Constructal entransy dissipation rate minimization for a heat generating volume cooled by forced convection

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The geometry of a heat generating volume cooled by forced convection is optimized by applying the entransy dissipation extremum principle and constructal theory, while the optimal spacing between the adjacent tubes and the optimal diameter of each tube are obtained based on entransy dissipation rate minimization. The results of this work show that the optimal constructs based on entransy dissipation rate minimization and maximum temperature difference minimization, respectively, are clearly different. For the former, the porosity of the volume of channels allocated to the heat generating volume is 1/2; while for the latter, the larger the porosity is, the better the performance will be. The optimal construct of the former greatly decreases the mean thermal resistance and improves the global heat transfer performance of the system compared with the optimal construct of the latter. This is identical to the essential requirement of the entransy dissipation extremum principle that the required heat transfer temperature difference is minimal with the same heat transfer rate (the given amount of heat generated in the heat generating volume) based on the entransy dissipation extremum principle.

entransy dissipation extremum principle, constructal theory, forced convection, entransy dissipation rate, generalized thermodynamic optimization

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The necessity to cool electronic devices escalates with the continued increase in chip integration level, packaging density and operating frequency, all leading to a steep increase in the generated heat density. Nowadays, the heat transfer optimization theory involves constructal theory [1–16], entropy generation minimization theory [11,14,17,18] and the field synergy principle [19–26]. Since Bejan studied the development of the street network [1], put forward the constructal theory and applied it to optimization problems involving heat conduction [3], constructal theory has been developing rapidly in simply flow systems [27–37]. Ordonez [27] considered a finite volume that consisted of many heat generating components with uniform rate of internal heat generation and optimized the equidistant internal flow channels that were cooled by natural and forced convection, by using the minimization of maximum tempera-

ture difference as the objective. Bello-Ochende and Bejan [28] enhanced the heat transfer rate by changing the shape of the channels in the heat generating volume. By taking the maximum heat transfer rate as the optimization objective and air as the heat transfer medium, Matos et al. [29,30] adopted staggered finned circular and elliptic tubes for forced convection and obtained the optimal spacing between the tubes. Walsh and Grimes [31] studied the structures of the flow channels to obtain the minimum requirements for forced convection cooling solutions. Muzychka et al. [32] discussed the heat transfer performances of different shapes of micro channels involving parallel plates, rectangles, ellipses, triangles and polygons including their squareness and roundness. Robbe and Sciubba [33] studied the optimal internal cooling geometry of a prismatic slab, and considered four kinds of distribution layout of the channels and their comparative diameters. Kim et al. [34] studied dendritic vascularization for countering intense

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heating from the side to control the maximum temperature of the system, and further considered the transient behavior of vascularized walls exposed to sudden heating [35]. Rocha et al. [36] put forward a constructal optimization method for designing tree-shaped vascular walls in multilayer slabs. Wang et al. [37] designed tree-shaped vascularization and developed a method to control the maximum temperature of the heat generating volume using nothing more than natural convection.

Refs. [27–37] only reflected the temperature limitation of the fins with heat transfer rate maximization (maximum thermal resistance minimization). Guo et al. [26,38] proposed a new physical quantity called “entransy” that describes the heat transfer ability (Ref. [39] referred to it as heat transfer potential capacity) and the entransy dissipation extremum principle to reflect global heat transfer performance. The physical meaning of entransy was further expounded through research into, for example, physical mechanisms of heat conduction and electro-thermal simulation experiments [40,41]. Many scholars [42–59] have shown great interest in and have also studied all kinds of heat transfer optimizations based on entransy dissipation. Wei et al. [60–65], Xie et al. [66–69] and Xiao et al. [70–74] combined the entransy dissipation extremum principle with constructal theory and conducted constructal optimizations for a series of heat transfer problems. Ref. [27] only required that the temperature of the electrical devices’ encapsulation should not exceed the temperature limitation, and did not consider how to most effectively remove the heat generated. Consequently, based on Ref. [27], this paper will combine constructal theory with the entransy dissipation extremum principle and re-optimize the uniformly heated volume cooled by forced convection, to obtain the optimal construct corresponding to the minimization of entransy dissipation rate (i.e. best heat transfer performance), and compare it with the construct corresponding to the minimization of maximum temperature difference.

1 Definition of entransy dissipation rate

Entransy is a new physical quantity reflecting the heat transfer ability of an object as defined in [38]. The entransy dissipation of a whole volume is deduced as [38]

$$\dot{E}_{\text{vh}\phi} = \int_V \dot{E}_{\text{h}\phi} dv = \int_V \dot{q} \cdot \nabla T dv, \quad (1)$$

where $\dot{q}$ is the thermal current density vector, and ∇T is the temperature gradient. The equivalent thermal resistance for multi-dimensional heat conduction problems with a specified heat flux boundary condition is given as follows [38]:

$$R_h = \dot{E}_{\text{vh}\phi} / \dot{Q}_h^2, \quad (2)$$

where $\dot{Q}_h$ is the thermal current.

The entransy dissipation extremum principle proposed by Guo et al. [38] is stated as follows: for a fixed boundary heat flux, the heat transfer process is optimized when the entransy dissipation is minimized (minimum temperature difference); while for a fixed boundary temperature, the heat transfer process is optimized when the entransy dissipation is maximized (maximum heat flux). The heat transfer problem belongs to the entransy dissipation extremum principle with a fixed boundary heat flux. The smaller the entransy dissipation rate, the better the heat transfer effect. That is, when the entransy dissipation rate is minimized, the mean thermal resistance in the heat transfer process will be minimized and the performance of the system will be optimal.

2 Structure of the heat generating volume

As shown in Figure 1, a finite volume that consists of many heat generating components can be regarded as a cuboid control-volume with a uniform rate of internal heat generation. The cuboid volume has a frontal area A , length L and heat generation rate q''' (W/m³) in a solid material of thermal conductivity k_s . The heat generated is removed from the volume by the coolant in multiple channels cooled by forced convection. The coolant is a single phase fluid with constant specific heat c_p and thermal conductivity k_f . The spacing between tube axes is S , and each tube has a length L and a diameter d .

One assumes that the arrangement of tube centers is square as shown in Figure 1. The heat generated inside the shaded square with side S is conducted entirely into the flow channel situated at its center. This shaded square element is based on the area A as constructed. The square perimeter

Figure 1 Model of heat generating volume [27].

is adiabatic. For ease of analysis [27], the square element can be substituted by a cylindrical element of equivalent diameter D as in Figure 1, and thus, $\pi D^2/4=S^2$ and the number of cylindrical elements is

$$n = \frac{A}{\pi D^2/4}. \quad (3)$$

The cylindrical perimeter is adiabatic with the same temperature. The total heat generated is

$$q = nq''\frac{\pi}{4}(D^2 - d^2)L. \quad (4)$$

Another simplifying assumption is that each cylindrical element is sufficiently slender ($D < L$) so that the axial conduction is negligible. That is, the heat generated in each annular cross-section ($q'''\pi(D^2-d^2)/4$) is conducted radially in a solid shell of thickness $(D-d)/2$. The heat flux q'' through the tube wall into the fluid can be determined by the following equation:

$$q = nq''\pi dL. \quad (5)$$

2.1 Large-diameter limit

The cylindrical element for this extreme condition is shown in Figure 2. The tube diameter d is sufficiently greater than the boundary layer thickness of the coolant so that the temperature difference between the fluid and the tube wall is negligible. The core region of the fluid is still as cold as at the inlet.

The conduction differential equation in the solid shell is

$$\frac{1}{r} \frac{dT}{dr} + \frac{d^2T}{dr^2} + \frac{q'''}{k_s} = 0, \quad (6)$$

with the boundary conditions:

$$k_s \left. \frac{dT}{dr} \right|_{r=d/2} = q'' \quad \text{and} \quad T|_{r=d/2} = T_w(x), \quad (7)$$

where $T_w(x)$ is the temperature of the tube wall. By solving the equation one can obtain:

$$T(x, r) - T_w(x) = \frac{q'''}{2k_s} \left[\frac{D^2}{4} \ln \left(\frac{2r}{d} \right) - \frac{r^2}{2} + \frac{d^2}{8} \right]. \quad (8)$$

Figure 2 Model of cylindrical element with large-diameter limit [27].

Because the heat flux in the tube wall can be regarded as uniform, the temperature difference of the tube wall is [8]

$$T_w(x) - T_{\min} = \frac{2.208q''x}{k_f Pr^{1/3} Re_x^{1/2}} \quad (Pr \geq 0.5, Re_x \leq 5 \times 10^5), \quad (9)$$

where $Pr = \nu/\alpha$, $Re_x = Ux/\nu$, and U serves as “free stream” velocity to the boundary layer [27]:

$$U = \left(\frac{d\Delta P}{2.656\rho\nu^{1/2}L^{1/2}} \right)^{2/3}. \quad (10)$$

The temperature difference of the cylindrical element is

$$T(x, r, \theta) - T_{\min} = \frac{qD^2}{4Ak_f} \left[3.058 \frac{Be^{-1/3}x^{1/2}}{d^{4/3}L^{1/6}} + \frac{d^2 + 2D^2 \ln \left(\frac{2r}{d} \right) - 4r^2}{4 \frac{k_s}{k_f} (D^2 - d^2)L} \right], \quad (11)$$

where $Be = \frac{\Delta PL^2}{\mu\alpha}$ is the dimensionless group pressure drop

[27]. The total entransy dissipation rate of the whole volume is

$$\begin{aligned} \dot{E}_{v\phi} &= n \int_0^{2\pi} \int_0^L \int_0^{\frac{D}{2}} q'' \frac{qD^2}{4Ak_f} \\ &\times \left[3.058 \frac{Be^{-1/3}x^{1/2}}{d^{4/3}L^{1/6}} + \frac{d^2 + 2D^2 \ln \left(\frac{2r}{d} \right) - 4r^2}{4 \frac{k_s}{k_f} (D^2 - d^2)L} \right] r dr dx d\theta \\ &= \frac{q^2 D^4}{4A(D^2 - d^2)^2 d^{4/3}} \left\{ \frac{2.039(D^2 - d^2)L^{1/3}}{k_f Be^{1/3}} \right. \\ &\quad \left. + \frac{d^{4/3}}{8k_s L} \left[2d^2 - 3D^2 + 4D^2 \ln \left(\frac{D}{d} \right) \right] \right\}. \quad (12) \end{aligned}$$

Its corresponding mean thermal resistance R_h and mean heat transfer temperature difference $\Delta \bar{T}$ are, respectively, presented as $\dot{E}_{v\phi}/q^2$ and $\dot{E}_{v\phi}/q$. The mean thermal resistance and mean heat transfer temperature difference are nondimensionalized as

$$\begin{aligned} \tilde{R}_h &= \frac{R_h}{L/(Ak_f)} \\ &= \frac{1}{4\phi(1-\phi)} \left[2.039 Be^{-1/3} \left(\frac{d}{L} \right)^{2/3} + K \left(\frac{d}{L} \right)^2 \right], \quad (13) \end{aligned}$$

$$\Delta \tilde{T} = \frac{\Delta \bar{T}}{qL/(Ak_f)} = \tilde{R}_h, \quad (14)$$

where $\phi = d^2/D^2$ is the porosity of the volume of channels allocated to the total cuboid volume, and

$$K = \frac{k_f}{2k_s} \left[\frac{\ln \phi^{-1/2} - 1/4}{(1-\phi)} - \frac{1}{2} \right]. \quad (15)$$

The factor in square brackets in eq. (15) is a 1st order number. In most applications $k_f/k_s \ll 1$ [27], so K can be expected to be considerably smaller than 1. Eq. (13) confirms the expectation that the dimensionless mean thermal resistance keeps increasing with increasingly larger diameter d (or D).

2.2 Small-diameter limit

The cylindrical element for this extreme condition is shown in Figure 3. The tube is sufficiently slender, so that its length is covered mainly by fully developed flow and the Nu number result [8] (i.e. $Nu_{q''} = h_x d/k_f = 4.364$) in the tube with uniform heat flux is obtained. According to the Newton cooling equation, one can obtain the temperature difference between the tube wall and the core of the fluid at any axial position.

$$T_w(x) - T_f(x) = \frac{q''}{h_x} = \frac{q'' d}{4.364 k_f}. \quad (16)$$

The first law of thermodynamics yields the differential equation:

$$\frac{\pi}{4} d^2 U \rho c_p dT_f = \pi d q'' dx, \quad (17)$$

with the boundary conditions:

$$T_f = T_{\min}, \quad x = 0. \quad (18)$$

In eq. (17) the mean velocity U is [27]

$$U = \frac{d^2 \Delta P}{32 \mu L}. \quad (19)$$

Solving eq. (17) yields the temperature of the fluid:

$$T_f(x) = \frac{128 q'' L^3}{d^3 Be k_f} x + T_{\min}. \quad (20)$$

The temperature of the solid material of the cylindrical element has the same form as in eq. (8). According to eqs. (8), (16) and (20) one can obtain the temperature difference

of the cylindrical element.

$$T(x, r, \theta) = \frac{D^2 q}{4 A k_f} \times \left[\frac{1}{4.364 L} + \frac{128 L^2}{Be d^4} x + \frac{d^2 + 2 D^2 \log\left(\frac{2r}{d}\right) - 4 r^2}{4 \frac{k_s}{k_f} L (D^2 - d^2)} \right] + T_{\min}. \quad (21)$$

The total entransy dissipation rate of the whole volume is

$$\begin{aligned} \dot{E}_{vh\phi} &= n \int_0^{2\pi} \int_0^L \int_0^{\frac{D}{2}} q'' \frac{D^2 q}{4 A k_f} \\ &\times \left[\frac{1}{4.364 L} + \frac{128 L^2}{Be d^4} x + \frac{d^2 + 2 D^2 \log\left(\frac{2r}{d}\right) - 4 r^2}{4 \frac{k_s}{k_f} L (D^2 - d^2)} \right] r dr dx d\theta \\ &= \frac{q^2 L}{4 A k_f \phi (1-\phi)} \left[\frac{64}{Be} \left(\frac{d}{L}\right)^{-2} + \frac{2\phi - 3 - 2 \ln \phi}{8(1-\phi)} \frac{k_f}{k_s} \left(\frac{d}{L}\right)^2 \right. \\ &\quad \left. + \frac{1}{4.364} \left(\frac{d}{L}\right)^2 \right]. \end{aligned} \quad (22)$$

Its corresponding mean thermal resistance R_h is presented as $\dot{E}_{vh\phi}/q^2$. The mean thermal resistance is nondimensionalized as

$$\begin{aligned} \tilde{R}_h &= \frac{\dot{E}_{vh\phi}}{q^2 L / (A k_f)} \\ &= \frac{1}{4\phi(1-\phi)} \left[\frac{64}{Be} \left(\frac{d}{L}\right)^{-2} + \left(K + \frac{1}{4.364}\right) \left(\frac{d}{L}\right)^2 \right]. \end{aligned} \quad (23)$$

From the above equation, if the following inequality is valid for a much smaller d/L :

$$\frac{64}{Be} \left(\frac{d}{L}\right)^{-2} \gg \left(K + \frac{1}{4.364}\right) \left(\frac{d}{L}\right)^2, \quad (24)$$

or

$$\frac{d}{L} \ll \left[\frac{64}{K + 0.23} \right]^{1/4} Be^{-1/4}, \quad (25)$$

in the small-diameter limit eq. (23) can be reduced as

$$\tilde{R}_h = \frac{16}{\phi(1-\phi) Be} \left(\frac{d}{L}\right)^{-2}. \quad (26)$$

From [27] one can obtain

$$\frac{d}{L} \ll \left[\frac{128}{0.4} \right]^{1/4} Be^{-1/4}. \quad (27)$$

Figure 3 Model of cylindrical element with small-diameter limit [27].

According to the comparison between eqs. (25) and (27), eq. (25) is valid because of the inequality $K \ll 1$. Eq. (26) confirms the expectation that for the given ϕ the dimensionless mean thermal resistance keeps decreasing with increasing diameter d (or D).

2.3 Optimal tube diameter and minimum mean thermal resistance

From the above analyses, in the large-diameter limit, the dimensionless mean thermal resistance keeps increasing with increasing diameter, while in the small-diameter limit, the dimensionless mean thermal resistance keeps decreasing with increasing diameter. Based on the intersection of asymptotes method [27], there exists an optimal tube diameter, d_{opt} , leading to minimum mean thermal resistance. Intersecting the two asymptotes (i.e. eqs. (13) and (26)) yields

$$\frac{2.039Be^{2/3}}{64} \left(\frac{d}{L} \right)_{\text{opt}}^{8/3} + \frac{KBe}{64} \left(\frac{d}{L} \right)_{\text{opt}}^4 = 1. \quad (28)$$

Solving eq. (28) yields

$$Be^{1/4} \left(\frac{d}{L} \right)_{\text{opt}} = F(K), \quad (29)$$

where F is the dimensionless function of K as shown in Figure 4. The optimal tube diameter and its corresponding dimensionless mean thermal resistance are

$$\left(\frac{d}{L} \right)_{\text{opt}} = F Be^{-1/4}, \quad (30)$$

and
$$\tilde{R}_{h,m} = \frac{16Be^{-1/2}}{\phi(1-\phi)F^2}. \quad (31)$$

Eq. (31) shows that $\tilde{R}_{h,m}$ first decreases, and then increases with the increase of ϕ . In other words, there exists an optimal ϕ or D/d leading to a minimum $\tilde{R}_{h,m}$. By optimizing

Figure 4 Optimal tube diameter $(d/L)_{\text{opt}}$ for this paper.

eq. (31) again one can obtain ϕ_{opt} or $(D/d)_{\text{opt}}$ and its corresponding twice-minimized dimensionless mean thermal resistance as

$$\phi_{\text{opt}} = 1/2, (D/d)_{\text{opt}} = \sqrt{2}, \quad (32)$$

and
$$\tilde{R}_{h,mm} = \frac{64Be^{-1/2}}{F^2}. \quad (33)$$

In eqs. (31) and (33), the subscripts, “m” and “mm”, denote once-optimized and twice-optimized dimensionless mean thermal resistances, respectively.

3 Comparison between different optimization objectives

The optimal tube diameter, d_{opt} , based on the minimization of maximum temperature difference is obtained by the following equation [27]:

$$\frac{3.057Be^{2/3}}{128} \left(\frac{d}{L} \right)_{\text{opt}}^{8/3} + \frac{K'Be}{128} \left(\frac{d}{L} \right)_{\text{opt}}^4 = 1, \quad (34)$$

where $K' = \frac{k_f}{2k_s} \left(\frac{\ln \phi^{-1/2}}{1-\phi} - \frac{1}{2} \right)$. Solving eq. (34) yields

$$Be^{1/4} \left(\frac{d}{L} \right)_{\text{opt}} = F'(K'), \quad (35)$$

where F' is the dimensionless function of K' as shown in Figure 5. The optimal tube diameter and its corresponding dimensionless mean thermal resistance are

$$\left(\frac{d}{L} \right)_{\text{opt}} = F' Be^{-1/4}, \quad (36)$$

and
$$\tilde{R}_{h,t} = \frac{16Be^{-1/2}}{\phi(1-\phi)F'^2}. \quad (37)$$

Table 1 lists the optimal constructs based on the minimi-

Figure 5 Optimal tube diameter $(d/L)_{\text{opt}}$ from [27].

Table 1 Optimal constructs for different optimization objectives

Optimization objectives	$(d/L)_{\text{opt}}$	$(D/d)_{\text{opt}}$	$\bar{R}_h$	$\Delta \bar{T}$
Minimization of entransy dissipation rate (this paper)	$FBe^{-1/4}$	$\sqrt{2}$	$64Be^{-1/2}/F^2$	$64Be^{-1/2}/F^2$
Minimization of maximum temperature difference ([27])	$F'Be^{-1/4}$	Better performance with even smaller value	$\frac{16Be^{-1/2}}{\phi(1-\phi)F'^2}$ a)	$\frac{16Be^{-1/2}}{\phi(1-\phi)F'^2}$ a)

a) These are derived from [27].

zation of entransy dissipation rates and maximum temperature differences. The results of [27] show that the performance improves as the porosity increases, or as D/d decreases. That is, when the minimization of maximum temperature difference is taken as the optimization objective, the better designs have relatively thin shells of heat generating material, and relatively higher volumetric heat generating rates (q''') for the total heat generation (q). This paper demonstrates the optimal constructal design, i.e. $\phi_{\text{opt}} = 1/2$ or $(D/d)_{\text{opt}} = \sqrt{2}$.

For visual comparison, $k_f/k_s=1/5$ and $\phi=1/5$ are set, and thus, $K=0.02$ and $K'=0.05$. Figures 4 and 5 show that $F=3.571$ and $F'=3.914$ are obtained. The calculations show that the optimal constructs based on entransy dissipation rate minimization and maximum temperature difference minimization are clearly different. The profiles of the optimized cylindrical elements based on entransy dissipation rate minimization and maximum temperature difference minimization are shown in Figure 6. The number of cylindrical elements for the former is much larger than that for the latter. Calculations show that the number of cylindrical elements for the former is 18 times that for the latter. Based on entransy dissipation rate minimization, the mean thermal resistance is decreased by 23.12%, and the heat transfer temperature difference is even smaller. Because the result based on the entransy dissipation extremum principle should be better, the heat transfer performance of the heat generating volume is improved.

4 Conclusions

The geometry of the heat generating volume cooled by forced convection is optimized by combining the entransy dissipation extremum principle with constructal theory. The optimal spacing between adjacent tubes and the optimal diameter of each tube are obtained based on the intersection of asymptotes method for minimizing the entransy dissipation rate. The optimized results and optimal constructs based on entransy dissipation rate minimization and maximum temperature difference minimization, respectively, are clearly different. For the former, the optimal design means $\phi_{\text{opt}} = 1/2$ or $(D/d)_{\text{opt}} = \sqrt{2}$; while for the latter, the better design means an increasing ϕ or a decreasing D/d . The results show that for the former the optimal construct greatly decreases the mean thermal resistance, and clearly improves

Figure 6 Optimal constructs (profiles of cylindrical elements) based on the two optimization objectives. (a) Minimization of entransy dissipation rate; (b) minimization of maximum temperature difference.

the global heat transfer performance of the heat generating volume. The mean thermal resistance as defined based on entransy dissipation rate reflects the global heat transfer performance. A lower mean thermal resistance means better heat transfer performance, lower mean temperature difference and a higher efficiency of heat transfer. The optimal construct corresponding to the minimization of maximum temperature difference shows the temperature limitation of the encapsulation of the electrical devices cooled by forced convection, does not reflect the global heat transfer performance. In this paper, the optimal construct corresponding to the minimization of entransy dissipation rate reflects the global heat transfer performance, and improves the global heat transfer performance of the system more efficiently. The constructal design corresponding to the minimization of entransy dissipation rate should be adopted for designing heat generating volumes cooled by forced convection when engineering for safe conditions. The work and results provided in this paper enrich the constructal theory and entransy dissipation extremum principle, and further develop the entransy dissipation extremum principle with some significance.

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