Numerical study of threshold gain behavior for a THz random laser in a two-dimensional active disordered medium with a three-level atomic system

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By numerically solving Maxwell's equations and rate equations in a two-dimensional (2D) active random media made of ruby grains with a three-level atomic system, the threshold gain behavior for a THz random laser is investigated. The spectral intensity variation with the pumping rate is calculated for both the transverse magnetic (TM) field and the transverse electric (TE) field. The computed results show that THz random lasing could occur in a 2D disordered medium for both the TM and TE cases. Further analysis reveals that the THz lasing threshold for TM fields is lower than that for TE fields.

random lasers, polarization, terahertz

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Since random laser action was first predicted by Letokhov in 1968 [1] and observed by Lawandy in 1994 [2], many experimental and theoretical studies have been performed [3–17]. To numerically simulate the propagation and localization of electromagnetic waves in such random media, the finite-difference time-domain (FDTD) method has been widely used. Moreover, various types of auxiliary differential equation FDTD (ADE-FDTD) approaches for the analysis of nonlinear and active systems have also been developed. Jiang and Soukoulis successfully reproduced random lasing in one-dimensional (1D) random media using a semi-classical model. In this model, FDTD method is coupled with the rate equation in a four-level energy structure [6]. Their methodology has been extended to the two-dimensional (2D) case.

Note that the search for efficient, high-power, inexpensive, and compact methods for the generation of coherent THz radiation has been a primary research thrust in modern optoelectronics and photonics [18–25]. For this, recently, a

1D theoretical model was built for active disordered media made of ruby grains with a three-level atomic system. These numerical results revealed that terahertz (THz) random lasing could occur in the 1D case under suitable conditions [17]. Although 1D random laser can reveal qualitative properties in a real random lasing system, there are still many physical mechanisms which cannot be investigated using a 1D model. Therefore, the 2D model has been widely used to research random lasing in the optical band [6,12–16].

We aim to construct a model to reveal THz random lasing effects in the 2D case. In this work, by numerically solving Maxwell's equations and rate equations in a 2D active random media made of ruby grains with a three-level atomic system, the threshold gain behavior of a THz random laser is investigated. Using this theory, numerical calculations are performed. The results show that THz random lasing could occur in a 2D disordered medium and that the lasing threshold for transverse magnetic (TM) fields is lower than that for transverse electric (TE) fields. We believe that these results will provide some valuable predictions that

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will motivate future experimental observations.

1 Theoretical model

Our model starts with a 2D square disordered medium with area, l^2 . It consists of circular particles with radius r , optical index n_2 , and surface-filling fraction Φ . This is shown in Figure 1(a). These disordered particles are randomly distributed in a background medium with refractive index n_1 . Using the given medium parameters (i.e. l , r , Φ , n_1 and n_2), a wide variety random media can be built. The medium parameters used in the following numerical calculations are: $l = 5$ cm, $r = 80$ μm , $\Phi = 40\%$, $n_1 = 1$, $n_2 = 10$. Note that the refractive index for THz radiation in ruby is twice that of the one for visible wavelengths [26].

For a TM field, Maxwell's equations are

$$\mu_0 \frac{\partial H_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \quad (1a)$$

$$\mu_0 \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \quad (1b)$$

$$\varepsilon_i \varepsilon_0 \frac{\partial E_z}{\partial t} + \frac{\partial P_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}, \quad (1c)$$

where E_z for TM field is the z component of the electric field: $\vec{E} = E_x \mathbf{e}_x + E_y \mathbf{e}_y + E_z \mathbf{e}_z$. In this case, $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are the unit vectors in the x , y and z directions, respectively. P_z is the z component of the polarization density vector: $\vec{P} = P_x \mathbf{e}_x + P_y \mathbf{e}_y + P_z \mathbf{e}_z$. In this case, P_x and P_y are the polarization density components in the x and y directions, respectively. n_i^2 and μ_0 are the electric permittivity and the magnetic permeability in a vacuum, respectively. $\varepsilon_i = n_i^2$ ($i=1, 2$).

If the active medium includes ruby, it can be described as a three-level atomic system. For such a system, the three-level rate equations are [17]

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} + \frac{N_3}{\tau_{31}} - N_1 W_p, \quad (2a)$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - \frac{N_2}{T} + \frac{E_z}{\hbar \omega_l} \cdot \frac{dP_z}{dt}, \quad (2b)$$

$$\frac{dN_3}{dt} = N_1 W_p - \frac{N_3}{\tau_{31}} - \frac{N_3}{\tau_{32}} + \frac{N_2}{T} - \frac{E_z}{\hbar \omega_l} \cdot \frac{dP_z}{dt}. \quad (2c)$$

Figure 1 (a) Schematic diagram of the 2D random medium made of ruby grains; (b) the energy levels for Cr^{3+} in ruby.

In the above rate equations, the system treated here is a simplified yet realistic three-level atomic system (Figure 1(b)) with energy levels E_1 , E_2 , E_3 , and their corresponding populations N_1 , N_2 and N_3 , respectively. W_p is the rate of external optical pumping at 693.9 nm that transfers atoms from the ground state (E_1) to level E_3 . $1/\tau_{ij}$ is the spontaneous decay rate for an atom in energy level E_i to level E_j . T is the time of flight between emission and re-absorption for a 29 cm^{-1} phonon, and it was chosen to be 1×10^{-9} s in our system [17]. $\omega_l = (E_3 - E_2)/\hbar = 2\pi \times 0.87$ THz is the transition frequency from level E_3 to level E_2 . The term, $(E/\hbar \omega_l) dP/dt$ is the classical expression for the instantaneous energy transfer divided by the energy per photon, and is equivalent to the stimulated transition probabilities that are more commonly used in the rate equation above.

The polarization density component (P_z) satisfies the following equation:

$$\frac{d^2 P_z}{dt^2} + \Delta \omega_l \frac{dP_z}{dt} + \omega_l^2 P_z = \kappa \Delta N E_z, \quad (3)$$

where $\Delta \omega_l = 1/\tau_{32} + 2/T_2$ is the actual linewidth (gain spectrum width) of the transition centered at angular frequency ω_l . T_2 is the collision time; κ is given by $\kappa = 6\pi \varepsilon_0 c_3 / \omega_l^2 \tau_{32}$; $\Delta N = N_2 - N_3$ is the population density difference between the populations in levels 2 and 3.

For a TE field, the corresponding equations are

$$\frac{\partial H_z}{\partial y} = \varepsilon_0 \varepsilon_i \frac{\partial E_x}{\partial t} + \frac{\partial P_x}{\partial t}, \quad (4a)$$

$$-\frac{\partial H_z}{\partial x} = \varepsilon_0 \varepsilon_i \frac{\partial E_y}{\partial t} + \frac{\partial P_y}{\partial t}, \quad (4b)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_0 \frac{\partial H_z}{\partial t}, \quad (4c)$$

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} + \frac{N_3}{\tau_{31}} - N_1 W_p, \quad (5a)$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - \frac{N_2}{T} + \frac{\vec{E}}{\hbar \omega_l} \cdot \frac{d\vec{P}}{dt}, \quad (5b)$$

$$\frac{dN_3}{dt} = N_1 W_p - \frac{N_3}{\tau_{31}} - \frac{N_3}{\tau_{32}} + \frac{N_2}{T} - \frac{\vec{E}}{\hbar \omega_l} \cdot \frac{d\vec{P}}{dt}, \quad (5c)$$

$$\frac{d^2 \vec{P}}{dt^2} + \Delta \omega_l \frac{d\vec{P}}{dt} + \omega_l^2 \vec{P} = \kappa \Delta N \vec{E}, \quad (6)$$

where $\vec{E} = E_x \mathbf{e}_x + E_y \mathbf{e}_y$ and $\vec{P} = P_x \mathbf{e}_x + P_y \mathbf{e}_y$ for a TE field.

The values used for these parameters in that calculations are: $T_2 = 2 \times 10^{-14}$ s, $\tau_{32} = 1.1 \times 10^{-9}$ s, $\tau_{21} = 3 \times 10^{-3}$ s, $\tau_{31} = 3 \times 10^{-6}$ s and $N_T = \sum_{i=1}^3 N_i = 1.6 \times 10^{25} \text{ m}^{-3}$.

To excite the system, we must introduce sources [12]. Therefore, a broadband Gaussian pulse with a random am-

plitude was used as an excitation source in our system. Then, using FDTD methods and Perfectly Matched Layer (PML) absorbing boundary condition, the electromagnetic fields can be calculated by solving the equations listed above. In the FDTD calculation, we discretize both space and time. The discrete space and time steps were chosen to be $\Delta x = 2 \times 10^{-6}$ m and $\Delta t \leq \Delta x / (2c)$, where Δt is taken to be 2×10^{-15} s. The pulse response was recorded over a time window of length $T_w = 5.2 \times 10^{-10}$ s at all nodes in the system. It was then Fourier transformed to obtain the intensity spectrum.

2 Numerical results

First, we calculated the spectral intensity for various pumping rates for the TM field (Figure 2). In each panel, two peaks are marked by their central wavelengths λ_{M1} and λ_{M2} . Note that every peak denotes a localized mode that is supported by the disordered medium. Hereinafter the subscripts M and E denote the TM and TE cases, respectively. From Figure 2(a), it can be seen that there are many discrete and weak peaks in the spectrum at a pumping rate of $W_p = 100$ s⁻¹. All of these peaks are within one order of magnitude in intensity. When the pumping rate is increased, the mode λ_{M1} dominates because it is amplified preferentially. Also, its spectral width becomes broader at $W_p = 0.9 \times 10^3$ s⁻¹ than those at lower pumping rates. With further pumping rate increases, the spectral width of mode λ_{M1} begins to decrease, but its peak intensity increases (Figure 2(c)). Then, another

mode λ_{M2} is also excited at $W_p = 5.0 \times 10^4$ s⁻¹ (Figure 2(d)). Finally, the peak intensities and widths of two modes reach to their steady-state values when the pumping rate reaches its largest values (Figure 2(e)).

Figure 3 is the counterpart to Figure 2 for the TE case. It can be seen from this figure that the spectra intensity and width evolution of the TE state is similar to that of the TM case.

To obtain more information on the threshold gain behavior for both the TM and TE modes, numerical calculations were performed at different pumping rates. From these calculations we can obtain the curves of the peak intensity vs the pumping rate. This is shown in Figure 4. The selected modes have the minimum lasing threshold for both the TM and TE modes. The wavelength of the TM mode is $\lambda_{M1} = 332.8$ μ m, and the wavelength of the TE mode is $\lambda_{E1} = 332.1$ μ m. Using a conventional method, the pump thresholds for the two modes can be measured from the intensity curves in Figure 4. For this calculation we obtained $W_M = 0.9 \times 10^3$ s⁻¹ and $W_E = 4.5 \times 10^3$ s⁻¹. This shows that the TM mode has a lower threshold than the TE mode. This conclusion is in accord with the experimental results presented by Ito of Shizuoka University in the optical band [12].

We provide a monochromatic pulse at 332.5 μ m to pump the medium at all sites to obtain the spatial distributions of both the TM and TE modes. Figure 5 shows that the two modes have different spatial distributions. The spatial distribution of the TM mode is much more centralized than that of the TE mode.

Figure 2 Spectral intensity vs. the wavelength (λ) for TM fields in the 2D random medium. (a) $W_p = 100$ s⁻¹; (b) $W_p = 0.9 \times 10^3$ s⁻¹; (c) $W_p = 1.0 \times 10^4$ s⁻¹; (d) $W_p = 5.0 \times 10^4$ s⁻¹; (e) $W_p = 1.0 \times 10^5$ s⁻¹.

Figure 3 Spectral intensity vs. wavelength (λ) for TE fields in the 2D random medium. (a) $W_p = 100$ s⁻¹; (b) $W_p = 0.9 \times 10^3$ s⁻¹; (c) $W_p = 1.0 \times 10^4$ s⁻¹; (d) $W_p = 5.0 \times 10^4$ s⁻¹; (e) $W_p = 1.0 \times 10^5$ s⁻¹.

Figure 4 Peak intensity of the mode with the minimum lasing threshold vs. the pumping rate for both the TE and TM cases.

Figure 5 Spatial distribution of the field amplitude corresponding to the marked modes. (a) TM mode; (b) TE mode.

To determine whether or not the above results can be universally applied, we performed many calculations for different combinations of system parameters. All the calculated results demonstrate that our conclusion is universal.

3 Conclusion

In summary, a numerical study of a two-dimensional disordered media made of ruby grains with a three-level atomic system was performed. The calculation results show that THz random lasing can occur in a 2D disordered medium, and that the lasing threshold of TM fields is lower than that of TE fields.

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