Effects of surface tension on bubble growth in an extensive uniformly superheated liquid

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The study of bubble growth in an extensive pool of liquid provides considerable insight into the mechanisms that play a role in bubble growth near a heated surface and in the cavitation phenomenon. This work focuses on analyzing the effects of surface tension on the growth rate for the thermally controlled stage of a single bubble in such a liquid. The conservation of energy equations, including the internal energy term for the bubble and that within boundary layer around it, are numerically solved. The complete temporal variations of the bubble in water and liquid nitrogen are investigated based on the assumption that the bubble growth is controlled only in sequence by inertia and heat. Thus, the two stages are subject to the continuity of the bubble growth, while the inertia-controlled stage is only formulated by the well-known Rayleigh solution. The thickness of the boundary layer around the bubble is also determined. The results are comparable with the Plesset-Zwick models and Forster-Zuber models, as well as available experimental data. It is found that the influence of internal energy on the rate of bubble growth is small enough to be ignored; however, the accumulative effects of the surface tension are significant and increase with a decrease in the degree of superheat.

surface tension, bubble growth, water, liquid nitrogen

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Study of bubble growth in an extensive pool of liquid provides considerable insight into the mechanisms that play a role in bubble growth near heated surfaces and in the cavitation phenomenon [1–3]. If a liquid in a large container is uniformly heated under constant pressure, when the temperature approaches saturation corresponding to the pressure, its surface will begin to boil. However, in the interior, the bubble cannot form easily, due to lack of the nuclei at that point. Thus, the liquid begins to form a metastable condition that has a certain degree of superheat as the liquid is further heated. This paper is devoted to study this fluid condition under the assumption that the liquid is uniformly superheated.

Strictly speaking, in uniformly superheated liquids, bubble growth can be divided into three stages [4]: the inertia-controlled stage, the heat-controlled stage, and a transitional

stage simultaneously controlled by both inertia and heat. The more general intermediate stage is fairly complex to analyze and there are few reports on it apart from that by Mikic et al. [5]. However, it is not too difficult to develop reasonably accurate analyses for the initial and final stages. In the initial stage, the bubble first forms and the vapor pressure nearly equals $P_{\text{sat}}(T_{\infty})$, while the temperature field for the bulk liquid is still in a uniform state and the temperature gradient has not been established. Thus, bubble growth is mainly controlled by the mechanical imbalance between bubble pressure and liquid pressure. Researchers focusing on this stage include Rayleigh, who proposed the Rayleigh model [6–8], Plesset, who proposed the Rayleigh-Plesset model [7,9], and Ellion [10], more information on whom can be found in [11]. For the heat-controlled stage, the pressure difference between the internal bubble and the exterior is $2\sigma/R$, according to the Laplace equation, which is

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relatively small. While the temperature at the interface will approach $T_{\text{sat}}(P_{\infty})$ as the liquid superheat near the interface is depleted to provide the latent heat of vaporization, the bubble grows mainly through vaporization, with the heat conducted from the nearby superheated liquid. Analytical treatments of this stage have been presented by Plesset-Zwick et al. [12], Zuber et al. [13], and Scriven [14]. These models provided significant insights into the mechanism of the bubble growth process. However, without exception, they neglected the internal energy variance of the bubble, because its inclusion into the energy equation significantly complicates the solution. Further, no quantitative analyses of the effect of surface tension have been studied to date.

The purpose of the present paper is to quantitatively investigate the effects of surface tension on bubble growth in an extensive, uniformly superheated liquid with consideration of the internal energy variance of the bubble for the heat-controlled stage. The contents are arranged as follows: First, the controlling equations for the inertia-controlled and heat-controlled stages are presented, and the well-known Rayleigh solution is given to maintain continuity of bubble growth. The equations, which include the internal energy term and the surface tension term for the second stage, are numerically solved in detail and a parameter k is introduced to quantitatively represent the total effects of surface tension on bubble growth. The thickness of the thermal boundary layer around the bubble is also computed. Second, taking the water as an example, the calculated temporal variance of the bubble radius is compared with the results of the Plesset-Zwick models and Forster-Zuber models, as well as the available experimental data. The effect of superheat on the bubble growth is then investigated. Finally, the obtained model is applied to liquid nitrogen and the same analyses as applied to water are also considered.

The following assumptions are made for the calculations in the paper:

(1) Because the bubble growth rate is of the order millimeters or centimeters per second, which approaches over three orders smaller than the local speed of sound, the fluid is considered to be incompressible.

(2) The temperature changes for both the liquid and the vapor are small, so the physical properties of the fluids are temperature-independent.

(3) The liquid viscous effect is ignored in the calculation process.

1 Inertia-controlled stage

The physical circumstances of interest are shown in Figure 1. At any instant during the growth process, the interface is located at $r=R$ and is moving with a velocity dR/dt relative to the laboratory reference frame. The pressure and temperature are P_v and T_v inside the bubble and P_{∞} and T_{∞} in the surrounding liquid, respectively. Initially, the liquid is

Figure 1 Schematic of bubble growing in superheated liquid.

undercooled ($T_{\infty} < T_{\text{sat}}(P_{\infty})$) but with heating, its temperature gradually increases to saturation ($T_{\infty} = T_{\text{sat}}(P_{\infty})$), where it is difficult to form a bubble in the interior of the liquid at this point because of the existence of surface tension. The fluid becomes superheated as heating is continued and begins to form bubbles in the presence of nuclei in the fluid. The distribution of the whole internal temperature of the liquid is still uniform at this moment, $T_v = T_{\infty}$. Assuming that the vapor inside the bubble has constant composition, no other non-condensable gas, and the bubble is spherical, the pressure inside the bubble is larger than the sum of the pressure outside the bubble and surface tension effects. The pressure difference inside and outside the bubble promotes the growth of the bubble, so this stage is also known as the pressure control stage.

Based on a spherical coordinate system with the center of the spherical bubble as the origin of coordinate, the Euler equation and continuity equation of liquid around the bubble can be written as follows, for an incompressible and inviscid fluid, ignoring the slip velocity between the bubble and liquid:

$$\frac{\partial u_r}{\partial \tau} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho_l} \frac{\partial P}{\partial r}, \quad (1)$$

$$\frac{1}{r^2} \left[\frac{\partial (r^2 u_r)}{\partial r} \right] = 0, \quad \text{where } u_{r=R} = \frac{dR}{dt}, \quad (2)$$

where u_r is the radial velocity of liquid at radius r . Integrating eq. (2) from $r=R$ to an arbitrary location r yields

$$u_r = \frac{dR}{dt} \left(\frac{R}{r} \right)^2. \quad (3)$$

Substituting eq. (3) into eq. (1) and integrating, yields

$$R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{P_1(R) - P_1(\infty)}{\rho_l}. \quad (4)$$

This equation is referred to as the Rayleigh equation [6–8, 15–17], where $P_1(R)$ is the pressure of liquid at $r=R$, $P_1(\infty)$ is the pressure of liquid at infinity, and $\ddot{R}$ is the second order derivative of the bubble radius with respect to time. With further integration of eq. (4), the following equation is obtained:

$$R(\tau) = R_0 + \sqrt{\frac{2(P_1(R) - P_1(\infty))}{3\rho_l}} \left(1 - \frac{R_0^3}{R^3}\right) \tau, \quad (5)$$

where R_0 is initial radius of the bubble, which is mainly affected by the nuclei and the initial superheat of the liquid. Generally, R_0 is relatively small, and thus, eq. (5) is simplified as

$$R(\tau) = \sqrt{\frac{2(P_1(R) - P_1(\infty))}{3\rho_l}} \cdot \tau. \quad (6)$$

We further note that $P_1(R)$ is related to the vapor pressure P_v inside the bubble by the Yong-Laplace equation:

$$P_v = P_1(R) + \frac{2\sigma}{R}. \quad (7)$$

For the inertia-controlled growth considered here, $2\sigma/R$ is usually much smaller than $P_v - P_1(\infty)$ and thus, can be ignored, resulting in $P_v = P_1(R)$. The following linear equation is used to evaluate the relationship between saturation pressure and temperature:

$$P_{\text{sat}} = NT_{\text{sat}} + M. \quad (8)$$

According to eqs. (7) and (8), eq. (6) is rewritten as

$$R(\tau) = \left(\frac{2N(T_v - T'_s)}{3\rho_l} \right)^{1/2} \cdot \tau, \quad (9)$$

where M and N can be obtained from fitting experimental data. T_v is the bubble temperature and is equal to T_∞ in this stage. T'_s is the saturation temperature at $P_1(\infty)$. The result indicates that the radius of bubble increases linearly with time during the inertia-controlled stage.

2 Heat-controlled stage

As the bubble grows, the temperature inside the bubble decreases and gradually equals the saturation temperature, corresponding to bubble pressure ($T_v = T_{\text{sat}}(P_v)$) as a result of vaporization, which is a little larger than T'_s because of the existence of the surface tension, resulting in $P_v > P_\infty$. Correspondingly, the pressure difference inside and outside the bubble decreases the same as the magnitude of the surface tension. However, the effects of the latter are larger than the former. At this stage, the bubble expands mainly by evaporation, with the latent heat transferred through a thin boundary layer, where there is a sharp temperature gradient in the radial direction. Outside this boundary layer zone, the liquid temperature can be considered to be constant [18].

The energy conservation equation for the bubble is

$$4\pi R^2 \cdot \lambda_1 \cdot \left(\frac{\partial T}{\partial r} \right)_{r=R} = 4\pi R^2 \frac{dR}{dt} \rho_v \gamma + \frac{4}{3} \pi R^3 c_{p,v} \rho_v \frac{dT_v}{dt}. \quad (10)$$

Combining eqs. (7) and (8), and noticing that $P_1(R) = P_1(\infty)$ in the heat-controlled stage, leads to

$$P_v = \left(P_1(\infty) + \frac{2\sigma}{R} \right) = NT_v + M. \quad (11)$$

The governing equation for the transport of heat in the boundary layer around the bubble is

$$\frac{2}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} = \frac{1}{\alpha_1} \left(\frac{\partial T}{\partial \tau} + u_r \frac{\partial T}{\partial r} \right). \quad (12)$$

By defining the excess temperature $\theta = T - T_l$, the boundary and initial conditions are

$$\begin{cases} \theta(R, \tau) = T(R, \tau) - T_l = T_v - T_l = \theta_w, \\ \theta(r, 0) = T(r, 0) - T_l = T_l - T_l = 0, \\ \theta(\infty, \tau) = T(\infty, \tau) - T_l = T_l - T_l = 0. \end{cases} \quad (13)$$

Eq. (12), together with eq. (3) and the boundary conditions of eq. (13), can be solved by the method of similarity transformation [19], so the obtained temperature distribution in the boundary layer is

$$\theta = T - T_l = c_1 \int_{\varepsilon_1}^{\varepsilon} \exp \left[- \left(\frac{D}{\varepsilon} + \varepsilon^2 + 2 \ln \varepsilon \right) \right] d\varepsilon + c_2, \quad (14)$$

where $\varepsilon = \frac{r}{2\sqrt{\alpha_1 \tau}}$, $\varepsilon_1 = \frac{R_1}{2\sqrt{\alpha_1 \tau_1}}$, $D = \frac{R^2 \dot{R}}{2\alpha_1 \sqrt{\alpha_1 \tau}}$; R_1 and τ_1 are the bubble radius and time at the end of the first growth stage, respectively. A large number of reports [5,12–14,18,20] have indicated that the bubble radius is proportional to the square root of time during the heat-controlled stage, and thus, the parameter D is approximately a constant. The boundary and initial conditions in eq.

(13) can be rewritten as $\begin{cases} \varepsilon = \varepsilon_1, \theta = \theta_w, \\ \varepsilon = 0, \theta = 0. \end{cases}$ Therefore, the

coefficient of the definite solution in eq. (14) can be obtained as

$$c_1 = -\theta_w \int_{\varepsilon_1}^{\infty} \exp \left[- \left(\frac{D}{x} + x^2 + 2 \ln x \right) \right] dx, \quad c_2 = T_v - T_l = \theta_w. \quad (15)$$

Eqs. (10), (11) and (14) form a closed set of equations. Substituting eqs. (11), (14) into (10) and integrating, setting

$$H = -\frac{D}{\varepsilon} - \varepsilon^2 - 2 \ln(\varepsilon), \quad H_1 = -\frac{D}{\varepsilon_1} - \varepsilon_1^2 - 2 \ln(\varepsilon_1), \text{ leads to}$$

$$\begin{aligned} & \frac{\lambda_1 (T_l - T_v)}{2\alpha_1 \rho_v \gamma} c \cdot \exp(H_1) (2\sqrt{\alpha_1 \tau} - 2\sqrt{\alpha_1 \tau_1}) \\ & = (R - R_1) - \frac{2\sigma c_{p,v}}{3N\gamma} \ln \left(\frac{R}{R_1} \right). \end{aligned} \quad (16)$$

While obtaining the above result, $T_v = T_{\text{sat}}(p_v)$ is applied. The coefficient c equals

$$c = 1 / \int_{\varepsilon_1}^{\infty} \exp \left[- \left(\frac{D}{x} + x^2 + 2 \ln x \right) \right] dx = 1 / \int_{\varepsilon_1}^{\infty} \exp(H) dx.$$

We further define $F = \frac{\lambda_1 \Delta T}{2\alpha_1 \rho_v \gamma} c \cdot \exp(H_1)$ and

$k_1 = \frac{(T_l - T_v)}{\Delta T}$, where ΔT is the superheat of the liquid. Eq. (16) can be simplified as

$$2k_1 F \sqrt{\alpha_1 \tau} = R - \frac{2\sigma c_{p,v}}{3N\gamma} \ln(R) + \frac{2\sigma c_{p,v}}{3N\gamma} \ln(R_1) - R_1 + 2k_1 F \sqrt{\alpha_1 \tau_1}. \quad (17)$$

It should be noted that when $k_1=1$, $T_v=T_1-\Delta T=T'_s$, implying that $P_v=P_1(R)=P_1(\infty)$, so $2\sigma/R=0$ according to eq. (7), meaning that the effect of surface tension is neglected. Therefore, k_1 represents the effect of surface tension on the temperature distribution in the thermal boundary layer, as well as the growth rate of the bubble. Actually, the pressure inside the bubble is slightly larger than the pressure outside, because of the effect of the surface tension, which causes (T_1-T_v) to be slightly less than the superheat ΔT of liquid.

Defining constant $A = \frac{2\sigma c_{p,v}}{3N\gamma} \ln(R_1) - R_1 + 2k_1 F \sqrt{\alpha_1 \tau_1}$, and rearranging eq. (17), one obtains

$$\begin{aligned} & \left(2k_1 k_2 F \sqrt{\alpha_1 \tau} \right) - \frac{2\sigma c_{p,v}}{3N\gamma} \ln \left(2k_1 k_2 F \sqrt{\alpha_1 \tau} \right) \\ &= R - \frac{2\sigma c_{p,v}}{3N\gamma} \ln(R), \end{aligned} \quad (18)$$

which leads to

$$\begin{cases} R = 2k_1 k_2 F \sqrt{\alpha_1 \tau}, \\ 2k_1 (k_2 - 1) F \sqrt{\alpha_1 \tau} + A = \frac{2\sigma c_{p,v}}{3N\gamma} \ln \left(2k_1 k_2 F \sqrt{\alpha_1 \tau} \right). \end{cases} \quad (19)$$

Eq. (19) implies that during the heat-controlled stage, the radius of the bubble is proportional to the square root of time. The same conclusion has been obtained by several other researches, such as Scriven [14]. In eq. (19), k_2 can be determined by numerical calculations and also relative to the effects of surface tension on the temperature inside the bubble, which implies the effect of internal energy variation on the bubble due to the surface tension, and the effect is zero if $k_2=1$. A cumulative parameter k is introduced to calculate the total effects of the surface tension:

$$k = k_1 k_2. \quad (20)$$

The closer k approaches one, the smaller are the effects of surface tension and it decreases to zero if $k=1$. The growth rate of the bubble is continuously transitioned from the first stage to the second stage and therefore R_1 and τ_1 at the end of the first stage, satisfy eq. (19), such that $R_1 = 2k_1 k_2 F \sqrt{\alpha_1 \tau_1}$, which leads to

$$\varepsilon_1 = \frac{R_1}{2\sqrt{\alpha_1 \tau}} = kF. \quad (21)$$

Namely,

$$\begin{cases} D = 2k^3 F^3, \\ H_1 = -3k^2 F^2 - 2\ln(kF). \end{cases} \quad (22)$$

Therefore, the function of bubble radius with time is

$$R = 2kF \sqrt{\alpha_1 \tau} = 2kc_s \cdot Ja \cdot \sqrt{\alpha_1 \tau}, \quad (23)$$

where $c_s = c \cdot \exp(H_1)/2$, Ja is the Jacobian number, defined as $Ja = c_1 \rho_1 \Delta T / \rho_v \gamma$; Eq. (23) is the function obtained for bubble growth rate with time during the heat-controlled stage.

3 Thickness of boundary layer

The thickness of the thermal boundary layer is defined as the distance from the interface to the point where the temperature drop is equal to 99% of the maximum temperature drop from the interface to infinite distance [21]. Defining the ratio ω as

$$\omega = \frac{R + \delta}{R} = \frac{r_{\max}}{R}, \quad (24)$$

where $r_{\max}$ is the radial distance from the center of the bubble to the outer boundary of the thermal boundary layer. With eqs. (14) and (24), the value of ω is determined by

$$\frac{\int_{\varepsilon_1}^{\omega k F} \exp(H) dx}{\int_{\varepsilon_1}^{\infty} \exp(H) dx} = 0.99. \quad (25)$$

The thickness of the thermal boundary layer δ is then obtained as $\delta = (\omega - 1)R$.

So far, eqs. (23) and (25) constitute the complete model of bubble growth for the heat-controlled stage, for which the solution is obtained by iteration.

4 The complete bubble growth process

Eqs. (9) and (23) describe the growth process of the bubble in an extensive uniformly superheated liquid for the inertia-controlled and heat-controlled stages, respectively, disregarding their transitional stage, expressed as

$$R(\tau) = \begin{cases} \left(\frac{2N(T_v - T'_s)}{3\rho_1} \right)^{1/2} \cdot \tau, & 0 \leq \tau \leq \tau_1, \\ 2kc_s \cdot Ja \cdot \sqrt{\alpha_1 \tau}, & \tau > \tau_1. \end{cases} \quad (26)$$

In view of the fact that the bubble growth process is continuous, the piecewise function eq. (26) must be continuous, so we can obtain the value of τ_1 as

$$\tau_1 = \frac{6\rho_1 \alpha_1}{N(\Delta T)} (kc_s \cdot Ja)^2. \quad (27)$$

5 Validation of the model

5.1 Example model

As the case of the water bubble has been widely studied by theoretical and experimental methods, it is adopted in this

paper for ease of comparison. The pressure of the water container is assumed to be one atmosphere. The solution procedure is as follows:

(1) Fitting the values of M , N in eq. (11) by the experimental data, for water, $N=4105.54$, $M=-309158.09$.

(2) Giving the physical properties at the saturated temperature plus half of the specified superheat. The values of water are shown in Table 1 with superheat of 3.1 K, and the values are also used for other superheat in the calculations, because the changes in physical parameters with temperature are very small.

(3) Substituting the obtained values into eqs. (9), (19), (22), (25) and (27), leading to

$$R(\tau) = \begin{cases} 2975.4\tau \text{ (mm)}, & 0 \leq \tau \leq 5.367 \mu\text{s}, \\ 7.451k_1k_2 \cdot \sqrt{\tau} \text{ (mm)}, & \tau > 5.367 \mu\text{s}, \end{cases} \quad (28)$$

$$\text{where } k_1 = \begin{cases} 0.939, & \tau_1 < \tau < 10 \text{ ms}, \\ 1, & \tau > 20 \text{ ms}, \end{cases} \quad k_2 = 0.985.$$

The value of $\omega=1.13$ is also determined using eq. (25), implying that the boundary layer thickness is small under such circumstances and its relationship with the bubble radius is

$$\delta = 0.13R. \quad (29)$$

As can be seen from the above equation, the temperature drop occurs only over a thin layer. The value is almost the same as the result calculated by Scriven [14], which indicates that the value of ω is less than 2.

5.2 Validation of model

Eq. (28) is plotted in Figure 2 for comparison with the experimental data of [12] and the calculated results of other two well-known models: the Plesset-Zwick model [11,12] and the Forster-Zuber model [16,20] for the heat-controlled stage. Here, the same time delay as experiments was adopted for matching the model's solutions with experimental data.

The Plesset-Zwick model:

$$R(\tau) = 2 \cdot \sqrt{\frac{3}{\pi}} \sqrt{\alpha_1 \tau} Ja. \quad (30)$$

The Forster-Zuber model:

$$R(\tau) = 2 \cdot \frac{\sqrt{\pi}}{2} \sqrt{\alpha_1 \tau} Ja. \quad (31)$$

Table 1 Physical properties of water for calculations^{a)}

The properties of liquid			
Density	Specific heat at constant P	Heat conductivity	Surface tension
958.4 kg/m ³	4217 J/(K kg)	0.6651 W/(K m)	0.05794 N/m
The properties of vapor			
Density	Specific heat at constant P	Latent heat of vaporization	
0.5891 kg/m ³	2060 J/(K kg)	2257 kJ/kg	

a) Conditions: $P=101.325$ kPa, $T=373.15$ K, $\Delta T=5.3$ K.

It should be pointed out that the expression of eq. (30) is exactly the leading term of the Plesset-Zwick solution and it is in error by approximately 10 to 30 percent for the smallest values of R [12]. Furthermore, eqs. (30) and (31) are obtained without consideration of the internal energy variance of the bubble. It can be seen that the calculated results with P-Z model, F-Z model and the present model are all greater to some extent than the experimental data. The calculated results of this model, which considers the effects of surface tension, are a little larger than the results of the F-Z model. Comparison with the results of this model without consideration of the effects of surface tension ($k=1$) shows that the current model is larger than the F-Z model. The deviation of these two results clearly shows the effects of surface tension on the bubble growth rate, i.e. surface tension always resists bubble growth. From the viewpoint of a long period, the comparison indicates that surface tension has substantial effects on bubble growth. The above conclusions are further proved in Figures 3 and 4, which show comparisons between the calculated results and the experimental data with different superheats.

Figure 2 Experimental and calculated water bubble radius vs. time with $\Delta T=3.1$ K.

Figure 3 Experimental and calculated water bubble radius vs. time with $\Delta T=4.5$ K.

Figure 4 Experimental and calculated water bubble radius vs. time with $\Delta T=5.3$ K.

From Figures 2–4, it is found that the theoretical curves from the three models are all greater to some extent than the experimental data and the higher the superheat, the greater the difference between them. The reasons are explained as follows. As pointed out above, these models are all based on the assumption that bubble growth is controlled only by the inertial and heat transfer in sequence, and neglect the transitional stage [5]. For the inertia-controlled stage, these models use the well-known Rayleigh solution [6–9,15–17]. The bubble growth rate in the inertia-controlled stage is proportional to time, while it is proportional to the square root of time during the heat-controlled stage. Thus, ignoring the transitional stage will lead to an increased initial radius in the heat-controlled stage. Therefore, the bubble radius calculated during the heat-controlled stage is larger than the experimental value, while it agrees well with experiments during the inertia-controlled stage. In addition, the calculations reveal that the duration, as well as the end radius of the bubble for the first stage, is proportional to the superheat, so larger superheat indicates a larger initial radius for the second stage.

The effects of liquid superheat on the bubble growth are illustrated in Figure 5. It is revealed that the difference in radius is almost proportional to the difference in superheat during the growth process. The bubble radius and time at the end of the inertia-controlled stage are mainly determined by the superheat and increases as the superheat increases.

Figure 6 shows the transient effects of surface tension on bubble growth with different superheats. It is found that the smaller the superheat, the greater are the instant effects of the surface tension on bubble growth, which is especially clear during the early period of the heat-controlled stage. Furthermore, all the curves indicate that the surface tension has significant instant effects on bubble variation prior to 100 μ s, while it quickly attenuates to a small effect during a short interval. However, from the viewpoint of long periods, the accumulation of the effects of surface tension is large and substantial.

Figure 5 Calculated water bubble radius vs. time with different superheats.

Figure 6 Effects of surface tension on the water bubble growth process at different superheats.

5.3 Extensive application of the model

Another common and interesting phenomenon concerns bubble formation and growth in cryogenic fluids. Here, liquid nitrogen is calculated with the model. The calculation processes are the same as the aforementioned descriptions and the fitted values of M and N in eq. (11) are -842112.25 and 12236.08 , respectively, for liquid nitrogen. Table 2 lists the physical properties of liquid nitrogen at a pressure of 1 bar and superheat of 8 K. The values were also applied to

Table 2 Physical properties of liquid nitrogen for calculations^{a)}

The properties of liquid			
Density	Specific heat at constant P	Thermal conductivity	Surface tension
806.6 kg/m ³	2042 J/(K kg)	0.1375 W/(K m)	0.008823 N/m
The properties of vapor			
Density	Specific heat at constant P	Latent heat of vaporization	
4.624 kg/m ³	1340 J/(K kg)	198.8 kJ/kg	

a) Conditions: $P=101.325$ kPa, $T=77.50$ K, $\Delta T=8$ K.

the relevant calculations with different superheats if the superheat changes did not exceed 10 K.

Figure 7 shows the changes in bubble radius in liquid nitrogen with time at different superheats, while Figure 8 shows the instant effect of surface tension during the bubble growth process. These results are very similar to those of water: (a) The bubble increases quickly during the inertia-controlled stage and then the rate of increase greatly decreases during the heat-controlled stage; (b) The bubble radius is larger with larger superheat than that with smaller superheat during the same time interval; (c) The instant effects of surface tension are important when the bubble radius is small, while it quickly decreases with the growth of bubble radius; and (d) The thickness of the boundary layer for both decreases as the superheat increases. Nevertheless, there were also some differences found between the results for water and liquid nitrogen: (a) The effects of surface tension on bubble growth for liquid nitrogen are much smaller than that for water, as a result of the much smaller value of surface tension of the former; and (b) The thickness of liquid nitrogen is slightly larger than that of water at the same

superheat, which is attributed to the relative larger ratio between thermal conductivity and latent heat of vaporization of liquid nitrogen.

6 Conclusions

The present paper solves the set of equations for bubble expansion in an extensive, uniformly superheated liquid with consideration of the internal energy of the bubble during the heat-controlled stage. The results from the inertia-controlled stage, which is calculated using the standard Rayleigh solution, were taken as the starting values for the heat-controlled stage. The calculated temporal curves of the bubble growth process for water with the model lay in between those of the Plesset-Zwick models and Forster-Zuber models. The bubble growth rate was mainly determined by the superheat during the growth process. The thickness of the thermal boundary layer around the bubble for both water and liquid nitrogen decreased as the superheat increased. At the same superheat, a larger ratio of thermal conductivity to the latent heat of vaporization results in a larger boundary layer thickness. From the viewpoint of a long period, the surface tension has considerable cumulative influence on the bubble radius, although the instant effect of surface tension on bubble growth rate reduces to a small enough value that it can be neglected over a very short time, say, 100 μs .

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Figure 7 Calculated bubble radius of liquid nitrogen vs. time with different superheats.

Figure 8 Effects of surface tension on the bubble growth process for liquid nitrogen at different superheats.

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