Constructal entransy dissipation rate minimization for “disc-to-point” heat conduction

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Based on constructal theory, “disc-to-point” heat conduction is optimized by minimizing the entransy dissipation rate whereby a critical point is determined that distributes the high-conductivity material according to optimized radial or branch patterns. The results show that the critical point is determined by the product of the thermal conductivity ratio of the two materials and the volume fraction of the high-conductivity material allocated to the entire volume. The notion of optimal heat transfer performance can be attributed to the disc based on the entransy dissipation extremum principle. Comparing the results based on EDR minimization (entransy dissipation rate minimization) with those based on MTD minimization (maximum temperature difference minimization), one finds that the performance derived from the two optimization procedures are different. When the product of the thermal conductivity ratio and volume fraction is 30, the critical point of the former procedure is that for which the nondimensional radius of the disc equals 1.75, while that of the latter procedure is that for which this radius of the disc equals 2.18. Comparing heat transfer performances from the two procedures, the mean heat transfer temperature difference is decreased more for the former procedure thereby receiving an improved performance quota.

constructal theory, entransy dissipation rate, “disc-to-point” heat conduction, generalized thermodynamic optimization

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The necessity to cool electronic devices grows ever higher as electronic devices become ever smaller. Convective and radiative means are not feasible because of the finite spaces involved with small-scaled electronic devices. Heat conduction appears as the best option. Since the constructal theory was put forward by Bejan and applied to optimization problems involving heat conduction [1], constructal theory has been developing rapidly [2–15] and has provided new research impetus into heat transfer problems [16–28]. Cooling electronic devices can be described as a “disc-to-point” heat conduction problem; essentially, how to determine the optimal distribution of a high-conductivity material through a given disc such that the heat generated in the disc is conveyed most effectively to a single point. Rocha et al. [29] determined the optimal structure and distribution of the

high-conductivity material by decreasing the thermal resistance via minimizing maximum temperature differences as the optimization criterion for heat conduction problems within a disc. da Silva et al. [30] studied a disc uniformly-slotted with radially-distributed rectangular blades and optimized the blade structures made of high-conductivity material by following the same optimization procedure. Rocha et al. [31] further validated their analytic solution of [29] with numerical simulations, optimized the tree channels with loops and compared heat conduction performances of radial-patterned and branch-patterned discs. However, constructal optimizations of “disc-point” heat conduction problems do not reflect global heat conduction performances by minimizing maximum thermal resistances (maximum temperature differences) as in [29–31]. From definitions of heat transfer potential capacity and heat transfer potential capacity dissipation function obtained from

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heat transfer theory, Guo et al. [32] pointed out that their physical meanings can be reinterpreted as a heat transfer ability measure and its dissipation rate in heat transfer processes respectively. Guo et al. [33] introduced definitively a new physical quantity called “entransy” that describes the heat transfer ability and proposed the entransy dissipation extremum principle stated as follows: for a fixed boundary heat flux, the conduction process is optimized when the entransy dissipation is minimized (minimum temperature difference), while for a fixed boundary temperature, the conduction process is optimized when the entransy dissipation is maximized (maximum heat flux). An equivalent thermal resistance for multi-dimensional heat conduction problems is defined based on entransy dissipation. An equivalent thermal resistance reflects an average heat transfer effect. The physical meaning of entransy was further expounded with research into, for example, physical mechanisms of heat conduction and electro-thermal simulation experiments [34–36]. Many scholars [37–59] have shown great interest and have also studied heat transfer optimizations based on minimizing entransy dissipation rate.

Wei et al. combined for the first time the entransy dissipation extremum principle with constructal theory and optimized rectangular [60] and triangular [61] elements. Results of optimization showed that constructs based on EDR minimization could decrease the mean temperature difference more effectively than those based on MTD minimization. The optimal constructs without the premise of optimized last-order construct based on EDR minimization were also obtained [62]. The reason for the difference between constant and discrete-variable cross-sectional conducting path elements based on EDR minimization was analyzed and a universal constructal optimization method through self-similar organization based on EDR minimization was obtained [63]. Wei et al. [64,65] further derived the mean temperature difference of an electromagnet and made a multi-disciplinary constructal optimization of the electromagnet by combining magnetic induction and heat transfer problems. Xie et al. optimized the heating unit with either a rectangular cavity [66] or a T-shaped cavity [67] based on constructal EDR minimization.

Ref. [29] illustrated a hierarchical strategy, in optimizing the structure and distribution of high-conductivity material of the heat generating disc based on constructal MTD minimization, and determined a critical point in which the high-conductivity material is distributed according to the optimized radial pattern or branch pattern that decreases the global thermal resistance of the entire disc. Based on [29], the structure and distribution of high-conductivity material of the disc will be optimized based on constructal EDR minimization, and the obtained results will be compared with those obtained in [29].

1 Definition of entransy dissipation rate

Entransy which is a new physical quantity reflecting heat transfer ability of an object was defined in [33] as

$$E_{vh} = \frac{1}{2} Q_{vh} T, \quad (1)$$

where Q_{vh} is the thermal capacity of an object with constant volume, T represents the thermal potential. The entransy dissipation function which represents the entransy dissipation per unit time and per unit volume is deduced [33] as

$$\dot{E}_{h\phi} = \dot{q} \cdot \nabla T, \quad (2)$$

where $\dot{q}$ is thermal current density vector, and ∇T is the temperature gradient. In steady-state heat conduction, $\dot{E}_{h\phi}$ can be calculated as the difference between the entransy input and the entransy output of the object, i.e.

$$\dot{E}_{h\phi} = \dot{E}_{h,in} - \dot{E}_{h,out}. \quad (3)$$

The entransy dissipation rate of the whole volume in the “volume-to-point” conduction is

$$\dot{E}_{vh\phi} = \int_v \dot{E}_{h\phi} dv. \quad (4)$$

The equivalent thermal resistance for multi-dimensional heat conduction problems with specified heat flux boundary condition is given as follows [33]:

$$R_h = \dot{E}_{vh\phi} / \dot{Q}_h^2, \quad (5)$$

where $\dot{Q}_h$ is the thermal current. The mean temperature difference for multi-dimensional heat conduction can be expressed as

$$\Delta \bar{T} = R_h \dot{Q}_h. \quad (6)$$

2 Radial-patterned disc

The cooling model of radial-patterned disc is shown in Figure 1 [29]. The radius of disc is R_0 and the thermal conductivity is k_0 . The heat current ($q = q''' \pi R_0^2$, where q''' is the heat generation rate) which is generated uniformly in the disc flows through the high-conductivity paths towards the center (T_0) of the disc. The thermal conductivity of the material is k_p and the thickness is D_0 . The number of the high-conductivity paths which are distributed uniformly over the disc is N . The temperature of the disc is above T_0 , and the rim is adiabatic.

According to the distribution of high-conductivity paths, the disc can be divided into a number of equal sectors, $N=2\pi R_0/(2H_0)$. The radial boundary (dotted line) of each

sector is adiabatic. Each sector as depicted in Figure 2 is a fundamental element and N sectors fit in a complete disc arrangement. One assumes $N \gg 1$ so that each sector is sufficiently slender to be approximated by an isosceles triangle of base $2H_0$ and height R_0 . The profile of the element is fixed

$$A_0 = H_0 R_0, \quad (7)$$

where H_0 and R_0 vary.

The fraction of high-conductivity material allocated to the disc is also fixed,

$$\phi_0 = \frac{D_0 R_0}{H_0 R_0} = \frac{D_0}{H_0} \ll 1. \quad (8)$$

The aspect ratio of the element, H_0/R_0 , and the number of the elements is unknown.

The ratio of the thermal conductivities is assumed to be large, $\tilde{k} = k_p/k_0 \gg 1$. The direction of the heat conduction in the element is considered to be parallel to the y -axis and that in the k_p blade parallel to the r -axis.

Figure 1 Radial-patterned disc [29].

Figure 2 Elemental sector [29].

The element has r -axis symmetry, so one only needs to consider the upper-half sector. The equation for heat conduction through the k_0 area is [1]

$$\frac{\partial^2 T}{\partial y^2} + \frac{q'''}{k_0} = 0, \quad (9)$$

with the boundary conditions

$$\partial T / \partial y = 0, \quad y = \frac{H_0}{R_0} (R_0 - r), \quad (10)$$

$$T = T(r), \quad y = 0, \quad (11)$$

where $T(r)$ is the temperature of the central k_p blade. Solving eq. (9) yields

$$T - T(r) = \frac{q'''}{k_0} \left[\frac{H_0}{R_0} (R_0 - r) y - \frac{y^2}{2} \right]. \quad (12)$$

The equation for heat conduction through the k_p blade is

$$\frac{\partial}{\partial r} (k_p D_0 \frac{\partial T}{\partial r}) + 2q''' \frac{H_0}{R_0} (R_0 - r) = 0, \quad (13)$$

with boundary conditions

$$\partial T / \partial r = 0, \quad r = 0, \quad (14)$$

$$T = T_0, \quad r = R_0. \quad (15)$$

Solving eq. (13) yields

$$T(r) - T_0 = -\frac{2q'''}{k_p D_0} \frac{H_0}{R_0} \left(\frac{R_0}{2} r^2 - \frac{r^3}{6} - \frac{R_0^3}{3} \right). \quad (16)$$

Eliminating $T(r)$ from eqs. (12) and (16) yields the distribution of temperature differences of the element for $y > 0$

$$T - T_0 = \frac{q'''}{k_0} \left[\frac{H_0}{R_0} (R_0 - r) y - \frac{y^2}{2} \right] - \frac{2q'''}{R_0 k_p \phi_0} \left(\frac{R_0}{2} r^2 - \frac{r^3}{6} - \frac{R_0^3}{3} \right). \quad (17)$$

The entransy dissipation rate of the element is

$$\begin{aligned} \dot{E}_{v,h\phi 0} &= 2 \int_0^{R_0} \int_0^{\frac{H_0}{R_0}(R_0-r)} q''' [T(r, y) - T_0] dr dy \\ &= q'''^2 A_0^2 k_0^{-1} \left(\frac{1}{6} \frac{H_0}{R_0} + \frac{8}{15 \tilde{k} \phi_0} \frac{R_0}{H_0} \right). \end{aligned} \quad (18)$$

By optimizing eq. (18) with respect to H_0/R_0 , the optimal H_0/R_0 , the corresponding minimal entransy dissipation rate, the mean temperature difference and maximum temperature difference are, respectively,

$$\left(\frac{H_0}{R_0} \right)_{\text{opt}} = \frac{4}{(5 \tilde{k} \phi_0)^{1/2}} \ll 1, \quad (19)$$

$$\dot{E}_{\text{vh}\phi 0,m} = \frac{4q''^2 A_0^2}{3\sqrt{5}} k_0^{-1} (\tilde{k}\phi_0)^{-1/2}, \quad (20)$$

$$\Delta \bar{T}_0 = \frac{4}{3\sqrt{5}} A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2} = 0.596 A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2}, \quad (21)$$

$$\Delta T_{0,\max}' = \frac{17}{6\sqrt{5}} A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2} = 1.267 A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2}. \quad (22)$$

However, the optimal aspect ratio of the element obtained in [29] based on MTD minimization was

$$\left(\frac{H_0}{R_0} \right)'_{\text{opt}} = \frac{2}{(3\tilde{k}\phi_0)^{1/2}} \ll 1. \quad (23)$$

Moreover, the corresponding mean temperature difference and the maximum temperature difference are, respectively,

$$\Delta \bar{T}'_0 = \frac{17}{15\sqrt{3}} A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2} = 0.654 A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2}, \quad (24)$$

$$\Delta T'_{0,\max} = \frac{2}{\sqrt{3}} A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2} = 1.155 A_0 q'' k_0^{-1} (\tilde{k}\phi_0)^{-1/2}. \quad (25)$$

From eqs. (19) and (23), one can see that the optimal construct of the element based on EDR minimization is different from that based on MTD minimization. The minimal values of the mean temperature difference of heat transfer and of the maximum temperature difference based on the former (solid line) and the latter (dotted line) minimizations are shown in Figure 3. One can see that the optimal construct based on the former can decrease the mean temperature difference to a great extent compared with the optimal construct based on the latter, with a clear improvement in the heat transfer performance.

The aspect ratio (eq. (19)) determines the number of elements which fit in a complete disc arrangement

$$N_{\text{opt}} = \frac{\pi}{4} (5\tilde{k}\phi_0)^{1/2} = 1.756 (\tilde{k}\phi_0)^{1/2} \gg 1, \quad (26)$$

Figure 3 The mean temperature difference and maximum temperature difference of element.

so eq. (19) shows that the optimal aspect ratio agrees with the above assumption.

The area of the entire disc is determined by eq. (26),

$$A = \pi R_0^2 = N A_0 = \frac{\pi}{4} (5\tilde{k}\phi_0)^{1/2} A_0. \quad (27)$$

And thus, the radius of the disc is

$$R_0 = \frac{A_0^{1/2}}{2} (5\tilde{k}\phi_0)^{1/4} = 0.748 A_0^{1/2} (\tilde{k}\phi_0)^{1/4}. \quad (28)$$

The corresponding dimensionless mean thermal resistance and dimensionless maximum thermal resistance of the entire disc are, respectively,

$$\tilde{R}_{h0} = \frac{\Delta \bar{T}}{q'' A / k_0} = \frac{16}{15 \pi \tilde{k}\phi_0}, \quad (29)$$

$$\tilde{R}_{t,h0} = \frac{\Delta T_{\max}}{q'' A / k_0} = \frac{34}{15 \pi \tilde{k}\phi_0}. \quad (30)$$

The radius of the disc is not determined but results from the optimization, and depends on A_0 , ϕ_0 and $\tilde{k}$. The product $\tilde{k}\phi_0$ is an important quantity. Large $\tilde{k}\phi_0$ signifies more high-conductivity material, a more slender element and a disc with larger radius. These properties will be discussed further in association with Figure 10.

The number of disc elements in the disc and the radius of the disc obtained in [29] based on MTD minimization were, respectively,

$$N'_{\text{opt}} = \frac{\pi}{2} (3\tilde{k}\phi_0)^{1/2} = 2.721 (\tilde{k}\phi_0)^{1/2} \gg 1, \quad (31)$$

$$R'_0 = (A_0 / 2)^{1/2} (3\tilde{k}\phi_0)^{1/4} = 0.931 A_0^{1/2} (\tilde{k}\phi_0)^{1/4}. \quad (32)$$

The corresponding dimensionless mean thermal resistance and dimensionless maximum thermal resistance are, respectively,

$$\tilde{R}_{t0} = \frac{\Delta T'_{\max}}{q'' A / k_0} = \frac{4}{3 \pi \tilde{k}\phi_0}, \quad (33)$$

$$\tilde{R}_{h,t0} = \frac{\Delta \bar{T}'}{q'' A / k_0} = \frac{34}{45 \pi \tilde{k}\phi_0}. \quad (34)$$

eqs. (24) and (34) are derived from the results in [29] by the author of this paper.

Comparing eq. (26) with (31), and (28) with (32) one can see that both the number of elements and the radius of the disc based on EDR minimization are smaller than those based on MTD minimization. That is, the A in eq. (33) or (34) is not equal to the A in eq. (29) or (30). Therefore, the mean thermal resistance and the maximum thermal resistance of the entire radial-patterned disc based on EDR minimization are not compared with those based on MTD

$$T - T_0 = \frac{q'''}{k_0} \left[\frac{H_1}{R_1} (R_1 - r) y - \frac{y^2}{2} \right] + \frac{q'''}{k_p D_1} \left(\frac{2}{3} A_1 + n A_0 \right) - \frac{q'''}{k_p D_1} \left[n A_0 r + \frac{2 H_1}{R_1} \left(\frac{R_1}{2} r^2 - \frac{r^3}{6} \right) \right]. \quad (44)$$

The entransy dissipation rate of central sector A_1 is

$$\begin{aligned} \dot{E}_{vh\phi c} &= 2 \int_0^{R_1} \int_0^{\frac{H_1}{R_1}(R_1-r)} q''' [T(r, y) - T_0] dr dy \\ &= \frac{q''' A_1}{30 k_p D_1} \left(\frac{5 \tilde{k} A_1^2 D_1}{R_1^2} + 16 R_1 A_1 + 20 n R_1 A_0 \right). \end{aligned} \quad (45)$$

The entransy dissipation rate of A_0 element is composed of the entransy dissipation rate introduced by the heat current $q''' A_0$ flowing through A_0 element and the entransy dissipation rate introduced by the heat current $q''' A_0$ flowing through high-conductivity blade of central sector A_1 . In fact, it can be derived from the definition of the entransy dissipation rate:

$$\begin{aligned} \dot{E}_{vh\phi p} &= \iint_{A_0} q''' (T - T_0) dr dy \\ &= q''' \iint_{A_0} (T - T_c) dr dy + q''' \iint_{A_0} (T_c - T_0) dr dy. \end{aligned} \quad (46)$$

The entransy dissipation rate introduced by the heat current $q''' A_0$ flowing through A_0 element (the first term on the right-hand side of eq. (46)) can be given by eq. (20). The entransy dissipation rate introduced by the heat current $q''' A_0$ flowing through high-conductivity blade of central sector (the second term on the right-hand side of eq. (46)) can be given by $q''' A_0 (T_c - T_0)$. Thus, the entransy dissipation rate of A_0 element is

$$\begin{aligned} \dot{E}_{vh\phi p} &= \frac{4 q''' A_0^2}{3 \sqrt{5}} k_0^{-1} (\tilde{k} \phi_0)^{-1/2} + q''' A_0 (T_c - T_0) \\ &= \frac{4 q''' A_0^2}{3 \sqrt{5}} k_0^{-1} (\tilde{k} \phi_0)^{-1/2} + A_0 \frac{q''' R_1}{k_p D_1} \left(\frac{2}{3} A_1 + n A_0 \right). \end{aligned} \quad (47)$$

The entransy dissipation rate of A sector is

$$\begin{aligned} \dot{E}_{vh\phi} &= \dot{E}_{vh\phi c} + n \dot{E}_{vh\phi p} = \\ &= \frac{q''' A_1}{30 k_p D_1} \left(\frac{5 \tilde{k} A_1^2 D_1}{R_1^2} + 16 R_1 A_1 \right) \\ &+ n A_0 q''' \left[\frac{4 A_0}{3 \sqrt{5}} k_0^{-1} (\tilde{k} \phi_0)^{-1/2} + \frac{R_1}{k_p D_1} \left(\frac{4}{3} A_1 + n A_0 \right) \right]. \end{aligned} \quad (48)$$

After some algebra, eq. (48) becomes

$$\begin{aligned} \dot{E}_{vh\phi} &= \frac{n q''' A_0^2 k_0^{-1}}{300 \tilde{R}^3 \tilde{D} (\tilde{k} \phi_0)^{\frac{7}{4}}} \left\{ -320 \sqrt{5} n \tilde{R}^5 (\tilde{k} \phi_0)^{\frac{1}{4}} \right. \\ &+ 256 \times 5^{\frac{1}{4}} n \tilde{R}^6 + 25 \times 5^{\frac{1}{4}} n^2 \tilde{D} (\tilde{k} \phi_0)^2 \\ &+ 40 \times 5^{\frac{1}{4}} n \tilde{R}^2 (\tilde{k} \phi_0) (3 \sqrt{5} n \tilde{D} \sqrt{\tilde{k} \phi_0} + 5) \\ &+ 40 \times 5^{\frac{1}{4}} n \tilde{R}^4 (\tilde{k} \phi_0)^{\frac{1}{2}} (7 \sqrt{5} + 2 n \tilde{D} \sqrt{\tilde{k} \phi_0}) \\ &- 20 \tilde{R}^3 (\tilde{k} \phi_0)^{\frac{3}{4}} [4 \sqrt{5} \tilde{D} (2 n^2 - 1) \sqrt{\tilde{k} \phi_0} + 35 n] \\ &\left. - 40 n \tilde{R} (\tilde{k} \phi_0)^{\frac{5}{4}} (\sqrt{5} + 5 n \tilde{D} \sqrt{\tilde{k} \phi_0}) \right\}, \end{aligned} \quad (49)$$

where $\tilde{D} = D_1 / D_0$ is the ratio of thicknesses of high-conductivity blades, and $\tilde{R} = R / A_0^{1/2}$ is the dimensionless radius of the disc. The corresponding mean temperature difference of heat transfer is

$$\begin{aligned} \Delta \bar{T} &= \frac{5^{\frac{1}{4}} q''' A_0 k_0^{-1}}{600 \tilde{R}^4 \tilde{D} (\tilde{k} \phi_0)^{\frac{3}{2}}} \left\{ -320 \sqrt{5} n \tilde{R}^5 (\tilde{k} \phi_0)^{\frac{1}{4}} \right. \\ &+ 256 \times 5^{\frac{1}{4}} n \tilde{R}^6 + 25 \times 5^{\frac{1}{4}} n^2 \tilde{D} (\tilde{k} \phi_0)^2 \\ &+ 40 \times 5^{\frac{1}{4}} n \tilde{R}^2 (\tilde{k} \phi_0) (3 \sqrt{5} n \tilde{D} \sqrt{\tilde{k} \phi_0} + 5) \\ &+ 40 \times 5^{\frac{1}{4}} n \tilde{R}^4 (\tilde{k} \phi_0)^{\frac{1}{2}} (7 \sqrt{5} + 2 n \tilde{D} \sqrt{\tilde{k} \phi_0}) \\ &- 20 \tilde{R}^3 (\tilde{k} \phi_0)^{\frac{3}{4}} [4 \sqrt{5} \tilde{D} (2 n^2 - 1) \sqrt{\tilde{k} \phi_0} + 35 n] \\ &\left. - 40 n \tilde{R} (\tilde{k} \phi_0)^{\frac{5}{4}} (\sqrt{5} + 5 n \tilde{D} \sqrt{\tilde{k} \phi_0}) \right\}. \end{aligned} \quad (50)$$

The fraction of high-conductivity material allocated to the disc is

$$\begin{aligned} \phi &= \frac{A_p}{\pi R^2} = \frac{N D_0 R_0 + \frac{N}{n} D_1 R_1}{\pi R^2} \\ &= \frac{(5 \tilde{k} \phi_0)^{1/4} \phi_0}{2 \tilde{R}} + \frac{\tilde{D} \phi_0}{n \tilde{R}} \left[\tilde{R} - \frac{(5 \tilde{k} \phi_0)^{1/4}}{2} \right]. \end{aligned} \quad (51)$$

Eq. (51) determines the relationship among n , $\tilde{D}$, $\tilde{R}$, $\tilde{k}$ and ϕ_0 . Because ϕ and $\tilde{k}$ are fixed (e.g. $\phi = 0.1$ and $\tilde{k} = 300$), the relation between $\tilde{D}$ and ϕ_0 is obtained when n and $\tilde{R}$ are given.

Figure 6 shows the dimensionless mean temperature difference ($\tilde{\bar{T}} = \Delta \bar{T} / (q''' A_0 / k_0)$) based on EDR minimization, the dimensionless maximum temperature difference ($\tilde{T} = \Delta T_{\max} / (q''' A_0 / k_0)$) based on MTD minimization, and $\tilde{D}$ versus ϕ_0 characteristics with $n = 2$ and $\tilde{R} = 4$. From

Figure 6 $\tilde{T}$, $\tilde{T}$ and $\tilde{D}$ versus ϕ_0 characteristics.

the figure, one can see that the optimal constructs corresponding to $\tilde{T}_{\min}$ and $\tilde{T}_{\min}$ respectively, are different from each other: the values of $\phi_{0,\text{opt}}$ are 0.122 and 0.092 respectively, the values of $\tilde{D}_{\text{opt}}$ are 1.3324 and 2.3726 respectively, and the values of $\tilde{T}$ corresponded to $\tilde{T}_{\min}$ and $\tilde{T}_{\min}$ are 0.1078 and 0.1147 respectively. The calculations show that the mean temperature difference of the A sector corresponding to the former decreases by 6.02% compared with that corresponding to the latter. The optimal construct based on EDR minimization can improve the heat transfer efficiency.

Figure 7 shows the effects of the dimensionless radius ($\tilde{R}$) on the optimal constructs ($\phi_{0,\text{opt}}$, $\tilde{D}_{\text{opt}}$, $\tilde{T}_{\min}$ or $\tilde{T}_{\min}$) based on EDR minimization (solid line) and MTD minimization (dotted lines) respectively. From the figure, one can see that when $\tilde{R}$ increases $\phi_{0,\text{opt}}$ increases, $\tilde{D}_{\text{opt}}$ decreases and $\tilde{T}_{\min}$ first decreases then increases based on EDR minimization, $\tilde{T}_{\min}$ increases, $\phi_{0,\text{opt}}$ decreases and $\tilde{D}_{\text{opt}}$ increases slowly based on MTD minimization. Evidently, the optimal constructs corresponding to $\tilde{T}_{\min}$ and $\tilde{T}_{\min}$ are different from each other. One can see also that when $\tilde{R}$ is small the values of $\tilde{D}_{\text{opt}}$ and $\phi_{0,\text{opt}}$ corresponding to $\tilde{T}_{\min}$ and $\tilde{T}_{\min}$ respectively, tend to be equal. Note that all the solid lines vanish below $\tilde{R} = 1.75$ and all the dotted lines vanish below $\tilde{R} = 2.18$. These phenomena will be explained in the following text.

Figure 7 shows the optimal number of peripheral elements:

$$N_{\text{opt}} = \pi \tilde{R} (5\tilde{k} \phi_{0,\text{opt}})^{1/4} / 2. \quad (52)$$

N_{opt} increases as $\tilde{R}$ increases (solid line in Figure 7), and there exists a similar law based on MTD minimization [29] (dotted line). Because $\phi_{0,\text{opt}}$ based on EDR minimization increases with n , one sees from eq. (52) that N_{opt} increases with n . The behavior is different from the relation based on MTD minimization where N_{opt} is independent of n . For the same $\tilde{R}$, N_{opt} corresponding to $\tilde{T}_{\min}$ is smaller than N_{opt} corresponding to $\tilde{T}_{\min}$. They tend to be equal as $\tilde{R}$ increases. Figure 7 also shows the optimal dimensionless radius of the central sector of the disc:

$$\tilde{R}_{1,\text{opt}} = \frac{R_{1,\text{opt}}}{A_0^{1/2}} = \tilde{R} - \frac{(5\tilde{k} \phi_{0,\text{opt}})^{1/4}}{2}. \quad (53)$$

When $\tilde{R}_{1,\text{opt}}$ shrinks to zero (hence $\phi_{0,\text{opt}} = \phi$), $\tilde{R} = 1.75$. The branch-patterned disc reduces to a radial-patterned disc. For this reason, the solid lines in Figure 7 vanish below $\tilde{R} = 1.75$. Similarly, the dotted lines in Figure 7 vanish below $\tilde{R} = 2.18$ based on MTD minimization [29]. $\tilde{R}_{1,\text{opt}}$ based on EDR minimization and $\tilde{R}_{1,\text{opt}}$ based on MTD minimization both increase and tend to be equal as $\tilde{R}$ increases. For the same $\tilde{R}$, the former is larger than the latter. The former increases as n increases, but the latter is independent of n .

Figure 8 shows the effects of the number (n) of tributaries on the optimal constructs based on EDR minimization (solid line) and MTD minimization (dotted line) respectively, with $\tilde{R} = 4$. From the figure, one can see that the optimal constructs ($\phi_{0,\text{opt}}$, $\tilde{D}_{\text{opt}}$, $\tilde{T}_{\min}$) of the former increase with n . For the latter, $\tilde{D}_{\text{opt}}$ also increases with n , but $\phi_{0,\text{opt}}$ and $\tilde{T}_{\min}$ are both independent of n and equal 0.092 and 0.3745, respectively. Comparing the solid lines with the dotted lines in the figure, one sees that $\phi_{0,\text{opt}}$ corresponding to the former is larger, but the $\tilde{D}_{\text{opt}}$ corresponding the former

Figure 7 Effect of dimensionless radius $\tilde{R}$ on the optimal construct.

Figure 8 Effect of the number of elemental branches on the optimal construct.

is smaller. A larger $\tilde{D}_{\text{opt}}$ means that an elemental insert (D_0) is thinner relative to the stem (D_1).

In each of the cases optimized in Figures 6–8 the elemental area and dimensionless radius of the disc were fixed. This means that the minimization of dimensionless mean thermal resistance of the entire disc is equivalent to the minimization of $\tilde{T}$:

$$\begin{aligned} \tilde{R}_{h1} &= \frac{\Delta \tilde{T}}{q''' \pi R^2 / k_0} = \frac{\tilde{T}}{\pi \tilde{R}^2} \\ &= \frac{5^{\frac{1}{4}}}{600 \pi \tilde{R}^6 \tilde{D} (\tilde{k} \phi_0)^{\frac{3}{2}}} \left\{ -320 \sqrt{5} n \tilde{R}^5 (\tilde{k} \phi_0)^{\frac{1}{4}} \right. \\ &\quad + 256 \times 5^{\frac{1}{4}} n \tilde{R}^6 + 25 \times 5^{\frac{1}{4}} n^2 \tilde{D} (\tilde{k} \phi_0)^2 \\ &\quad + 40 \times 5^{\frac{1}{4}} n \tilde{R}^2 (\tilde{k} \phi_0) (3 \sqrt{5} n \tilde{D} \sqrt{\tilde{k} \phi_0} + 5) \\ &\quad + 40 \times 5^{\frac{1}{4}} n \tilde{R}^4 (\tilde{k} \phi_0)^{\frac{1}{2}} (7 \sqrt{5} + 2 n \tilde{D} \sqrt{\tilde{k} \phi_0}) \\ &\quad - 20 \tilde{R}^3 (\tilde{k} \phi_0)^{\frac{3}{4}} [4 \sqrt{5} \tilde{D} (2n^2 - 1) \sqrt{\tilde{k} \phi_0} + 35n] \\ &\quad \left. - 40 n \tilde{R} (\tilde{k} \phi_0)^{\frac{5}{4}} (\sqrt{5} + 5 n \tilde{D} \sqrt{\tilde{k} \phi_0}) \right\}. \end{aligned} \quad (54)$$

Because $\tilde{T}_{\text{min}}$ is almost proportional to $\tilde{R}$, the minimized dimensionless mean thermal resistance ($\tilde{R}_{h1,\text{min}} = \tilde{T}_{\text{min}} / (\pi \tilde{R}^2)$) is almost proportional to $\tilde{R}^{-1}$. $\tilde{R}_{h1,\text{min}}$ depends on $\tilde{R}$, $\tilde{k}$, ϕ and n . However, the minimized dimensionless maximum thermal resistance ($\tilde{R}_{t1,\text{min}} = \tilde{T}_{\text{min}} / (\pi \tilde{R}^2)$) [29] is independent of n as shown in Figure 9. Based on the same elemental area and the same amounts and properties of conductive materials ($\phi, \tilde{k}$), one compares $\tilde{R}_{h1,\text{min}}$ with $\tilde{R}_{h0}$ for different n as shown in Figure 9 (solid line). The dimensionless radius of the radial-patterned disc is (cf. eq. (28))

$$\tilde{R} = (5 \tilde{k} \phi)^{1/4} / 2. \quad (55)$$

This dimensionless radius $\tilde{R}$ is fixed when $\tilde{k}$ and ϕ are fixed. The dimensionless radius $\tilde{R}$ of a branch-patterned disc in Figure 4 can be free to vary. This is why in Figure 9 $\tilde{R}_{h0}(\tilde{R})$ is a point and $\tilde{R}_{h1,\text{min}}(\tilde{R})$ is a curve. From the solid lines in the figure, one can see that $\tilde{R}_{h1,\text{min}}(\tilde{R})$ increases with n . When $\tilde{R}$ exceeds 1.75, the global resistance is smaller when the high-conductivity material is distributed according to the optimized branched pattern. That is, $\tilde{R} = 1.75$ is a critical point that whether the high-conductivity material is distributed according to the optimized radial pattern or branched pattern. Tables 1 and 2 list the key results of parameters of some $\tilde{R}$ points. Based on EDR minimization, radial-patterned disc has 10 elemental sectors for $\tilde{R} = 1.75$, and the branch-patterned disc has 30 peripheral elements for $\tilde{R} = 5$ and 64 peripheral elements for $\tilde{R} = 10$.

Table 1 Optimal construct based on EDR minimization (this paper)

$\tilde{R}$	n	$\tilde{R}_{h0}$	$\tilde{R}_{h1,\text{min}}$	N_{opt}	$\tilde{R}_{t,\text{opt}}$	$\tilde{D}_{\text{opt}}$	$\phi_{0,\text{opt}}$
1.75	—	0.0113	—	10	—	—	0.1
5	2	—	0.0018	30	3.1138	1.1674	0.1350
	4	—	0.0028	32	3.0351	1.5548	0.1590
10	2	—	0.0014	64	7.9429	0.8003	0.1910
	4	—	0.0020	64	7.9402	1.5861	0.1920

Figure 9 Dimensionless mean thermal resistances of the radial-patterned and branch-patterned discs and the results in [29].

Table 2 Optimal construct based on MTD minimization (ref. [29])

$\tilde{R}$	n	$\tilde{R}_{t0}$	$\tilde{R}_{t1,\text{min}}$	N_{opt}	$\tilde{R}_{t,\text{opt}}$	$\tilde{D}_{\text{opt}}$	$\phi_{0,\text{opt}}$
2.18	—	0.0141	—	15	—	—	0.1
5	2	—	0.0065	34	2.8966	2.5159	0.0870
	4	—	0.0065	34	2.8966	5.0317	0.0870
10	2	—	0.0058	64	7.9938	2.9730	0.0720
	4	—	0.0058	64	7.9938	5.9460	0.0720

Based on Figure 9 (solid line), Figure 10 (solid line) shows the effects of $\tilde{k}$ and ϕ on the dimensionless mean thermal resistance with $n=2$, $30 \leq \tilde{k} \leq 1000$ and $0.01 \leq \phi \leq 0.1$. The dotted lines reproduce the results in [29]. The point $\tilde{R}_{h0}(\tilde{R})$ and the curve $\tilde{R}_{h1,min}(\tilde{R})$ in Figure 9 with $n=2$ is a special case of Figure 10. Figure 10 includes all these results with $n=2$. From the solid lines in the figure, one can see that when the dimensionless radius $\tilde{R}$ exceeds that of the optimized branch-patterned disc, lower mean thermal resistance is achieved when the high-conductivity material is distributed according to the branch pattern. The mean thermal resistance can also be decreased and the heat transfer performance is improved by increasing $\tilde{k}\phi$.

Figure 11 shows the comparison between the mean thermal resistances based on both EDR and MTD minimizations, with different n . The dotted lines which are derived from the results of [29] vanish below $\tilde{R} = 2.18$. From the figure, one can see that the difference between the two mean thermal resistances decreases as $\tilde{R}$ decreases. The former is smaller than the latter and clearly improves the heat transfer performance of the disc with the larger value of $\tilde{R}$.

Figure 10 Effects of $\tilde{k}$ and ϕ on dimensionless mean thermal resistances of the radial- and branch-patterned discs and the results of [29].

Figure 11 Dimensionless mean thermal resistances based on $\tilde{R}_h$ and $\tilde{R}_l$.

4 Conclusions

The entransy dissipation extremum principle provides new academic justification and criterion for heat transfer optimizations. This paper imposes minimum temperature differences for given heat flux that satisfies the minimum entransy dissipation principle. When the entransy dissipation rate is minimal, the mean temperature difference of heat transfer is minimal and the heat transfer performance is optimal. The constructal problem of cooling a disc was analyzed and discussed based on EDR minimization. The results showed that the construct of optimal heat transfer performance of the system can be designed based on this entransy dissipation extremum principle. From Figures 3, 6–11, one can see that the difference between the optimal constructs for the two objectives is evident. $\phi_{0,opt}$ corresponding to the former is larger than $\phi_{0,opt}$ corresponding to the latter, and $\tilde{D}_{opt}$ corresponding to the former is smaller than $\tilde{D}_{opt}$ corresponding to the latter for a given $\tilde{R}$. When

$\tilde{R}$ increases, $\phi_{0,opt}$ increases, $\tilde{D}_{opt}$ decreases, and $\tilde{T}_{min}$ first decreases and then increases based on EDR minimization; $\tilde{T}_{min}$ increases, $\phi_{0,opt}$ decreases and $\tilde{D}_{opt}$ increases slowly

based on MTD minimization. When $\phi=0.1$ and $\tilde{k}=300$, for the former $\tilde{R}=1.75$ is the critical point determining whether the high-conductivity material is distributed according to radial or branch patterns, and for the latter, $\tilde{R}=2.18$ is the critical point. These are all the differences for the two optimization objectives, one based on EDR minimization and the other on MTD minimization. Additionally, compared with optimal construct based on the latter, the optimal construct based on the former can decrease the mean temperature difference of heat transfer to a greater extent, and clearly improves the heat transfer performance. The constructal optimization based on MTD minimization amounted to limiting maximum temperatures, and the constructal optimization based on EDR minimization amounted to decreasing the mean temperature difference of heat transfer of the disc and improving the heat transfer performance. Therefore, the optimal constructal design corresponding to the minimal mean thermal resistance should be adopted by limiting maximum temperature.

When cooling ever larger heat-generating discs, the second order assembly of the branch-patterned disc can be considered based on the work of this paper. The sector with tip angle β in Figure 5 is composed of a number of optimized A sectors as discussed in section 3. Deductions process of entransy dissipation rate of these second-order assemblies are similar as in section 3. The curve of the mean thermal resistance of the second order assembly will intersect the existing curve (Figure 9) of the mean thermal resistance of the first order assembly, and the intersecting point will be the critical point that the high-conductivity material

should be distributed according to the first order assembly or the second order assembly. The work in this paper has fully described the effect of EDR minimization. Based on EDR minimization, the constructal optimization of the second (or higher) order assembly of the disc can be further performed for heat conduction.

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