, we calculate the average value of the displacements at all time points of the same phase over all vibration cycle, so an average displacement curve is obtained. In Figure 2, the bottom curve of each panel corresponds to the source bead displacement, whereas the other 10 curves are for those of the other 10 beads according to their position above the source bead. The peak points for the different beads are marked with squares.

From Figure 2, we can see that as Γ increases the behavior of the C-A chain is similar to that of a single-section chain consisting of “hard” beads; both make transitions from a condensed to a fluidized phase [7,8]. When $\Gamma = 1$, all beads in the chain have the same phase as the source bead and still stay in contact with the source bead. As Γ increases to 5, each bead begins to have a different phase. The beads begin to move independently from each other, and away from the source bead.

The order of the bead sections were then interchanged with 5 aluminum beads at the bottom and 5 cuprum beads on the top; we refer to this arrangement as the “A-C chain”. The behavior of the A-C chain is completely different from C-A chains; a similar analysis is presented in Figure 3. The same frequency of $f = 15 \text{ Hz}$ is maintained, but the displacements are averaged over two driving cycles.

From Figure 3, we can see that as Γ increase the beads in the A-C chain remain in contact with each other unlike the behavior in the C-A chain. When Γ increases to a certain value, the period for the motion of chain is double the driving period (Figure 3(c)). This is similar to a single-section chain consisting of “soft” beads [7,8].

To characterize the phase differences in bead motion for both arrangements, we give in Figure 4 the maximum phase differences of displacement peaks of all beads (abbreviated to “phase-shift”) p_i under different Γ . From this figure, we can see that for the C-A chain p_i increases as Γ increases; for $\Gamma \leq 2$, p_i is smaller. This indicates that all beads move basically as a whole. As Γ increases from 2 to 3, p_i increases rapidly; a further increase in Γ induces asymptotic behavior

Figure 1 The diagram of the experiment system.

Figure 2 Mean displacement as function of time for a C-A chain at $f=15$ Hz. (a) $\Gamma=1$; (b) $\Gamma=3$; (c) $\Gamma=5$.

Figure 3 Mean displacement as function of time for the A-C chain at $f=15$ Hz. (a) $\Gamma=1$; (b) $\Gamma=3$; (c) $\Gamma=5$.

Figure 4 Phase-shift of two arrangements for $f=15$ Hz.

to a constant p_i . That is, the phase-shift mainly takes place for Γ in the range of 2 to 3. In contrast, we can see under the same vibration acceleration that p_i for the A-C chain is obviously smaller. We observe that as Γ increases from 4 to 5, the movement period of A-C chain increases from one driving period to double the driving period (Figure 3(c)), and there is a distinct decrease in p_i .

To describe the separation between beads, we define the mean dilation of the granular chain as $Di = (Z_n - Z_1 - L)/L$, where $L=0.012$ m is the original length of each section in repose. Z_n and Z_1 are the heights of top and bottom beads in the chain at any given time. Calculations for both heavy and light bead sections are performed. The experimental results are shown in Figure 5 for Γ in the range 1–5. From this figure, we can see for the C-A chain that Di of the light section on top is always larger than for the heavy section under it. Moreover, Di almost vanishes for both heavy and light sections, indicating that gaps between the beads almost vanish. This agrees with the conclusion of Figure 4. As Γ increases, Di increases for both heavy and light sections. Over the range $\Gamma=2-3$, Di increases fastest, which reaffirms the conclusion in Figure 4. For $\Gamma \geq 4$, Di still increases, but with a lower rate. This is different from the phase-shift behavior. From this we can see that, for the C-A chain in a fluidized state, as Γ increases, bead separation will be enhanced and the propagating velocity of the vibration increases with excitation acceleration. By comparison, we can see that Di for the A-C chain are much smaller than those for the C-A chain indicating that the beads are closer in the former than in the latter. As Γ increases, the mean dilation of the light section at the bottom increases slowly, and is always smaller than that of heavy section. This trend is different from C-A chain. As Γ increases from 4 to 5, dilation distinctly decrease for both heavy and light sections. This suggests that a “constriction” occurs in the A-C chain, i.e. all beads become closer to each other in accordance with the result in Figure 4.

Figure 6 presents the mean displacement of the center of

Figure 5 Mean dilation for two arrangements at $f=15$ Hz.

Figure 6 Center-of-mass mean displacement for the two arrangements at $f=15$ Hz.

mass (relative to the rest position) as Γ increases. For the C-A chain with $\Gamma=1$, the heavy- and light-section displacements are the same; with $\Gamma>1$, the respective positions of the center-of-mass rise. The displacement for the light section is always larger than that of the heavy section. For the A-C chain, the displacements also increase as Γ increases. By comparison, we can see given $\Gamma=1, 2$ and 5 that the heavy sections in both arrangements have similar displacements, and as Γ increases from 2 to 5, the displacement of the heavy section in the C-A chain increases with a constant rate, whereas in the A-C chain the rate is higher in the range from 2 to 3, thereafter steadily declining. There are great differences between the displacements of light sections in two arrangements. The displacement of the light section in the A-C chain is significantly smaller than that in the C-A chain, and also always smaller than that in the heavy section.

We also calculate the energy of both arrangements for $f=15$ Hz. The results in Figure 7 are the average over 204 driving cycles of the sum of the kinetic and gravitational potential energies of all beads for a given set of parameters.

Figure 7 Energy of two arrangements for $f=15$ Hz.

From Figure 7 we can see that the energy in the granular chains increases as Γ increases. By comparison, we can see that the heavy sections of both arrangements have similar energy, the difference being that the energy of the heavy section in the A-C chain increases faster for a Γ -range of 2–3 than in other ranges, whereas that in the C-A chain increases almost uniformly. This agrees with the conclusion of displacements in Figure 6. The difference in energy between the light sections is great by comparison; the energy of the light section in the A-C chain is generally lower than that in the C-A chain. Comparing the total energy of all beads in each arrangement, we find that under the same excitation parameters, the total energy in the A-C chain is generally lower than that in the C-A chain; for lower Γ values, the difference is not significant, but if Γ is increased to higher values, the difference becomes greater.

Luding et al. [7,8] used stainless steel (“hard”) and aluminum (“soft”) beads in constructing two single-section chains, one “hard” chain and the other a “soft” chain. Their experimental conditions are similar to ours. The behavior of the “hard” (“soft”) chain is similar to our C-A (A-C) chain. Based on previous work [7–9], that the “soft” single-section chain cannot be fluidized is due to the energy dissipation being greater through collisions between “soft” beads. If we increase the number of beads, there will be more collisions, and hence energy dissipation increases. Thus even a “hard” single-section chain cannot be fluidized even if the number of beads is large enough. This is the reason why we chose 10 beads for each chain in our experiment. From Figure 7 we can see that both types of composite chains have different energy. We propose that this is due to the existence of “an acoustic-diode” effect at the interface between bead sections. As in [2–6], in a one-dimensional granular chain, when acoustic waves propagate from heavy to light bead sections, almost all the energy will be transmitted through the interface; for reverse propagation, most of the energy will be reflected. Because the vibration source is at the bot-

tom, the energy in a C-A chain will be transmitted from bottom to top without reflection almost. For the hard heavy beads, the energy dissipation is less. For soft light beads, the energy dissipation is limited as there are only a small number of beads. Thus the total energy of a C-A chain is higher, which leads to ideal fluidization. The A-C chain is quite different from C-A chain in comparison. The energy from the bottom source transmits upwards through the light section; most of it will be reflected by the interface with only a little energy being transmitted into the heavy section. Thus the majority of the energy is confined in the light section and partly dissipated through collisions between “soft” beads. The energy of the light section in A-C chain, as well as the total energy, is significantly lower than that in the C-A chain; the A-C chain, like the “soft” single-section chain, moves almost as a whole under continuous vibration.

Generally speaking, the “acoustic-diode” effect forms an “energy trap”, localizes the energy in a region, and then releases and dissipates it gradually. Under continuous vibration this effect changes the transmission and distribution of energy, and results in a different energy dissipation processes for the different arrangements. As vibrational isolators, granular materials are good buffers. The underlying principle is energy dissipation by bead collisions. From our experiments one can see that a bi-composite chain consisting of “hard” and “soft” beads have better isolating properties for vibration than a “hard” single-section chain. Unidirectional isolation, resulting from single-direction energy transmission, cannot occur in “soft” single-section chains.

3 Conclusions

We found experimentally that under continuous vibration the C-A bi-composite chain of beads is similar to a single-section chain consisting of beads with high coefficient of restitution. As the vibration acceleration increased, the bead dynamics changes from a condensed phase to a fluidized phase. However, the A-C bi-composite chain is similar to a single-section chain consisting of beads with a low coefficient of restitution. Even under high accelerations, the chain still moves almost as a whole, the reason being the difference in energy. The “acoustic-diode” effect at the interfaces between two types of beads sections causes higher energy dissipation in the A-C chain. This provides a new method for vibrational isolating using granular materials, and therefore has potential in various applications. Also, one-dimensional granular chains form a basic model for granular materials in general, and could provide a perspective in future research of more complex granular systems. In a 2D or 3D granular system, if the system consists of more than one type of bead with different parameters, questions arise such as how will the beads interact with each other, and what kind of influence will the interaction have on the whole system? These questions are still to be studied.

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