Exact solutions of dispersion equation for MHD waves with short-wavelength modification

CHEN Ling^{1,2} & WU DeJin^{1*}

¹Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China;

²Graduate University of Chinese Academy of Sciences, Beijing 100012, China

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Dispersive magnetohydrodynamic (MHD) waves with short-wavelength modification have an important role in transforming energy from waves into particles. In this paper, based on the two-fluid mode, a dispersion equation, including the short-wavelength effect, and its exact solution are presented. The outcome is responsible for the short-wavelength modification versions of the three ideal MHD modes (i.e. the fast, slow and Alfvén). The results show that the fast and Alfvén modes are modified considerably by the short-wavelength effect mainly in the quasi-parallel and quasi-perpendicular propagation directions, respectively, while the slow mode can be affected by the short-wavelength effect in all propagation directions. On the other hand, the dispersive modification occurs primarily in the finite- β regime of $0.001 < \beta < 1$ for the fast mode and in the high- β regime of $0.1 < \beta < 10$ for the slow mode. For the Alfvén mode, the dispersive modification occurs from the low- β regime of $\beta < 0.001$ through the high- β regime of $\beta > 1$.

plasma, MHD waves, short-wavelength effect, dispersion relation

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It is well known that there are three ideal magnetohydrodynamic (MHD) modes: the fast, slow and Alfvén waves in the low-frequency limit (much lower than the ion cyclotron frequency ω_{ci}) and the long-wavelength limit (much longer than the ion Larmor radius ρ_i). When wavelengths reduce to the characteristic length scales, such as λ_e and ρ_i , where λ_e is the electron inertial length, the ions are free while the electrons are tightly bound to magnetic field lines because their Larmor radius is much smaller than that of the ions (i.e. $\rho_e \ll \rho_i$). This leads to the presence of spatial charge and the dispersion of the MHD waves. In the case of the Alfvén wave, the dispersive wave is called the kinetic Alfvén wave (KAW). In the KAW, the perturbed electric fields have a non-zero component parallel to the background magnetic field that may cause efficient heating and acceleration of the plasma particles [1–4].

KAWs have been extensively discussed because of their potential importance in the particle energization of plasmas.

Also, they have been applied to laboratory, space and astrophysical plasmas, such as tokamak plasma heating [5–7], auroral electron acceleration [8–11], solar coronal plasma heating [12–14], and abnormal heating of heavy ions in the extended corona [15,16]. On the other hand, the other two modes, the fast and slow waves, have been given little attention. Some recent studies on the MHD turbulence in interstellar and interplanetary spaces show that the energy cascades primarily by developing small-scales structures perpendicular to the local field, with $k_\perp \gg k_\parallel$ [17–20]. Large values of $k_\perp$ could be the consequence of refraction [21,22], resonant absorption [23–25], and turbulent cascade [26]. This result is supported by numerical simulations of magnetized turbulence with a dynamically strong mean field [27,28]. *In situ* measurements of turbulence in the solar wind and observations of interstellar scintillation also show similar evidence for the theoretical results above [29–34].

For a warm plasma with a high- β of $\beta_{i(e)} \sim 1$, which is the case for the solar wind and the magnetosheath, the compression Alfvén wave is coupled with the acoustic wave and leads

*Corresponding author (email: djwu@pmo.ac.cn)

to fast and slow magnetosonic waves. In particular, the fact that the ion thermal speed is $v_{Ti} \sim v_A$ (the Alfvén velocity) indicates that the ion gyroradius ρ_i is comparable to the ion inertial length λ_i . This implies that the short-wavelength effect can considerably modify the dispersion relations of the fast and slow magnetosonic waves, as well as modifying the dispersion relation of the shear Alfvén wave, such as the case of KAWs. Some theoretical studies of MHD turbulence cascades in the solar wind show that the fluctuations in the MHD modes and their collisionless Landau damping are responsible for the steeping of the MHD turbulence spectrum at higher wavenumbers- $k_\perp$ (i.e. short-wavelengths) [26,35–38]. Recently, similar low-frequency phenomena has been investigated by many authors, for example, the ionospheric dynamo [39], shock arrival time [40], the response of magnetosphere to interplanetary shock [41,42], suprathermal particle events [43], distribution of field-aligned current carried by Alfvén waves [44], the behavior of plasma armature [45] and MHD processes of flight vehicles [46–48].

In this paper, we derive an exact dispersion equation, based on the two-fluid model, which contains all three MHD modes with the short-wavelength modification. Furthermore, based on the dispersion equation, we present the dispersion relations, which are the versions, with short-wavelength modification, of the three modes: the fast and slow magnetosonic waves and the shear Alfvén wave. The effects of the propagation angle, the short-wavelength modification and the plasma beta on the three dispersive waves are also discussed.

1 Dispersion equation in the two-fluid model

In a proton-electron plasma, which is magnetized by a uniform magnetic field $\mathbf{B}_0$ along the z direction, the complete set of two-fluid equations can be written as follows:

$$(\partial_t + \mathbf{v}_i \cdot \nabla)n_i + n_i \nabla \cdot \mathbf{v}_i = 0, \quad (1)$$

$$(\partial_t + \mathbf{v}_e \cdot \nabla)n_e + n_e \nabla \cdot \mathbf{v}_e = 0, \quad (2)$$

$$(\partial_t + \mathbf{v}_i \cdot \nabla)\mathbf{v}_i = \frac{e}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \frac{1}{m_i n_i} \nabla p_i, \quad (3)$$

$$(\partial_t + \mathbf{v}_e \cdot \nabla)\mathbf{v}_e = -\frac{e}{m_e}(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \frac{1}{m_e n_e} \nabla p_e, \quad (4)$$

$$(\partial_t + \mathbf{v}_i \cdot \nabla)(p_i n_i^{-\gamma_i}) = 0, \quad (5)$$

$$(\partial_t + \mathbf{v}_e \cdot \nabla)(p_e n_e^{-\gamma_e}) = 0, \quad (6)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad (7)$$

$$\nabla \times \mathbf{B} = \mu_0 e(n_i \mathbf{v}_i - n_e \mathbf{v}_e) + \epsilon_0 \mu_0 \partial_t \mathbf{E}, \quad (8)$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ the magnetic field, ∂_t the partial derivative with respect to the time t , ∇ the 3-dimensional spatial gradient operator, e the elementary charge, μ_0 the permeability of free space, ϵ_0 the permittivity of free space, $m_{e(i)}$, $n_{e(i)}$, $\mathbf{v}_{e(i)}$, $p_{e(i)}$, and $\gamma_{e(i)}$ are the electron (ion) mass, density, velocity, thermal pressure, and polyindex, respectively, α is

the angle between the wave vector $\mathbf{k}$ and the ambient magnetic field $\mathbf{B}_0$ (i.e. the z axis).

We mark the unperturbed and perturbed quantities with the subscript 0 and the prefix δ , respectively. For one-dimensional plane waves propagating in the x - z plane, with wave vector $(k_x, 0, k_z)$ and the frequency ω , all the perturbed quantities vary in the form: $\delta f \propto \exp[-i(\omega t - k_x x - k_z z)]$. The linearizing perturbed velocities of the ions and electrons can be obtained as follows:

$$\delta \mathbf{v}_{i\perp} - \left(\frac{\omega}{\omega_{ci}}\right)^2 \delta \mathbf{v}_{iz} = \frac{1}{B_0^2} \left(\delta \mathbf{E} \times \mathbf{B}_0 - \frac{i \delta p_i}{q_i n_{i0}} \mathbf{k} \times \mathbf{B}_0 \right) - \frac{i q_i \omega}{m_i \omega_{ci}^2} \left(\delta \mathbf{E} - \frac{i \delta p_i \mathbf{k}}{q_i n_{i0}} \right), \quad (9)$$

$$\delta \mathbf{v}_{e\perp} - \left(\frac{\omega}{\omega_{ce}}\right)^2 \delta \mathbf{v}_{ez} = \frac{1}{B_0^2} \left(\delta \mathbf{E} \times \mathbf{B}_0 - \frac{i \delta p_e}{q_e n_{e0}} \mathbf{k} \times \mathbf{B}_0 \right) - \frac{i q_e \omega}{m_e \omega_{ce}^2} \left(\delta \mathbf{E}_z - \frac{i \delta p_e \mathbf{k}_z}{q_e n_{e0}} \right), \quad (10)$$

where the unperturbed velocity $\mathbf{v}_0 = 0$ and the low-frequency limit ($\omega \ll \omega_{ci}$) have been used. We neglect the contribution of the polarization drifts of the electrons (the last parenthesis term in the transverse components of (10)) compared to that of the ions. The pressure perturbations may be obtained from eqs. (5) and (6):

$$\delta p_i = \gamma_i T_{i0} \delta n_i, \quad \delta p_e = \gamma_e T_{e0} \delta n_e, \quad (11)$$

where the density perturbations δn_i and δn_e follow from eqs. (1) and (2):

$$\delta n_i = n_{i0} \frac{\mathbf{k} \cdot \delta \mathbf{v}_i}{\omega}, \quad \delta n_e = n_{e0} \frac{\mathbf{k} \cdot \delta \mathbf{v}_e}{\omega}. \quad (12)$$

Following Wu [49], for the low-frequency and the nonrelativistic cases, in which the Maxwell's displacement current term, on the right-hand side of eq. (8), may be neglected, equating Faraday's Law and Ampere's Law yields a dispersion tensor of the form $\Lambda \cdot \mathbf{E} = 0$. The dispersion equation can then be obtained by calculating $|\Lambda| = 0$ as follows:

$$\Omega^6 - A\Omega^4 + B\Omega^2 - C = 0, \quad (13)$$

where the coefficients

$$A = K_A + \frac{k^2}{k_z^2} + C; \quad B = \frac{k^2}{k_z^2} K_A + 2C; \quad C = \frac{\beta}{1 + Q + k_\perp^2 \lambda_e^2 k_z^2}. \quad (14)$$

In the expressions above, the parameter

$$K_A \equiv \frac{1 + Q + k_\perp^2 \rho_s^2}{1 + Q + k_\perp^2 \lambda_e^2} \quad (15)$$

is the coefficient of the KAW dispersion relation, $\Omega^2 = K_A$ [3,39], in a low- β plasma, where $\Omega \equiv \omega/(k_z v_A)$ is the wave frequency in units of Alfvén frequency ($k_z v_A$). The other parameters are $v_s \equiv \sqrt{(\gamma_i T_{0i} + \gamma_e T_{0e})/m_i}$: the ion acoustic

speed, $\rho_s \equiv v_s/\omega_{ci}$: the ion acoustic gyroradius, $\lambda_e \equiv c/\omega_{pe}$: the electron inertial length, $\omega_{pe} \equiv \sqrt{n_0 e^2/m_e \epsilon_0}$: the electron plasma frequency, $\beta \equiv v_s^2/v_A^2 = \beta_i + \beta_e$: the ratio of the total thermal pressure to the magnetic pressure, $Q \equiv m_e/m_i$: the ratio of the electron mass to the ion mass, and $k^2 = k_\perp^2 + k_z^2$: the total wavenumber. Finally, we note that in the derivation of the dispersion eq. (13), we have used the quasi-neutrality condition $n_i \approx n_e$.

2 Solutions with the short-wavelength modification

The dispersion eq. (13) can be rewritten as the standard form of the Cardan equation, that is

$$(\Omega^2 - \Omega_0^2)^3 + p(\Omega^2 - \Omega_0^2) + q = 0, \quad (16)$$

where

$$p = B - 3\Omega_0^4, \quad q = -2\Omega_0^6 + B\Omega_0^2 - C \quad (17)$$

with the parameter $\Omega_0^2 = A/3$. When the discriminant $\Delta \equiv q^2/4 + p^3/27 < 0$, the Cardan eq. (16) has three real roots as follows:

$$\Omega_F^2 = \Omega_0^2 + 2R \cos \frac{\theta}{3}, \quad (18)$$

$$\Omega_S^2 = \Omega_0^2 + 2R \cos \frac{\theta + 2\pi}{3}, \quad (19)$$

and

$$\Omega_A^2 = \Omega_0^2 + 2R \cos \frac{\theta + 4\pi}{3}, \quad (20)$$

where

$$R = \sqrt{\Omega_0^4 - \frac{B}{3}}; \quad \theta = \arccos \frac{-q}{2R^3}. \quad (21)$$

In the long-wavelength limit of $k_\perp^2 \lambda_e^2$ and $k_\perp^2 \rho_s^2 \ll 1$, one has

$$A \approx 1 + (1 + \beta) \frac{k^2}{k_z^2}, \quad B \approx (1 + 2\beta) \frac{k^2}{k_z^2}, \quad C \approx \beta \frac{k^2}{k_z^2}, \quad (22)$$

and $K_A \approx 1$. As a consequence, from eqs. (18)–(20), the three roots of the Cardan eq. (16) in the long-wavelength limit are

$$\Omega_F^2 \approx \frac{1 + \beta}{2} \left[1 + \sqrt{1 - \frac{4\beta}{(1 + \beta)^2} \frac{k_z^2}{k^2}} \right] \frac{k^2}{k_z^2}, \quad (23)$$

$$\Omega_S^2 \approx \frac{1 + \beta}{2} \left[1 - \sqrt{1 - \frac{4\beta}{(1 + \beta)^2} \frac{k_z^2}{k^2}} \right] \frac{k^2}{k_z^2}, \quad (24)$$

and

$$\Omega_A^2 \approx 1. \quad (25)$$

These are the dispersion relations for the ideal MHD modes (i.e. the fast and slow magnetosonic waves and the shear Alfvén wave). Therefore, the three roots, in eqs. (18)–(20),

of the Cardan eq. (16) are the modified versions, including the short-wavelength effects, of the dispersion relations for the three ideal MHD waves.

Figure 1 plots the three roots in eqs. (18)–(20) versus the propagation angle α for the fixed parameters $k_\perp \rho_s = 0.8$ and $\beta = 0.5$. From the top down, the curves correspond to the fast, Alfvén and slow modes, respectively. For the sake of comparison, the corresponding ideal MHD solutions are presented as dashed lines in Figure 1.

From Figure 1, we see that the fast and Alfvén modes are considerably modified by the short-wavelength effect in the quasi-parallel and quasi-perpendicular directions, respectively. The slow mode, however, obviously departs from its ideal MHD version from $\alpha = 0^\circ$ through $\alpha = 90^\circ$ due to the short-wavelength effect. The parallel phase speed of the fast mode increases by a factor of ~ 2 in the quasi-parallel direction of $\alpha < 20^\circ$ and that of the Alfvén mode increases by a factor of ~ 1.5 in the quasi-perpendicular direction of $\alpha > 50^\circ$. For the slow mode, the parallel phase speed decreases by a factor of ~ 2 in the quasi-parallel direction of $\alpha < 20^\circ$ and a slightly lower factor of ~ 1.5 in the quasi-perpendicular direction of $\alpha > 50^\circ$ due to the short-wavelength effect.

Figure 2 shows the dependence of the dispersion relations for the fast, Alfvén and slow modes (from the top down) on the perpendicular wavenumber ($k_\perp \rho_s$) for the quasi-parallel ($\alpha = 15^\circ$ for the left column) and quasi-perpendicular ($\alpha = 60^\circ$ for the right column) propagating cases. The fixed parameter $\beta = 0.5$ has been used and the dashed lines plot the corresponding ideal MHD solutions for the sake of comparison.

From Figure 2, we have found, for all cases, the modification of the dispersion relations to be negligibly small

Figure 1 The dispersion relations versus the propagation angle. Panels, from the top down, present the fast, Alfvén and slow modes, respectively, where the dashed lines plot the corresponding ideal MHD solutions, where the parameters $k_\perp \rho_s = 0.8$ and $\beta = 0.5$ have been used.

Figure 2 The dispersion relations versus the perpendicular wavenumber. Panels, from the top down, present the fast, Alfvén and slow modes, respectively, where the dashed lines plot the corresponding idea MHD solutions for the sake of comparison and the left and right columns correspond to the cases of quasi-parallel ($\alpha = 15^\circ$) (a) and quasi-perpendicular ($\alpha = 60^\circ$) (b) propagations, respectively.

when $k_\perp \rho_s < 0.1$ and to increase quickly with $k_\perp \rho_s$ when $k_\perp \rho_s > 0.1$. This clearly implies that the dispersion modification is caused by the short-wavelength effect in the perpendicular direction. The short-wavelength effect leads to the increase of the parallel phase speed for both the fast and Alfvén modes, but the decrease for the slow mode.

On the other hand, the plasma β parameter (the ratio of thermal to magnetic pressures) can remarkably influence the dispersion modification as well. Figure 3 shows the dependence of the dispersion relations for the fast, Alfvén and slow modes (from the top down) on the plasma β parameter for the quasi-parallel ($\alpha = 15^\circ$ for the left column) and quasi-perpendicular ($\alpha = 60^\circ$ for the right column) propagating cases. We have used the fixed perpendicular wavenumber $k_\perp \rho_s = 0.8$ and have plotted the corresponding ideal MHD solutions with dashed lines for the sake of comparison.

From Figure 3, we find that, for the fast mode, the dispersion modification mainly occurs in the finite- β regime of $0.001 < \beta < 1$ for the quasi-parallel propagating case (see the top panel of the left column). For this case, the parallel phase speed is considerably higher than its ideal MHD solution, whereas the modifications are negligibly small for the quasi-perpendicular propagating case (see the top panel of the

right column), as shown in the top panel of Figure 1. On the other hand, for the slow mode, the dispersion modification occurs mainly in the high- β regime of $0.1 < \beta < 10$ for both the quasi-parallel and quasi-perpendicular propagating cases, in which the parallel phase speed is lower than its ideal MHD solution (see the bottom panels in Figure 3).

For the Alfvén mode, however, the matter is more complex. In the low- β regime of $\beta < 0.001$, the parallel phase speed is remarkably lower than its ideal MHD solution, but is higher than its ideal MHD solution in the finite- and high- β regimes of $0.001 < \beta < 10$. For the extremely high- β regime of $\beta > 100$, the parallel phase speed of the Alfvén mode approaches its ideal MHD solution as β increases. As shown in Figure 1, the modification in the quasi-perpendicular propagation direction is much larger than that in the quasi-parallel propagation direction in the finite- and high- β regimes of $0.001 < \beta < 10$.

Incidentally, it is worth noting that, for the quasi-parallel propagating case (see the left column in Figure 3), both the fast and Alfvén modes encounter the Landau resonance of electrons near $\beta \approx m_e/m_i < 10^{-3}$ due to $v_A \approx v_{Te}$ (the electron thermal speed), where the flow description is invalid and the kinetic theory becomes necessary.

Figure 3 The dispersion relations versus plasma beta parameter. Panels, from the top down, present the fast, Alfvén and slow modes, respectively, where the dashed lines plot the corresponding idea MHD solutions for the sake of comparison and the left and right columns correspond to the cases of quasi-parallel ($\alpha = 15^\circ$) (a) and quasi-perpendicular ($\alpha = 60^\circ$) (b) propagations, respectively.

3 Low- β and quasi-perpendicular propagation limits

In the low- β limit of $\beta \rightarrow 0$, one has approximately, from eq. (14):

$$A \cong K_A + \frac{k^2}{k_z^2}, \quad B \cong K_A \frac{k^2}{k_z^2}, \quad C = \frac{\beta}{1 + Q + k_\perp^2 \lambda_e^2} \frac{k^2}{k_z^2} \cong 0. \quad (26)$$

Consequently, the dispersion eq. (13) can be approximated as

$$\Omega^2 \left(\Omega^2 - \frac{k^2}{k_z^2} \right) (\Omega^2 - K_A) \cong \frac{\beta}{1 + Q + k_\perp^2 \lambda_e^2} \frac{k^2}{k_z^2} \cong 0. \quad (27)$$

Its three roots are

$$\Omega_S^2 = 0; \quad \Omega_F^2 = k^2/k_z^2; \quad \Omega_A^2 = K_A. \quad (28)$$

This indicates that in the low- β limit the slow mode is ordered out and the fast and Alfvén modes are reduced to the compressive Alfvén and the KAW modes, respectively.

On the other hand, in the quasi-perpendicular propagation limit of $k_\perp^2 \gg k_z^2$ (i.e. $k_z^2/k^2 \rightarrow 0$), one has approximately,

from eq. (14):

$$A \cong \frac{1 + Q + k_\perp^2 \lambda_e^2 + \beta k^2}{1 + Q + k_\perp^2 \lambda_e^2} \frac{k^2}{k_z^2}, \quad B \cong \frac{1 + Q + k_\perp^2 \rho_s^2 + 2\beta k^2}{1 + Q + k_\perp^2 \lambda_e^2} \frac{k^2}{k_z^2},$$

$$C \cong \frac{\beta}{1 + Q + k_\perp^2 \lambda_e^2} \frac{k^2}{k_z^2}. \quad (29)$$

Consequently, the dispersion eq. (13) can be approximated as

$$\Omega^4 - \frac{1 + Q + k_\perp^2 \rho_s^2 + 2\beta}{1 + Q + k_\perp^2 \lambda_e^2 + \beta} \Omega^2 + \frac{\beta}{1 + Q + k_\perp^2 \lambda_e^2 + \beta} \frac{k^2}{k_z^2} \cong 0$$

$$\cong \Omega^6 \frac{1 + Q + k_\perp^2 \lambda_e^2}{1 + Q + k_\perp^2 \lambda_e^2 + \beta} \frac{k_z^2}{k^2} \cong 0. \quad (30)$$

Its two roots,

$$\Omega_{A,S}^2 = \frac{1}{2} \frac{1 + Q + k_\perp^2 \rho_s^2 + 2\beta}{1 + Q + k_\perp^2 \lambda_e^2 + \beta} \times \left[1 \pm \sqrt{1 - 4 \frac{1 + Q + k_\perp^2 \lambda_e^2 + \beta}{(1 + Q + k_\perp^2 \rho_s^2 + 2\beta)^2} \beta} \right], \quad (31)$$

are the coupling modes of the Alfvén and slow modes. In particular, for the low- β limit of $\beta \ll 1$, one has

$$\Omega_A^2 \cong K_A; \quad \Omega_S^2 \cong \frac{\beta}{1 + Q + k_{\perp}^2 \rho_s^2 + 2\beta}. \quad (32)$$

This indicates that the two coupling modes are decoupled into the KAW and ion-acoustic modes. The fast mode is ordered out in the quasi-perpendicular propagation limit.

4 Summary and conclusion

The MHD (fast, Alfvén and slow) waves are intrinsic and ubiquitous fluctuation processes in space and astrophysical plasmas and play an important role in their dynamical coupling and energy transportation. In particular, it has been widely accepted that the dispersive small-scale MHD waves could be responsible for the dissipation of wave energies, as well as particle acceleration and plasma heating, which occur continually in various eruptive phenomena from terrestrial aurora and solar flares, to cosmic rays. The dispersive small-scale Alfvén wave, called KAW, has been extensively investigated and applied to plasma energization processes [1–4]. In analogous works, however, little attention has been paid to the fast and slow modes. Recent investigations on the MHD turbulence have proposed that the compressible fluctuations in the fast and slow modes, as well as the KAWs, may need to be taken further into consideration when referring to the steeping of the spectrum at higher wavenumbers [17–20,26]. In this paper, the dispersive MHD waves, with the short-wavelength modification, are self-consistently revised on the basis of the two-fluid model. The dispersion equation, including the short-wavelength effect, and its three roots, are presented as the short-wavelength modification versions of the three ideal MHD (fast, Alfvén and slow) modes.

The results show that the fast mode is considerably modified by the short-wavelength effect in the quasi-parallel propagation direction and the parallel phase speed increases by a factor of ~ 2 in the quasi-parallel propagation direction of $\alpha < 20^\circ$. Moreover, the dispersion modification occurs mainly in the finite- β regime of $0.001 < \beta < 1$.

In contrast to the fast mode, the Alfvén mode is modified by the short-wavelength effect primarily in the quasi-perpendicular propagation direction and the parallel phase speed increases by a factor of ~ 1.5 in the quasi-perpendicular propagation direction of $\alpha > 50^\circ$. In the low- β regime of $\beta < 0.001$, the parallel phase speed is remarkably lower than its ideal MHD solution, whereas the parallel phase speed is higher than its ideal MHD solution in the finite- and high- β regimes of $0.001 < \beta < 10$. Furthermore, in the extremely high- β regime of $\beta > 100$, the parallel phase speed of the Alfvén mode approaches its ideal MHD solution.

Different from both the fast and Alfvén modes, the slow mode clearly departs from its ideal MHD solution between $\alpha = 0^\circ$ and $\alpha = 90^\circ$, due to the short-wavelength effect.

The parallel phase speed decreases by a factor of ~ 2 in the quasi-parallel propagation direction of $\alpha < 20^\circ$ and a slightly lower factor of ~ 1.5 in the quasi-perpendicular propagation direction of $\alpha > 50^\circ$. Moreover, the dispersion modification occurs predominantly in the high- β regime of $0.1 < \beta < 10$.

Finally, our results show further that, in the low- β limit of $\beta \rightarrow 0$, the slow mode is ordered out, the fast mode reduces to the compressive Alfvén mode, and the shear Alfvén mode leads to KAWs. Conversely, in the quasi-perpendicular propagation limit of $k_{\perp}^2 \gg k_z^2$, the fast mode is ordered out and the shear Alfvén and slow modes become coupled.

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