Wireless Pers Commun (2013) 72:2305–2320
DOI 10.1007/s11277-013-1151-7

Soft Decision Error Assisted Layered Multiuser Detectors for MIMO 2D Spread MC DS-CDMAs

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Abstract In this paper, we present two layered multiuser detectors (MUDs) for MIMO frequency-time-domain (FT-domain) multi-carrier (MC) direct sequence code division multiple access (DS-CDMA) systems with an antenna array at the base station. We assume that multiple users are active and individually utilize multiple transmit antennas in the MC DS-CDMA system. The users are organized into groups, and each user is assigned a unique Time-domain (T-domain) signature code. Moreover, users in the same group share a unique F-domain signature code. As a result, they can exploit the T-domain and F-domain signature codes to spread their multiple symbols in parallel, and then transmit the FT-domain spread signals from the corresponding multiple antennas over the fading channels to the base station. However, because of the non-ideal channel effect and/or the use of non-orthogonal signature codes, the FT-domain spread MC DS-CDMA system is affected by multiple access interference (MAI) in the same way as CDMA-like systems. To mitigate the effects of MAI and improve the system's performance, we propose two layered MUDs that exploit the layered soft decision errors. Specifically, in a trade-off between the performance and the computational complexity, only the soft decision errors of one user/one group are used in the proposed layered MUDs. The results of simulations and a complexity analysis demonstrate that the

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This work was supported by National Science Council of R.O.C. under contract NSC 99-2221-E-019-026 and NSC 100-2221-E-019-051.

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proposed layered MUDs outperform existing approaches and the computational complexity is modest.

Keywords Layered detector · Multi-carrier direct sequence code division multiple access · Multiple input multiple output · Soft decision error

1 Introduction

Because of the non-ideal channel effect and/or the non-orthogonality of signature codes, wireless communication systems are affected by interference, such as multiple access interference (MAI) [1]. In fact, the interference is the major limitation on the performance of wireless systems [2]. As a result, a great deal of research effort has been invested in solving the problem over the last two decades [2–6]. One of the most promising solutions is the multiuser detector (MUD) [2] because it has the potential to mitigate MAI. For example, it has been shown that the maximum likelihood (ML) MUD [2] can achieve an optimum performance in terms of the bit error rate (BER); however, the computational complexity increases exponentially with the number of users. To reduce the prohibitive complexity, several sub-optimal schemes have been proposed [2], e.g., the decorrelating detector (DD) [7], the minimum mean square error (MMSE) MUD [8], successive interference cancellation (SIC) [9], and parallel interference cancellation [10]. Recently, the substantial benefits of multiple-input multiple-output (MIMO) schemes [11], such as the provision of spatial diversity and spatial multiplexing (SM) gain, have been exploited by researchers searching for methods to improve the performance of wireless systems [12,13]. Moreover, to guarantee the high transmission rate of SM-based MIMOs, Foschini et al. proposed a layered detector, called Vertical Bell Laboratories Layered Space-Time (VBLAST) [14]. The detector is a variant of order SIC (OSIC), which performs the ordering, interference nulling and symbol detection steps iteratively to estimate the transmitted data in a symbol-by-symbol manner.

VBLAST's superior detection performance has generated a great deal of research interest, and the method has been applied in several multiuser MIMO-like systems [15–18]. For example, Layered Space-Time (LAST) MUD, a variant of VBLAST for symbol detection in the SM-based MIMO code division multiple access (CDMA) system, was proposed in [17]. The approach assumes that users are arranged in groups, and each user is assigned a unique time-domain (T-domain) signature code to spread his/her multiple symbols concurrently via multiple antennas. Similar to VBLAST, LAST MUD performs the ordering, interference nulling and symbol detection steps to estimate the transmitted data iteratively in a symbol-layer by symbol-layer manner. However, like VBLAST, the symbol-based LAST MUD also incurs a huge computational overhead when implementing a symbol-based layered space-time detection mechanism. To reduce the computational complexity, a user-based LAST MUD was proposed in [18]. It estimates the transmitted data in a user-layer by user-layer fashion, but the performance deteriorates. Specifically, as the length of the signature codes decreases, the performance of the LAST MUDs proposed in [17] and [18] degrades significantly.

Recently, a new multi-carrier (MC) direct sequence CDMA (DS-CDMA) scheme, called frequency-time-domain (FT-domain) spread MC DS-CDMA, was proposed in [19]. In FT-domain spread MC DS-CDMA systems, each user exploits his/her unique T-domain signature code to spread the transmitted symbol, and then copies the T-domain spread signal to each sub-carrier to be multiplied by the corresponding entry of an F-domain signature code. The two-domain (2D) spreading mechanism enables FT-domain spread MC DS-

CDMAs to provide the advantages of conventional MC DS-CDMAs, such as robustness to inter-symbol interference (ISI) [20], as well as improved performance. Because of their superior performance, FT-domain spread MC DS-CDMAs have generated a great deal of research interest [21–23]. There will be increasing demand for advanced QoS in future communications systems; hence, developing technologies that exploit MIMO mechanisms to enhance the performance of FT-domain spread MC DS-CDMA systems is a crucial and interesting issue. However, research into a hybrid of MIMO and FT-domain spread MC DS-CDMA technologies has received relatively little attention up to now.

In an attempt to bridge this research gap, we present two layered MUDs for MIMO frequency-time-domain (FT-domain) multi-carrier (MC) direct sequence code division multiple access (DS-CDMA) systems. Under our approach, we assume multiple antennas are deployed at each user's transmitter and the base station's receiver. Moreover, like the approaches in [17] and [18], users are arranged in groups, and each user is assigned a unique T-domain signature code. We also assume that users in the same group share a unique F-domain signature code. Each user then utilizes his T-domain and F-domain signature codes to spread multiple symbols in parallel; and concurrently transmits the multiple FT-domain spread signals from his multiple transmit antennas over the Rayleigh fading channel to the base station's receive antennas. However, because of the non-orthogonality of the signature codes, the performance of the proposed system is affected by MAI in the same way as other CDMA-like systems. To resolve the problem, we present a user-based layered MUD that exploits one user's soft decision errors to help detect symbols in a user-layer by user-layer manner. In addition, to further reduce the computational complexity, we propose a group-based layered MUD that utilizes one group's soft layer decision errors to facilitate symbol estimation in a group-layer by group-layer manner. The results of simulations and a complexity analysis demonstrate that the proposed layered MUDs outperform existing approaches and the computational complexity is reasonable.

The remainder of this paper is organized as follows. In Sect. 2, we introduce the system model. We describe the proposed user-based and group-based layered MUDs in Sects. 3 and 4 respectively; and in Sect. 5, we present the results of the simulations and complexity analysis. Section 6 contains some concluding remarks.

2 System Model

We consider the uplink of a synchronous multiuser MIMO FT-domain spread MC DS-CDMA system, as shown in Fig. 1. The M_R receive and M_T transmit antennas are deployed at the base station and each user respectively. Following [17] and [18], users are organized into G groups; and, for simplicity, we assume that the number of users in each group is equal to K. Furthermore, each user is assigned a unique T-domain signature code and also shares a unique F-domain signature code with other users in the same group. We also assume that the lengths of the T-domain and F-domain signature codes are N_t and N_f respectively.

Let $\mathbf{t}_{g,k}$ and $\mathbf{f}_g$ be the corresponding T-domain and F-domain signature codes for the kth user in the gth group, where $1 \leq g \leq G$ and $1 \leq k \leq K$. Initially, each user utilizes his/her unique T-domain signature code to spread the M_T binary phase shift keying (BPSK) modulated symbols. Then, the T-domain spread signals are copied to the sub-carriers to be multiplied by the corresponding entry of the F-domain signature code. After processing by the Inverse Fast Fourier Transform (IFFT) and the insertion of a cyclic prefix (CP) [19], the FT-domain spread signals are transmitted in parallel over the frequency selective Rayleigh fading channels to the M_R receive antennas at the base station. For ease

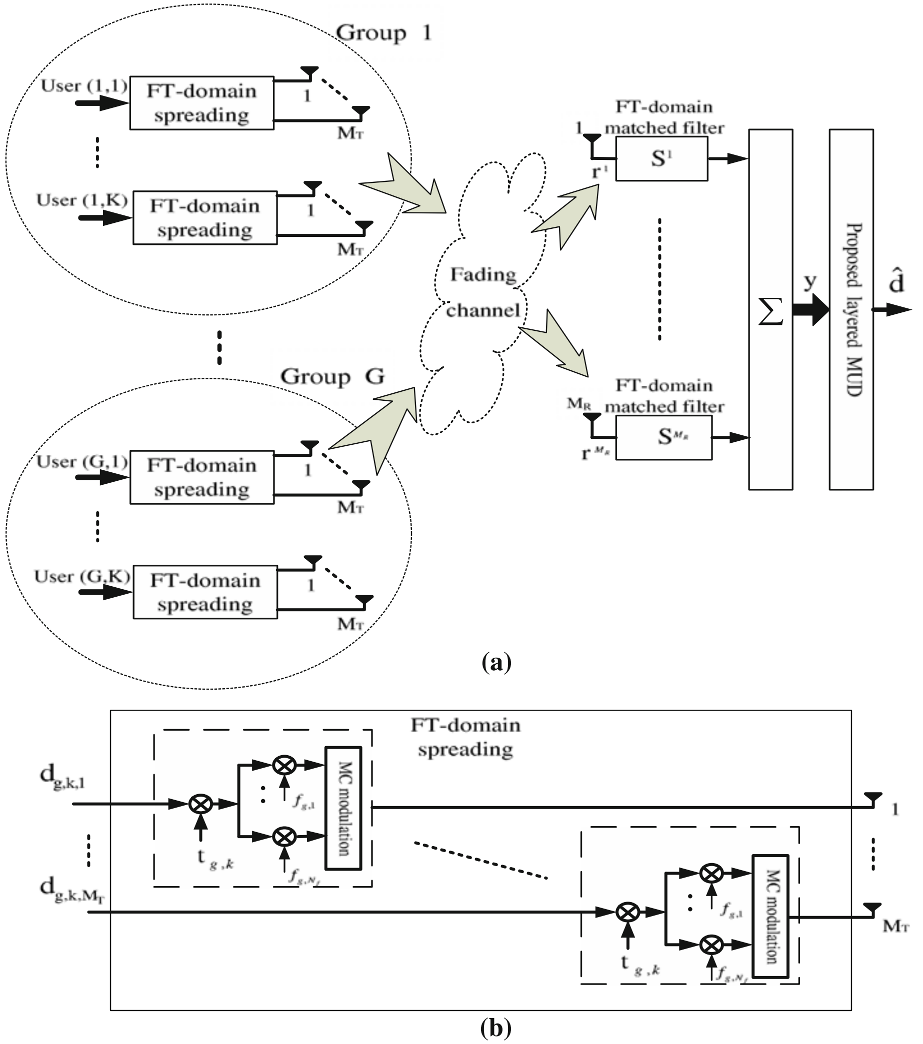

Fig. 1 **a** Block diagram of the uplink of a synchronous multiuser MIMO FT-domain spread MC DS-CDMA; **b** the structure of the FT-domain spreading mechanism

of derivation, we assume that the lengths of the fading channels are equal to L for all users. We also let $\{h^m_{g,k,i}(1), \ldots h^m_{g,k,i}(L)\}$ be the fading channel coefficients between the ith transmit antenna of the kth user in the gth group and the mth receive antenna at the base station, where $1 \le g \le G$, $1 \le k \le K$, $1 \le i \le M_T$, and $1 \le m \le M_R$. In addition, the corresponding $N_f \times N_f$ F-domain channel's transfer function matrix is denoted as $\mathbf{H}^m_{g,k,i} = diag\Big(\mathcal{FFT}[h^m_{g,k,i}(1), \ldots h^m_{g,k,i}(L), \mathbf{0}^T_{N_f-L}]\Big)$, where $\mathcal{FFT}(\cdot)$, $\mathbf{0}_U$, and $(\cdot)^T$ represent the FFT operation, a $U \times 1$ zero vector, and the transpose operation respectively. For simplicity, we assume that after removing the CP and applying the FFT mechanism, the received signal will be perfect. Then, the received signal of the mth receive antenna at the base station can be expressed as follows:

$$\mathbf{r}^m = \sum_{g=1}^{G}\sum_{k=1}^{K}\sum_{i=1}^{M_T}\Big((\mathbf{H}_{g,k,i}^m \mathbf{f}_g^m) \otimes \mathbf{t}_{g,k}\Big) d_{g,k,i} + \mathbf{n}^m, \quad 1 \le m \le M_R, \tag{1}$$

where $\otimes$ denotes the Kronecker product operator [24]; $d_{g,k,i}$ is the BPSK modulated symbol of the kth user in the gth group transmitted via the ith transmit antenna, where $1 \le g \le G$, $1 \le k \le K$, and $1 \le i \le M_T$; and $\mathbf{n}^m$ is the additive white Gaussian noise (AWGN) vector with $\mathcal{N}(0, \sigma^2 \mathbf{I}_{N_f N_t})$, where $\mathbf{I}_U$ is the $U \times U$ identity matrix. Furthermore, for brevity, let $\tilde{\mathbf{f}}_{g,k,i}^m = \mathbf{H}_{g,k,i}^m \mathbf{f}_g$ be the associative effective F-domain signature code; and let $\mathbf{s}_{g,k,i}^m = \tilde{\mathbf{f}}_{g,k,i}^m \otimes \mathbf{t}_{g,k}$ be the effective FT-domain signature code between the base station's mth receive antenna and the ith transmit antenna of the kth user in the gth group, where $1 \le g \le G$, $1 \le k \le K$, $1 \le i \le M_T$, and $1 \le m \le M_R$. Then, (1) can be re-written as

$$\mathbf{r}^m = \sum_{g=1}^{G}\sum_{k=1}^{K}\sum_{i=1}^{M_T} \mathbf{s}_{g,k,i}^m d_{g,k,i} + \mathbf{n}^m, \quad 1 \le m \le M_R. \tag{2}$$

To develop the proposed layered MUDs, we first stack the M_T symbols transmitted by the kth user in the gth group to form the transmitted symbol vector $\mathbf{d}_{g,k} = [d_{g,k,1}, d_{g,k,2}, \ldots, d_{g,k,M_T}]^T$. Then, we can re-write (2) as

$$\mathbf{r}^m = \sum_{g=1}^{G}\sum_{k=1}^{K} \mathbf{S}_{g,k}^m \mathbf{d}_{g,k} + \mathbf{n}^m, \quad 1 \le m \le M_R, \tag{3}$$

where $\mathbf{S}_{g,k}^m = [\mathbf{s}_{g,k,1}^m \ \mathbf{s}_{g,k,2}^m \ \cdots \ \mathbf{s}_{g,k,M_T}^m]$ is the corresponding user-based $N_f N_t \times M_T$ effective FT-domain spreading matrix between the mth receive antenna at the base station and the kth user in the gth group; and $1 \le g \le G$, $1 \le k \le K$, and $1 \le m \le M_R$. Similarly, by stacking the gth group's K transmitted symbol vectors to form the group's corresponding $KM_T \times 1$ transmitted symbol vector, denoted as $\mathbf{d}_g = [\mathbf{d}_{g,1}^T \ \mathbf{d}_{g,2}^T \ \cdot \ \mathbf{d}_{g,K}^T]^T$, we can re-write (3) as

$$\mathbf{r}^m = \sum_{g=1}^{G} \mathbf{S}_g^m \mathbf{d}_g + \mathbf{n}^m, \quad 1 \le m \le M_R, \tag{4}$$

where $\mathbf{S}_g^m = [\mathbf{S}_{g,1}^m \ \mathbf{S}_{g,2}^m \ \ldots \ \mathbf{S}_{g,K}^m]$ is the corresponding group-based $N_f N_t \times KM_T$ effective FT-domain spreading matrix between the mth receive antenna and the gth group. In a similar manner, we stack the G effective FT-domain spreading matrices and the transmitted symbol vectors as $\mathbf{S}^m = [\mathbf{S}_1^m \ \mathbf{S}_2^m \ \ldots \ \mathbf{S}_G^m]$ and $\mathbf{d} = [\mathbf{d}_1^T \ \mathbf{d}_2^T \ \ldots \ \mathbf{d}_G^T]^T$ respectively. Using $\mathbf{S}^m$ and $\mathbf{d}$, (4) can be re-written as

$$\mathbf{r}^m = \mathbf{S}^m \ \mathbf{d} + \mathbf{n}^m, \quad 1 \le m \le M_R. \tag{5}$$

Finally, following [17] and [18], we pass the M_R received signals $\mathbf{r}^m$, $m = 1, \ldots, M_R$ through the corresponding effective FT-domain matched filter matrices $\mathbf{S}^m$, $m = 1, \ldots, M_R$. Then, the sufficient statistics of the received signal of the M_R receive antennas at the base station can be represented by

$$\begin{aligned} \mathbf{y} &= \sum_{m=1}^{M_R} \mathbf{S}^{m^H} \mathbf{r}^m \\ &= \mathbf{C}\ \mathbf{d} + \mathbf{v}. \end{aligned} \tag{6}$$

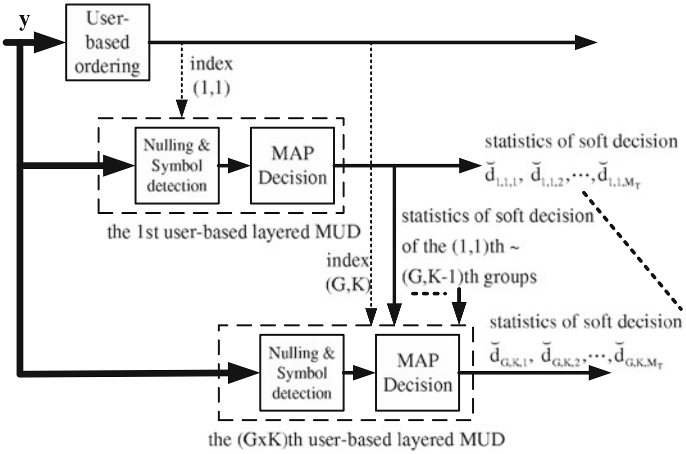

Fig. 2 The structure of the proposed user-based layered MUD

We define $\mathbf{C} = \sum_{m=1}^{M_R} \mathbf{S}^{m^H}\mathbf{S}^m$, which represents the $GKM_T \times GKM_T$ effective space-frequency-time (SFT) code correlation matrix, where $(\cdot)^H$ denotes the Hermitian operation [24]; and $\mathbf{v}$ is the corresponding Gaussian noise vector with $\mathcal{N}(0, \sigma^2\mathbf{C})$.

3 The Proposed User-Based Layered MUD

In this section, we describe the proposed user-based layered MUD, which exploits the previous user's soft decision errors to enhance the performance, as shown in Fig. 2. For ease of derivation, we assume that all the system information, such as the channel parameters and the users' signature codes, is available at the base station. We developed the proposed MUDs for performance enhancement because the VBLAST-related approaches in [14,17], and [18] generally assume that the corresponding interference in each layered detection stage can be estimated exactly and then removed from the received signal. The drawback with the assumption is that it ignores the error propagation effect, which in turn degrades the performance. To address the problem, the proposed user-based layered MUD utilizes one user's soft decision errors to improve the system's performance. Like other VBLAST-related systems, the proposed approach employs three key mechanisms: ordering, nulling, and symbol detection, which we describe in detail below.

3.1 User-Based Ordering

In VBLAST [14] and the symbol-based LAST MUD [17], the ordering mechanism must be implemented symbol by symbol, which incurs a huge computational overhead. To resolve the problem, we propose a user-based ordering method that exploits the users' effective SFT code correlation matrices for ordering. The effective SFT code correlation matrix of the kth user in the gth group can be expressed as

$$||\mathbf{C}_{g,k}||_F, \quad g = 1, \ldots, G, \text{ and } k = 1, \ldots, K, \tag{7}$$

where $\mathbf{C}_{g,k} = \sum_{m=1}^{M_R} \mathbf{S}_{g,k}^{m^H}\mathbf{S}_{g,k}^m$ is the corresponding $M_T \times M_T$ effective SFT code correlation matrix in which $\mathbf{S}_{g,k}^m$ is as defined in (3); and $|| \cdot ||_F$ is the Frobenius norm notation [24].

Because we assume that the coefficients of the fading channels remain fixed in a data frame and change frame by frame [19], the ordering mechanism can be implemented in one pass in a frame period. As a result, the huge computational complexity of existing VBLAST-like schemes can be reduced substantially.

3.2 User-Based Nulling and Symbol Detection

Without loss of generality, we assume that after applying the above ordering mechanism, the indices of the detection order are $(1, 1), (1, 2), \ldots, (g, k), \ldots, (G, K)$, where (g, k) denotes the index of the kth user in the gth group. In the following, we consider symbol detection for the (g, k)th users after the $(1, 1)$th, $\ldots$, $(g, k-1)$th users' corresponding symbol detection mechanisms have been applied and the interference contributed by the $(1, 1)$th, $\ldots$, $(g, k-2)$th users' has been removed from the received signal. Then, using (6), the sufficient statistics of the residual received signal can be expressed as

$$\breve{\mathbf{y}}_{g,k-1} = \breve{\mathbf{C}}_{g,k-1}\breve{\mathbf{d}}_{g,k-1} + \breve{\mathbf{v}}_{g,k-1}, \quad 1 \le g \le G, \text{ and } \ 1 \le k \le K, \tag{8}$$

where $\breve{\mathbf{C}}_{g,k-1}$ is the sub-block matrix derived by removing the corresponding columns and rows of the $(1, 1)$th, $\ldots$, $(g, k-2)$th users from the effective SFT code correlation matrix $\mathbf{C}$ in (6). In addition, $\breve{\mathbf{d}}_{g,k-1} = [\breve{\mathbf{e}}^T_{g,k-1}, \ \mathbf{d}^T_{g,k}, \ \mathbf{d}^T_{g,k+1}, \ldots, \ \mathbf{d}^T_{G,K}]^T$ is the corresponding residual symbol vector, where

$$\breve{\mathbf{e}}_{g,k-1} = \mathbf{d}_{g,k-1} - \tilde{\mathbf{d}}_{g,k-1} \tag{9}$$

is the soft decision error of the $(k-1)$th user in the gth group; $\tilde{\mathbf{d}}_{g,k-1} = [\breve{d}_{g,k-1,1}, \ \breve{d}_{g,k-1,2}, \ldots, \breve{d}_{g,k-1,M_T}]^T$ represents the corresponding statistics of the soft decisions [25]; and $\breve{\mathbf{v}}_{g,k-1}$ is the Gaussian noise vector. Note that because $\breve{\mathbf{e}}_{g,k-1}$ is the soft decision error of the $(k-1)$th user in the gth group, we only need to remove the corresponding estimated signals of the $(1, 1)$th, $\ldots$, $(g, k-2)$th users. Then, using (8), we utilize the minimum mean square error (MMSE) detector to estimate the symbols of the kth user in the gth group as follows:

$$arg \min_{\breve{\mathbf{W}}_{g,k}} E\left[||\mathbf{d}_{g,k} - \breve{\mathbf{W}}^H_{g,k}\breve{\mathbf{y}}_{g,k-1}||^2\right], \quad g = 1 \ldots G, \quad \text{and} \quad k = 1 \ldots K. \tag{10}$$

Here, $\breve{\mathbf{W}}_{g,k}$ is the $P \times M_T$ MMSE detection matrix for the kth user in the gth group, where we let $P = (G * K - ((g-1) * K + k - 2)) * M_T$; and $E[\cdot]$ and $|| \cdot ||$ are the expectation operation and the Euclidean norm respectively [24]. As a result, applying the gradient in (10) with respect to $\breve{\mathbf{W}}_{g,k}$ and setting it to zero yields

$$\breve{\mathbf{W}}_{g,k} = \left(E\{\breve{\mathbf{y}}_{g,k-1}\breve{\mathbf{y}}^H_{g,k-1}\}\right)^{-1}\left(E\{\breve{\mathbf{y}}_{g,k-1}\mathbf{d}^T_{g,k}\}\right). \tag{11}$$

For simplicity, we assume that the transmitted symbols are BPSK modulated. The results reported in this section can be easily extended to other modulation schemes. We also assume that the transmitted BPSK symbols are i.i.d., and the transmitted symbols and noise are mutually uncorrelated [2]. Based on the above assumptions and substituting (8) into (11), the MMSE detector in (11) can be re-written as

$$\begin{aligned} \breve{\mathbf{W}}_{g,k} &= \left(\breve{\mathbf{C}}_{g,k-1}E\{\breve{\mathbf{d}}_{g,k-1}\breve{\mathbf{d}}^T_{g,k-1}\}\breve{\mathbf{C}}^H_{g,k-1} + \sigma^2\breve{\mathbf{C}}_{g,k-1}\right)^{-1}\breve{\mathbf{C}}^{(g,k)}_{g,k-1}, \\ & g = 1 \ldots G, \quad \text{and} \quad k = 1 \ldots K, \end{aligned} \tag{12}$$

where $\breve{\mathbf{C}}^{(g,k)}_{g,k-1}$ is the corresponding effective SFT code correlation sub-block matrix of $\breve{\mathbf{C}}_{g,k-1}$ for the M_T transmitted symbols of the kth user in the gth group. Moreover,

$$E\{\breve{\mathbf{d}}_{g,k-1}\breve{\mathbf{d}}^T_{g,k-1}\} = \begin{bmatrix} E\{\breve{\mathbf{e}}_{g,k-1}\breve{\mathbf{e}}^T_{g,k-1}\} & \mathbf{0}_{M_T\times(P-M_T)} \\ \mathbf{0}_{(P-M_T)\times M_T} & \mathbf{I}_{P-M_T} \end{bmatrix} \tag{13}$$

and

$$[E\{\breve{\mathbf{e}}_{g,k-1}\breve{\mathbf{e}}^T_{g,k-1}\}]_{i,j} = \begin{cases} 1-(\breve{d}_{g,k-1,i})^2, & i=j, \\ (\breve{d}_{g,k-1,i})(\breve{d}_{g,k-1,j}), & \text{otherwise}, \end{cases}$$

where $[\mathbf{X}]_{i,j}$ is the (i,j)th entry of matrix $\mathbf{X}$, $\mathbf{1} \leq \mathbf{i}, \ \mathbf{j} \leq \mathbf{M_T}$; and $\breve{d}_{g,k-1,i}$ denotes the statistics of the soft decision for the ith transmitted symbol of the $(k-1)$th user in the gth group. Next, we derive the expression for the statistics of soft decision $\breve{d}_{g,k-1,i}$. First, using the MMSE detector $[\breve{\mathbf{W}}_{g,k}]_{:,i}$, we estimate the symbol transmitted by the ith transmit antenna of the kth user in the gth group. Then, we derive the detector output as follows:

$$\breve{z}_{g,k,i} = [\breve{\mathbf{W}}_{g,k}]^H_{:,i}\breve{\mathbf{y}}_{g,k-1}, \quad g=1\ldots G, \ k=1\ldots K, \quad \text{and} \quad i=1\ldots M_T, \tag{14}$$

where $[\mathbf{X}]_{:,i}$ denotes the ith column vector of matrix $\mathbf{X}$. Furthermore, $\breve{z}_{g,k,i}$ in (14) is the corresponding output of the MMSE detector. In general, it can be assumed that the distribution of the MMSE detector's output is approximately Gaussian with $\mathcal{N}(\breve{m}_{g,k,i}, \breve{\sigma}^2_{g,k,i})$ [1], where

$$\breve{m}_{g,k,i} = E\{\breve{z}_{g,k,i}d_{g,k,i}\} = [\breve{\mathbf{W}}_{g,k}]^H_{:,i}[\breve{\mathbf{C}}^{(g,k)}_{g,k-1}]_{:,i}, \tag{15}$$

$$\breve{\sigma}^2_{g,k,i} = \mathrm{var}\{[\breve{\mathbf{W}}_{g,k}]^H_{:,i}\breve{\mathbf{v}}_{g,k-1}\} = \sigma^2[\breve{\mathbf{W}}_{g,k}]^H_{:,i}\breve{\mathbf{C}}_{g,k-1}[\breve{\mathbf{W}}_{g,k}]_{:,i}. \tag{16}$$

Furthermore, based on the maximum a posteriori (MAP) method, the soft decision $\breve{\lambda}(d_{g,k,i})$ for the transmitted symbol $d_{g,k,i}$ can be expressed as follows [1,25]:

$$\breve{\lambda}(d_{g,k,i}) = \log\frac{p(\breve{z}_{g,k,i}|d_{g,k,i}=+1)}{p(\breve{z}_{g,k,j}|d_{g,k,i}=-1)} = \frac{2\times\breve{z}_{g,k,i}\times\breve{m}_{g,k,i}}{\breve{\sigma}^2_{g,k,i}}, \quad g=1\ldots G, \\ k=1\ldots K, \ \text{and} \quad i=1\ldots M_T. \tag{17}$$

The corresponding statistics of the soft decision are $\breve{d}_{g,k,i} = E\{d_{g,k,i}\} = tanh(\frac{1}{2}\breve{\lambda}(d_{g,k,i}))$. Using (17), the M_T soft decisions of the kth user in the gth group are estimated in parallel. The resulting decisions are used to estimate the corresponding soft decision error $\breve{\mathbf{e}}_{g,k}$, which is then used to perform the detection and nulling steps for the $(k+1)$th user in the gth group. Therefore, the proposed user-based layered MUD estimates the transmitted symbols in a user-layer by user-layer manner.

4 The Proposed Group-Based MUD

In this section, we consider the proposed group-based layered MUD, which is designed to reduce the computational complexity in a group-layer by group-layer manner. Like the user-based layered MUD, the proposed approach is comprised of three mechanisms: ordering, nulling, and symbol detection; and it only utilizes the previous group's soft decision errors to enhance the BER performance. We describe the mechanisms in detail below.

4.1 Group-Based Ordering

Similar to the ordering process described in Sect. 3, the effective SFT code correlation matrices of the G groups are used to rank the detection order, which is given by

$$||\mathbf{C}_g||_F, \quad g = 1, \ldots, G, \tag{18}$$

where $\mathbf{C}_g = \sum_{m=1}^{M_R} \mathbf{S}_g^{m^H} \mathbf{S}_g^m$ is the effective SFT code correlation matrix of the gth group, and $\mathbf{S}_g^i$ is as defined in (4). Note that, like the user-based layered MUD, the ordering mechanism in (18) only needs to be applied once in a frame period.

4.2 Group-Based Nulling and Symbol Detection

Without loss of generality, after applying the group-based ordering in (18), the indices of the detection order are $1, 2, \ldots, G$. In addition, we perform group-based nulling and symbol detection for the gth group under the assumption that symbol detection for the 1th, ..., $(g-1)$th groups has been completed and the interference contributed by the 1th, ..., $(g-2)$th groups has been estimated and removed form the received signal. Then, the sufficient statistics of the residual received signal can be expressed as

$$\bar{\mathbf{y}}_{g-1} = \bar{\mathbf{C}}_{g-1}\bar{\mathbf{d}}_{g-1} + \bar{\mathbf{v}}_{g-1}, \quad g = 1, \ldots G, \tag{19}$$

where $\bar{\mathbf{C}}_{g-1}$ is the sub-block matrix derived by removing the corresponding rows and columns of the 1th, 2th, ..., $(g-2)$th groups from the effective SFT code correlation matrix $\mathbf{C}$. Furthermore, $\bar{\mathbf{d}}_{g-1} = [\bar{\mathbf{e}}_{g-1}^T, \ \mathbf{d}_g^T, \ldots, \ \mathbf{d}_G^T]^T$, which contains the $KM_T \times 1$ soft decision error vector $\bar{\mathbf{e}}_{g-1}$ for the $(g-1)$th group and the $(G-g+1)KM_T \times 1$ residual symbol vectors, $\mathbf{d}_g, \ldots, \mathbf{d}_G$; and $\mathbf{v}_{g-1}$ is the corresponding Gaussian noise vector. Note that we define the soft decision error vector $\bar{\mathbf{e}}_{g-1} = \mathbf{d}_{g-1} - \check{\mathbf{d}}_{g-1}$, where $\check{\mathbf{d}}_{g-1} = [\bar{d}_{g-1,1,1}, \ \bar{d}_{g-1,1,2}, \ldots, \ \bar{d}_{g-1,K,M_T}]^T$ are the statistics of the soft decisions for the $(g-1)$th group. Similar to the user-based layered MUD (10)–(13), the corresponding MMSE detector for the gth group can be formulated as follows

$$\bar{\mathbf{W}}_g = \left(\bar{\mathbf{C}}_{g-1}E\{\bar{\mathbf{d}}_{g-1}\bar{\mathbf{d}}_{g-1}^T\}\bar{\mathbf{C}}_{g-1}^H + \sigma^2\bar{\mathbf{C}}_{g-1}\right)^{-1}\bar{\mathbf{C}}_{g-1}^{(g)}, \tag{20}$$

where

$$E\{\bar{\mathbf{d}}_{g-1}\bar{\mathbf{d}}_{g-1}^T\} = \begin{bmatrix} E\{\bar{\mathbf{e}}_{g-1}\bar{\mathbf{e}}_{g-1}^T\} & \mathbf{0}_{KM_T\times(Q-KM_T)} \\ \mathbf{0}_{(Q-KM_T)\times KM_T} & \mathbf{I}_{Q-KM_T} \end{bmatrix}; \tag{21}$$

and

$$E\{\bar{\mathbf{e}}_{g-1}\bar{\mathbf{e}}_{g-1}^T\} = diag\left(1-\bar{d}_{g,1,1}^2, 1-\bar{d}_{g,1,2}^2, \ldots, 1-\bar{d}_{g,K,M_T}^2\right), \tag{22}$$

where $Q = (G-g+1)KM_T$; and $\bar{\mathbf{C}}_{g-1}^{(g)}$ is the sub-block matrix of $\bar{\mathbf{C}}_{g-1}$ for the gth group. Then, the corresponding output of the MMSE detector for the kth user in the gth group transmitted by the ith transmit antenna can be expressed as

$$\bar{z}_{g,k,i} = [\bar{\mathbf{W}}_g]_{:,l}^H\bar{\mathbf{y}}_g, \quad g = 1\ldots G, \ k = 1\ldots K, \ \text{and} \quad i = 1\ldots M_T, \tag{23}$$

where $l = (k-1)M_T + i$. We also assume that the distribution of the output of the MMSE detector, $\bar{z}_{g,k,i}$, is approximately Gaussian with $\mathcal{N}(\bar{m}_{g,k,i}, \bar{\sigma}^2_{g,k,i})$, where

$$\bar{m}_{g,k,i} = E\{\bar{z}_{g,k,i} d_{g,k,i}\} = [\bar{\mathbf{W}}_g]^H_{:,l}[\bar{\mathbf{C}}^{(g)}_{g-1}]_{:,l}, \text{ and} \tag{24}$$

$$\bar{\sigma}^2_{g,k,i} = \text{var}\{[\bar{\mathbf{W}}_g]^H_{:,l}\bar{\mathbf{v}}_{g-1}\} = \sigma^2[\bar{\mathbf{W}}_g]^H_{:,l}\bar{\mathbf{C}}_{g-1}[\bar{\mathbf{W}}_g]_{:,l}, \tag{25}$$

Then, for the kth user in the gth group, the soft decision $\bar{\lambda}(d_{g,k,l})$ transmitted by the ith transmit antenna can be expressed as

$$\bar{\lambda}(d_{g,k,i}) = \log \frac{p(\bar{z}_{g,k,i}|d_{g,k,i} = +1)}{p(\bar{z}_{g,k,i}|d_{g,k,i} = -1)} = \frac{2 \times \bar{z}_{g,k,i} \times \bar{m}_{g,k,i}}{\bar{\sigma}^2_{g,k,i}}, \quad g = 1 \ldots G, \quad k = 1 \ldots K, \text{ and } \quad i = 1 \ldots M_T. \tag{26}$$

The corresponding statistics of the soft decision are $\bar{d}_{g,k,i} = E\{d_{g,k,i}\} = tanh(\frac{1}{2}\bar{\lambda}(d_{g,k,i}))$. Note that by using (19)–(26), the group-based layered MUD obtains the soft decisions of the gth group in parallel. The decisions are used to construct the corresponding soft decision error vector $\bar{\mathbf{e}}_g$, which is then utilized to detect the next layered symbols for the $(g+1)$th group. Therefore, the proposed group-based layered MUD is implemented in a group-layer by group-layer manner.

5 Simulations and Discussion

We conducted a number of simulations and a complexity analysis to assess the performance of the proposed schemes. It is assumed that (1) the channels in the simulations are frequency selective Rayleigh fading, (2) the length of the channels, L, is 3. In addition, the signal-to-noise-ratio (SNR) is defined as the ratio of the symbol energy E_b to the noise power σ^2, i.e., E_b/σ^2. For simplicity, we utilize the BPSK symbol modulation scheme in the simulations; however, the results can be generalized to other modulation methods. To compare the performance, we consider five algorithms: the zero-forcing (ZF) MUD [19], the symbol-based LAST MUD [17], the user-based LAST MUD [18], the proposed user-based layered MUD, and the proposed group-based layered MUD.

We investigate the effects of various lengths of the F-domain and T-domain signature codes on the performance of the above five schemes. First, we consider a scenario with the following system settings: ($G = 4$, $K = 3$, $M_T = 2$, $M_R = 4$, $N_f = 9$, $N_t = 8$). The corresponding bit error rate (BER) results versus the SNR are shown in Fig. 3a. From the figure, we observe that the proposed user-based layered MUD outperforms the other four schemes; while the proposed group-based scheme outperforms both of the LAST MUD schemes and the ZF scheme. The LAST MUD and ZF schemes are subject to more serious error-flooring problems than the proposed layered MUDs. Therefore, utilizing the soft decision error in the proposed schemes helps improve the system performance.

Next, we change $N_f = 9$ to 12 and leave the other parameter settings unchanged. From the results, shown in Fig. 3b, we observe that the performance of each of the five schemes is only slightly better than that under $N_f = 9$. Even though the performance gain is not significant, we can still conclude that utilizing the soft decision error enables the proposed MUDs to outperform the other three schemes. Once again, the LAST MUD schemes and the ZF scheme are affected by serious error-flooding problems.

In the next simulation, we use the system parameters shown in Fig. 3a, but $N_t = 12$ instead of 8. The results are shown in Fig. 4a. We observe that the BER performance results of the

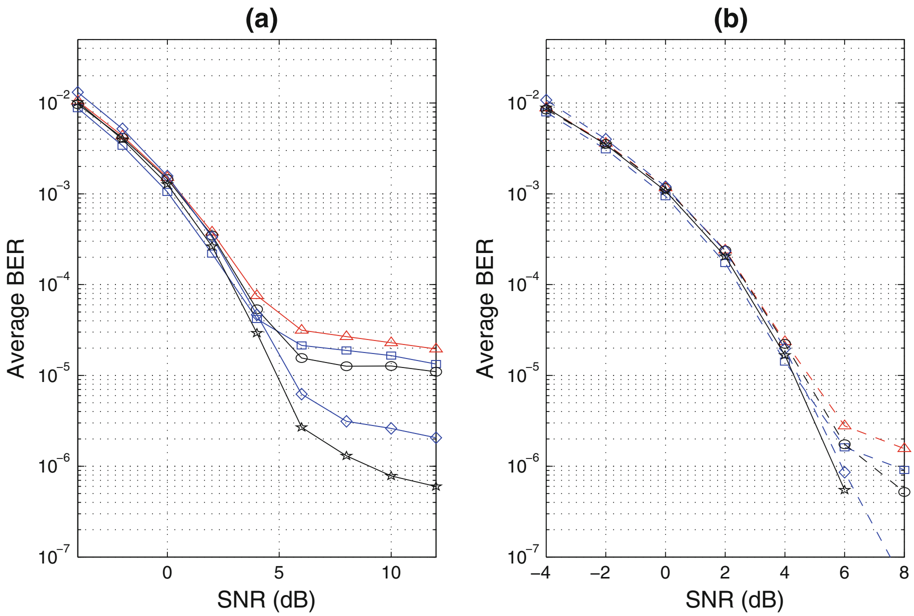

Fig. 3 Comparison of the BER versus the SNR with $G = 4$, $K = 3$, $M_T = 2$, $M_R = 4$, **a** $N_f = 9$, $N_t = 8$, **b** $N_f = 12$, and $N_t = 8$ (*triangle* ZF, *square* user-based LAST MUD, *circle* symbol-based LAST MUD, *diamond* proposed group-based layered MUD, *star* proposed user-based layered MUD)

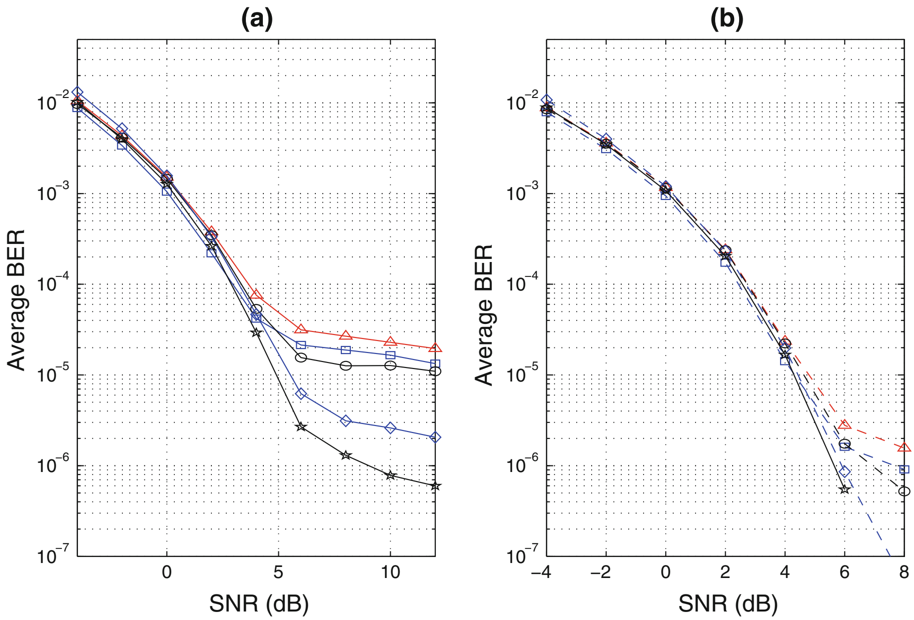

Fig. 4 Comparison of the BER versus the SNR with $G = 4$, $K = 3$, $M_T = 2$, $M_R = 4$, **a** $N_f = 9$, $N_t = 12$, **b** $N_f = 12$, and $N_t = 16$ (*triangle* ZF, *square* user-based LAST MUD, *circle* symbol-based LAST MUD, *diamond* proposed group-based layered MUD, *star* proposed user-based layered MUD)

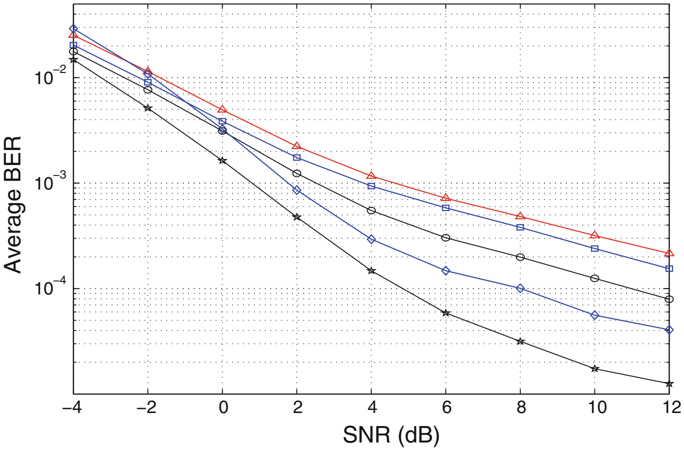

Fig. 5 Comparison of the BER versus the SNR with $G = 6$, $K = 3$, $M_T = 2$, $M_R = 4$, $N_f = 9$, and $N_t = 8$ (*triangle* ZF, *square* user-based LAST MUD, *circle* symbol-based LAST MUD, *diamond* proposed group-based layered MUD, *star* proposed user-based layered MUD)

five schemes in Fig. 4a are all significantly better than those in Fig. 3a–b. This implies that longer T-domain signature codes are more effective in increasing the signal space and thereby mitigating the MAI effect in an FT-domain spread MC DS-CDMA system. Furthermore, we change the lengths of the F-domain and T-domain signature codes from ($N_f = 9$, $N_t = 8$) to ($N_f = 12$, $N_t = 16$) concurrently, but leave the values of the other parameters as shown Fig. 3a. From the results, presented in Fig. 4b, we observe that all five schemes achieve significant performance gains over the results shown in Figs. 3a, b and 4a. Hence, for all five schemes, to mitigate the MAI effect, increasing the lengths of the F-domain and T-domain signature codes concurrently is more effective than increasing them at different times. In addition, from Fig. 4a, b, we can draw the same conclusions as those drawn from Fig. 3a. The results in Figs. 3a, b and 4a, b also show that, in contrast to the LAST MUD and ZF MUD schemes, the proposed two layer MUDs are more robust against short signature codes.

Next, we consider three scenarios to evaluate the proposed schemes for robustness to MAI. Because the number of transmitted symbols increases, MAI usually becomes more serious. Yang and Wang [21] demonstrated that short signature codes are suitable for 2D spread MC DS-CDMA systems; therefore we use the system parameters shown in Fig. 3a, but change ($G = 4$–6), ($K = 3$–4), and ($M_T = 2$–3) for the three scenarios. The parameter settings are as follows: ($G = 6$, $K = 3$, $M_T = 2$, $M_R = 4$, $N_f = 9$, $N_t = 8$), ($G = 4$, $K = 4$, $M_T = 2$, $M_R = 4$, $N_f = 9$, $N_t = 8$), and ($G = 4$, $K = 3$, $M_T = 3$, $M_R = 4$, $N_f = 9$, $N_t = 8$). The corresponding results are shown in Figs. 5, 6, and 7 respectively. Note that the total numbers of transmitted symbols for the three scenarios are $G \times K \times M_T = 6 \times 3 \times 2 = 36$, $4 \times 4 \times 2 = 32$, and $4 \times 3 \times 3 = 36$ respectively. From the results in Figs. 5, 6 and 7, we observe that the proposed user-based layered MUD outperforms the other schemes. Meanwhile, the proposed group-based layered MUD performs better than the ZF scheme and the LAST MUD schemes in the SNR >2 dB scenarios. In summary, the proposed two layer MUDs utilize soft decision errors effectively and help mitigate the effects of MAI.

Next, we analyze the computational complexity of the five compared schemes. Because computing the inversions of the matrices usually dominates the complexity overhead, we focus on the computations. The user-based layered MUD needs $2P^3$ complex multiplications

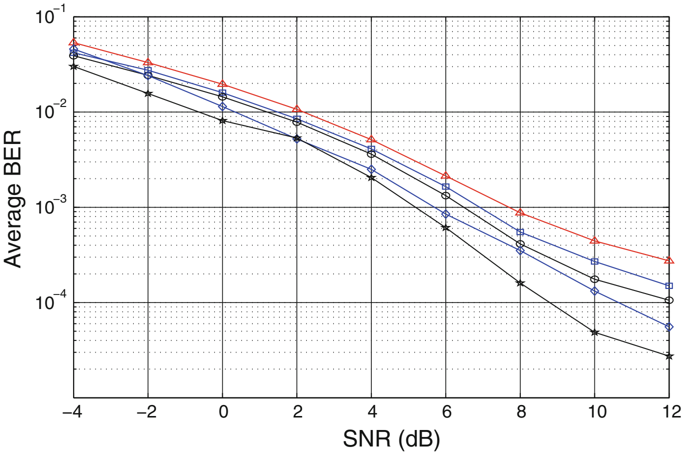

Fig. 6 Comparison of the BER versus the SNR with $G = 4, K = 4, M_T = 2, M_R = 4, N_f = 9$, and $N_t = 8$ (*triangle* ZF, *square* user-based LAST MUD, *circle* symbol-based LAST MUD, *diamond* proposed group-based layered MUD, *star* proposed user-based layered MUD)

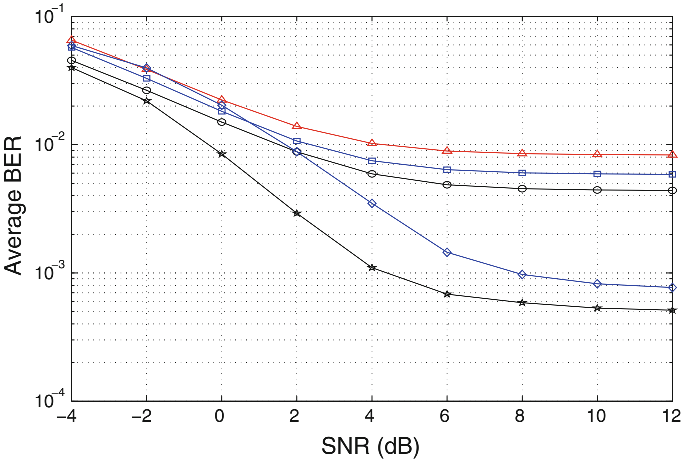

Fig. 7 Comparison of the BER versus the SNR with $G = 4, K = 3, M_T = 3, M_R = 4, N_f = 9$, and $N_t = 8$ (*triangle* ZF, *square* user-based LAST MUD, *circle* symbol-based LAST MUD, *diamond* proposed group-based layered MUD, *star* proposed user-based layered MUD)

and additions (CMAs) [24] to compute the inversion of the $P \times P$ matrix in (12) for the kth user in the gth group, $k = 1 \dots K$ and $g = 1 \dots G$, where $P = (G*K-((g-1)*K+k-2))*M_T$ is as defined in Section 3. Hence, the total number of CMAs required to compute the inversions of the matrices of the user-based layered MUD is $\sum_{g=1}^{G}\sum_{k=1}^{K} 2P^3$. Similarly, the group-based layered MUD needs $2Q^3$ CMAs to compute the inversion of a $Q \times Q$ matrix for the gth group; hence, it requires $\sum_{g=1}^{G} 2Q^3$ CMAs, where $Q = (G-g+1)KM_T,\ g = 1 \dots G$. The symbol-based LAST MUD, user-based LAST MUD, and ZF scheme need $\sum_{i=0}^{GKM_T-1}(GKM_T - i)^3$, $\sum_{i=0}^{GK-1}(GKM_T - i * M_T)^3$, and $(GKM_T)^3$ respectively. For ease of reference, we summarize the above complexity expressions in Table 1 and present the practical results in Table 2 by substituting the values of the settings in the above scenarios.

Table 1 Comparison of the computational complexity of the proposed schemes

MUDs	Complex multiplications/additions
ZF	$(GKM_T)^3$
Symbol-based LAST MUD	$\sum_{i=0}^{GKM_T-1}(GKM_T - i)^3$
User-based LAST MUD	$\sum_{i=0}^{GK-1}(GKM_T - i * M_T)^3$
Proposed user-based layered MUD	$\sum_{g=1}^{G}\sum_{k=1}^{K} 2P^3$
Proposed group-based layered MUD	$\sum_{g=1}^{G} 2Q^3$

Table 2 Comparison of the computational complexity under various values of G, K, and M_T

Detectors	$G = 4$, $K = 3$, $M_T = 2$	$G = 6$, $K = 3$, $M_T = 2$	$G = 4$, $K = 4$, $M_T = 2$	$G = 4$, $K = 3$, $M_T = 3$
ZF	13824	46656	32768	46656
Symbol-based LAST MUD	90000	443556	278784	443556
User-based LAST MUD	48672	233928	147968	164268
Proposed user-based layered MUD	97344	467856	295936	328536
Proposed group-based layered MUD	43200	190512	102400	145800

From Table 2, we observe the ZF method requires the smallest number of CMAs. However, the simulation results show that it yields the worst BER performance among the five schemes. The symbol-based LAST MUD and the user-based MUD require similar numbers of CMAs, but the latter achieves the best BER performance among the five schemes. The CMAs of the group-based layered MUD are higher than those of the ZF method, but lower than those of the two LAST MUDs and the user-based MUD. Therefore, based on the above results, we conclude that the group-based layered MUD is more feasible in practice.

6 Conclusions

We have proposed two soft decision error assisted layered MUDs for the uplink of frequency-time-domain spread MC DS-CDMA systems. First, we presented a user-based layered MUD that utilizes the previous user's soft decision error to mitigate the effects of MAI. Then, to reduce the computational complexity of practical implementations, we proposed a group-based layered MUD that utilizes the previous group's soft decision error. The results of simulations and a complexity analysis demonstrate that the user-based layered MUD outperforms the group-based layered MUD and three existing schemes. However, the group-based layered MUD is a viable alternative because its performance is similar to that of the user-based layered MUD, and its computational overhead is reasonable.

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