Spatial Link Scheduling for SCDMA Multihop Cellular Networks: A Cross Layer Framework

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Abstract Probability of error based Spatial Code Division Multiple Access scheduling algorithm is presented in this paper to systematically reuse the orthogonal CDMA codes in a given cell for Multihop Cellular Network. We assign and reuse the CDMA codes to peer-to-peer links such that the probability of error in all scheduled links are below certain threshold. The proposed scheduling algorithm PoE-LinkSchedule involves two phases. In the first phase we present a scheduling metric “Probability of Error (PoE)” as a function of first and second order statistics of wireless channel coefficients between nodes. The second phase presents a graph theoretical as well as PoE based centralized scheduling algorithm. For a graph of network with n number of nodes, U number of links and θ thickness, the proposed scheduling algorithm has computational complexity of $O(Un\log n + Un\theta)$ as opposed to $O(U^U)$ in the case of exhaustive search algorithm. The performance of the proposed algorithm is evaluated in terms of *spatial reuse* and end-to-end throughput. We show that the proposed algorithm has considerably higher end-to-end throughput and higher *spatial reuse* compared to existing link scheduling algorithms.

Keywords Scheduling · Multihop cellular networks · Spatial reuse

1 Introduction

Multi-hop cellular networks (MCNs) combine the best characteristics of both centralised cellular and distributed ad hoc networks at the same time reduces the issues associated with

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both the networks individually [1–4]. In MCN mobile terminals transmit packets to base stations as well as to other mobile stations using multiple hops with less transmission range. Due to this short range communications MCN enhances the throughput, user capacity and energy efficiency [5]. This paper addresses the problem of link scheduling for CDMA based MCN to increase the spatial reuse. By link scheduling, we mean assignment of proper CDMA codes to peer-to-peer links and reuse them at farther distance links such that communications over all links are successful in probability of error sense. CDMA access mechanism with orthogonal spreading codes is considered.¹ However, it can be shown that the number of orthogonal waveforms that can be designed within a given bandwidth (W) and time duration (T) is limited ($O(WT)$). Furthermore, since the transmissions in MCN are divided into many smaller hops, there could be many simultaneous smaller range transmissions in a given time than a conventional single hop cellular networks [6]. Therefore, it is of paramount importance to devise a scheduling strategy to reuse the orthogonal CDMA codes in a given cell to maximize the systems user capacity. Once the link scheduling information is available we can find source to destination path along the scheduled links using some routing algorithms so that the spatial reuse will be maximized in the network. We term the CDMA system where the CDMA codes are reused in a given cell as Spatial-CDMA (SCDMA) system. The problem of determining an optimal link schedule for a general multihop network is NP-complete [7,8]. Hence, we present a suboptimal method for link scheduling in SCDMA MCNs. The significant contributions of this paper are:

- (1) SNR based graph theoretical algorithms as well as brute-force computations for link scheduling in SCDMA wireless systems often lead to high probability of error in the links. To overcome this problem, we propose a link scheduling algorithm PoE-LinkSchedule for SCDMA MCN with probability of error criterion. Since the probability of error is a widely used performance metric for digital systems, it is appropriate to use it as a criterion for scheduling.
- (2) Peer-to-peer physical layer probability of error is formulated by assuming independent and identically distributed (i.i.d) Rayleigh multipath channel (frequency selective) between nodes. Only the statistical knowledge of the channel coefficients between mobile nodes is assumed in the probability of error formulation instead of complete channel knowledge. The above formulated physical layer probability of error will be used as a criterion in MAC layer link scheduling. Hence, we have a cross layer scheduling scheme.
- (3) We introduce spatial reuse as an important performance metric and argue that a high value of spatial reuse directly translates to higher throughput over many links.

2 Related Work

A Spatial Time Division Multiple Access (STDMA) link scheduling is reported in [7], shows that the tree networks can be scheduled optimally and oriented graphs can be scheduled near-optimally. Link scheduling algorithms under the physical interference model are derived in [9,10]. In [11], the authors investigate the time complexity of scheduling a set of communication requests in an arbitrary network. A general framework for the max–min scheduling problem in static wireless networks is proposed in [12]. A cross layer framework for multiple access problem in a contention based wireless network is derived in [13]. The

¹ Orthogonal CDMA codes are considered since it gives tight bounds on resources. The scheduling strategy with non orthogonal CDMA codes with fixed correlation coefficient is under investigation.

design of simple distributed dynamic routing algorithms and scheduling policies based upon link state information is proposed in [14]. The problem of determining the jointly optimal end-to-end communication rates, routing, power allocation and transmission scheduling for wireless networks is considered in [15]. A code reuse scheme based on a heuristic method is proposed in [8]. Variable Spreading Factor (VSF) code allocation protocol for maximizing throughput in CDMA based ad-hoc networks is analyzed in [16]. Our proposed algorithm PoE-LinkSchedule is considerably different from all the above work. It is a cross layer approach with physical layer probability of error used as metric.

3 Scheduling Metric Formulation

Consider a DS-CDMA system with U transmitting links. All the nodes are assumed to transmit at same power P . Multipath channel is considered for wireless links between nodes. Partial channel knowledge, namely the first order and second order statistics of the channel between nodes are assumed to be available at the base station. Logical channel is divided into Control Channels (CCH) and Traffic Channels (TCH). CCH handles only signaling, while TCH carries speech and data traffic. Base station is the centralized scheduler. Scheduling information is transmitted from base station to the nodes through CCH. We formulate the scheduling metric i.e. link (peer-to-peer) probability of error by conditioning on the transmitted bits at the link. The transmit node of link u transmits BPSK bit $b_u(i)$, with amplitude A_u , in i th bit interval. The length of signaling interval for each user is T_{bit} . Assume that the transmit node of link u is assigned with a spreading waveform $c_u(\cdot)$ as shown in Fig. 1 and $\mathbf{s}_u = [s_{u0}, s_{u1}, \dots, s_{uN-1}]^T$ denotes the spreading sequence corresponding to the spreading waveform $c_u(\cdot)$. Then,

$$c_u(t) = \sum_{k=0}^{N-1} s_{uk} \text{rect}(t - kT_c), \quad u = 1, 2, \dots, U \quad (1)$$

where T_c is the chip period, $\text{rect}(t)$ is a rectangular waveform with unit amplitude in $[0, T_c]$ and N is the processing gain of the system. The baseband signal for the u th link in the i th bit interval can now be expressed as

$$x_u(t) = A_u b_u(i) c_u(t - iT_{bit}), \quad iT_{bit} \leq t < (i+1)T_{bit} \quad (2)$$

Fig. 1 Proposed system model for communication between links

Assume that $x(t)$ is sampled ($x[n]$) at T_c (chip rate sampling), then

$$x[n] = A_u b_u(n_N) \tilde{s}_u[n] \quad (3)$$

where $n_N = \lfloor \frac{n}{N} \rfloor$ because of chip rate sampling (note that $NT_c = T_{bit}$) and $\tilde{s}_u[n] = s_{u((n) \bmod(N))}$. The multipath wireless channel is modeled as a Finite Impulse Response (FIR) filter and the channel coefficients are assumed to be constant over one bit period. The channel gain for the u th link at the i th bit interval is denoted as $\mathbf{h}_u[l][i]$ which is of length L_h^u for all i . The elements of the channel FIR filter ($h_u[l][n]$) are assumed to be complex Gaussian with both real and imaginary parts following the *i.i.d.* Gaussian distribution. The noise ($\eta_u[n]$) is assumed to be *i.i.d.* zero mean Additive White Gaussian Noise (AWGN). The received signal at user u can now be written as

$$r_u[n] = \mathbf{h}_u[.][n] \otimes x[n] + \eta_u[n] \quad (4)$$

where $\otimes$ denotes the convolution operation and $h_v[.][n]$ ($v \neq u$) is the channel coefficient between any other node (interfering nodes) to the receiver of link u . Converting $r_u[n]$ into a parallel stream of N samples (number of chips), we obtain $\mathbf{r}_u[n] = [r_u[iN], \dots, r_u[iN + N - 1]]^T$. Rake receiver is used in the link's receiver as shown in Fig. 1. The received signal after rake receiver filtering is

$$y_u[i] = \sum_{f=1}^F s_u^{(f)T} \mathbf{r}_u[i] = \sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} r_u[iN + k] \quad (5)$$

where F is the number of fingers and $s_{uk}^{(f)}$ is the sampled version of PN code corresponding to f th finger. From (4) and (5)

$$\begin{aligned} y_u[i] &= \sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} h_u[m][j] \sum_{v=1}^U A_v b_v[j] \tilde{s}_v[iN + k - m] \\ &+ \sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \eta_u[iN + k] \end{aligned} \quad (6)$$

where

$$j = \left\lfloor \frac{iN + k - m}{N} \right\rfloor \quad (7)$$

$\sum_{m=0}^{L_h^u-1}$ gives the Inter Symbol Interference (ISI) term due to the multipath channel while $\sum_{v=1}^U$ is the Multiple Access Interference (MAI) component due to loss of orthogonality of the spreading codes. Since BPSK constellations are used for input data, the decision statistics is given by $\Re(y_u[i]) = y_u^R[i]$. We compute the conditional probability of error ($P_{E|\mathbf{B}[i]}$) conditioned on transmitted bit vector sequence $\mathbf{B}[i] = \mathbf{b}[i], \mathbf{b}[i-1], \dots$, where $\mathbf{b}[i] = [b_1[i], b_2[i], \dots, b_U[i]]^T$ is the vector of bits transmitted at time instant i in links $1, 2, \dots, U$. The mean of the m th channel coefficient at i th bit interval is defined as

$$\gamma_u[m][i] = E[h_u[m][i]] \quad (8)$$

and the second order statistics of the channel coefficients are

$$C_{u,v}[m_1, m_2][i_1, i_2] = E[(h_u[m_1][i_1] - \gamma_u[m_1][i_1])(h_v^*[m_2][i_2] - \gamma_v^*[m_2][i_2])] \quad (9)$$

$$\tilde{C}_{u,v}[m_1, m_2][i_1, i_2] = E[(h_u[m_1][i_1] - \gamma_u[m_1][i_1])(h_v[m_2][i_2] - \gamma_v[m_2][i_2])] \quad (10)$$

where $*$ denotes complex conjugate. The conditional mean ($\mu_{y_u^R|\mathbf{B}[i]}$) of the decision statistic ($y_u^R[i]$) is given by

$$\mu_{y_u^R|\mathbf{B}[i]} = E \left[\Re \left(\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U h_v[m][j] A_v b_v[j] \tilde{s}_v[iN+k-m] + \sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \eta_u[iN+k] \right) \right] \quad (11)$$

By using the fact that $E[\Re(a)] = \Re(E[a])$ for any a , $E(\eta_u[iN+k]) = 0$, and except $h_u[m][j]$, $\eta_u[iN+k]$ all other quantities are deterministic, the above equation can be written as

$$\mu_{y_u^R|\mathbf{B}[i]} = \Re \left[\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U \gamma_v[m][j] A_v b_v[j] \tilde{s}_v[iN+k-m] \right] \quad (12)$$

Note that $\mu_{y_u^R|\mathbf{B}[i]}$ is conditioned on transmitted bit and it is a function of mean of the channel coefficient $\gamma_u[m][j]$.

Without loss of generality we can assume that channel $h_u[.][.]$ and noise $\eta_u[.]$ follow independent distribution. Now by using the fact that $\text{var}(a+b) = \text{var}(a) + \text{var}(b)$ when a and b are independent, the conditional variance of the decision statistics ($\sigma_{y_u^R|\mathbf{B}[i]}^2$) is

$$\sigma_{y_u^R|\mathbf{B}[i]}^2 = \text{var} \left[\Re \left(\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U h_v[m][j] A_v b_v[j] \tilde{s}_v[iN+k-m] \right) \right] + \text{var} \left[\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \eta_u[iN+k] \right] \quad (13)$$

Since the receiver noises are assumed to be zero mean *i.i.d* AWGN with equal variance (i.e., σ) and also the spreading coefficient $s_{uk}^{(f)}$ is either $+1$ or -1 and deterministic, we can get

$$\text{var} \left[\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \eta_u[iN+k] \right] = N \times F \frac{\sigma^2}{2} \quad (14)$$

where we have used linearity property of variance operation over independent random variables. Let

$$\sigma_{y_u^R|\mathbf{B}[i]}^2 = \text{var} \left[\Re \left(\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U h_v[m][j] A_v b_v[j] \tilde{s}_v[iN+k-m] \right) \right] \quad (15)$$

This can be rewritten as

$$\sigma_{y_u^R|\mathbf{B}[i]}^2 = \text{var} \left[\Re \left(\sum_{f,k,m,v} h_v[m][j] \phi_v[f, k, m, v] \right) \right] \quad (16)$$

where $\phi_u[f, k, m, v] = s_{uk}^{(f)} A_v b_v[j] \tilde{s}_v[iN + k - m]$ is deterministic variable, the index j is given in (7) and the sum $\sum_{f,k,m,v} = \sum_{f=1}^F \sum_{k=0}^{N-1} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U$. Since $\phi_u[f, k, m, v]$ is deterministic variable and by using the fact that $\text{var}(a) = E[a - E(a)]^2$, (16) can be written as

$$\begin{aligned} \sigma_{\tilde{y}_u^R|\mathbf{B}[i]}^2 &= E \left[\left(\Re \left[\sum_{f,k,m,v} (h_u[m][j] - \gamma_u[m][j]) \phi_u[f, k, m, v] \right] \right)^2 \right] \\ &= E \left[\Re \left(\sum_{f_1,k_1,m_1,v_1} (h_u[m_1][j_1] - \gamma_u[m_1][j_1]) \phi_u[f_1, k_1, m_1, v_1] \right) \right. \\ &\quad \left. \cdot \Re \left(\sum_{f_2,k_2,m_2,v_2} (h_u[m_2][j_2] - \gamma_u[m_2][j_2]) \phi_u[f_2, k_2, m_2, v_2] \right) \right] \quad (17) \end{aligned}$$

where

$$j_1 = \left\lfloor \frac{iN + k_1 - m_1}{N} \right\rfloor, \quad j_2 = \left\lfloor \frac{iN + k_2 - m_2}{N} \right\rfloor \quad (18)$$

For notational convenience let us assume

$$\begin{aligned} h_1 &= h_u[m_1][j_1], & \gamma_1 &= \gamma_u[m_1][j_1], & \phi_1 &= \phi_u[f_1, k_1, m_1, v_1] \\ h_2 &= h_u[m_2][j_2], & \gamma_2 &= \gamma_u[m_2][j_2], & \phi_2 &= \phi_u[f_2, k_2, m_2, v_2] \end{aligned} \quad (19)$$

Since the channel coefficients of different links are *i.i.d.*, from (17), (19) we can get

$$\sigma_{\tilde{y}_u^R|\mathbf{B}[i]}^2 = E \left[\sum_{\substack{f_1,k_1,m_1,v_1 \\ f_2,k_2,m_2,v_2}} \Re[(h_1 - \gamma_1)\phi_1] \Re[(h_2 - \gamma_2)\phi_2] \right] \quad (20)$$

By using the fact that $\Re(a) = \frac{a+a^*}{2}$ for any complex number a , the above equation can be rewritten as

$$\sigma_{\tilde{y}_u^R|\mathbf{B}[i]}^2 = \frac{1}{4} E \left[\sum_{\substack{f_1,k_1,m_1,v_1 \\ f_2,k_2,m_2,v_2}} (h_1\phi_1 - \gamma_1\phi_1 + h_1^*\phi_1^* - \gamma_1^*\phi_1^*) (h_2\phi_2 - \gamma_2\phi_2 + h_2^*\phi_2^* - \gamma_2^*\phi_2^*) \right] \quad (21)$$

Since ϕ_1 and ϕ_2 are deterministic the above equation will become

$$\sigma_{\tilde{y}_u^R|\mathbf{B}[i]}^2 = \frac{1}{4} \left[\sum_{\substack{f_1,k_1,m_1,v_1 \\ f_2,k_2,m_2,v_2}} \tilde{C}_{v_1,v_2} \phi_1 \phi_2 + \tilde{C}_{v_1,v_2}^* \phi_1^* \phi_2^* + C_{v_1,v_2} \phi_1 \phi_2^* + C_{v_1,v_2}^* \phi_1^* \phi_2 \right] \quad (22)$$

where $C_{v_1, v_2} = C_{v_1, v_2}[m_1, m_2][j_1, j_2]$ and $\tilde{C}_{v_1, v_2} = \tilde{C}_{v_1, v_2}[m_1, m_2][j_1, j_2]$ can be calculated using (9) and (10) respectively. From (14), (22) the conditional variance can be written as

$$\sigma_{y_u^R|\mathbf{B}[i]}^2 = \frac{1}{4} \left[\sum_{\substack{f_1, k_1, m_1, v_1 \\ f_2, k_2, m_2, v_2}} \tilde{C}_{v_1, v_2} \phi_1 \phi_2 + \tilde{C}_{v_1, v_2}^* \phi_1^* \phi_2^* + C_{v_1, v_2} \phi_1 \phi_2^* + C_{v_1, v_2}^* \phi_1^* \phi_2 \right] + N \times F \frac{\sigma^2}{2} \quad (23)$$

We have derived expressions for the conditional mean (12) and conditional variance (23) of $y_u^R[i]$. The conditional probability of error of u th link as a function of above derived conditional mean and conditional variance is [17]

$$P_{E|\mathbf{B}[i]}[i] = Q \left(\frac{b_u[i] \mu_{y_u^R|\mathbf{B}[i]}[i]}{\sigma_{y_u^R|\mathbf{B}[i]}[i]} \right) \quad (24)$$

Let us denote the conditional probability of error of u th link between nodes i and j as P_{Eij} . The code assignment and reuse will be done such that $P_{Eij} < \beta \forall i, j \in n$ where β is the probability of error threshold and n is the number nodes in the network.

4 System Model and Proposed Link Schedule Algorithm

Let us consider a SCDMA MCN network $\Phi(\mathcal{V}, \mathcal{E})$ with n nodes, where $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ is the set of vertices/nodes and $\mathcal{E}$ is the set of edges/links between nodes. A link schedule for the SCDMA network is denoted by $\Psi(\mathbf{C}, \mathcal{M}_1, \dots, \mathcal{M}_C)$, where

- $\mathbf{C} = \{c_i\}$ set of CDMA codes available for link schedule
- $|\mathbf{C}| :=$ size of $\mathbf{C}$, $|\mathcal{E}| :=$ U size of $\mathcal{E}$, $|\mathcal{V}| :=$ size of $\mathcal{V}$
- $\mathcal{M}_i =$ set of transmitter–receiver pairs which can communicate concurrently using same CDMA code c_i i.e. $\{t_{i,1} \rightarrow r_{i,1}, \dots, t_{i,|\mathcal{M}_i|} \rightarrow r_{i,|\mathcal{M}_i|}\}$
- $|\mathcal{M}_i| :=$ size of $\mathcal{M}_i$

If node v_k is within node v_j 's *communication range*, then there is a *communication edge* from v_j to v_k , denoted by $v_j \xrightarrow{c} v_k$. Thus, the mapping from network $\Phi(\cdot)$ to communication network $\mathcal{G}_c(\mathcal{V}_c, \mathcal{E}_c)$ can be described as follows:

$$D(j, k) \leq R_c \Rightarrow v_j \xrightarrow{c} v_k \in \mathcal{E}_c, \quad v_k \xrightarrow{c} v_j \in \mathcal{E}_c \quad (25)$$

where R_c is the communication range [18] and $D(j, k)$ is the distance between nodes v_j and v_k . i.e. $\mathcal{G}_c(\mathcal{V}_c, \mathcal{E}_c)$ consists of nodes who have communication link with atleast one of its neighbour. The schedule $\Psi(\cdot)$ is then designed from the graph $\mathcal{G}_c(\cdot)$. The scheduling problem is to assign CDMA code to nodes such that the communication over links are successful and simultaneously maximize the *spatial reuse* of the CDMA codes in the network. Specifically, an SCDMA link scheduling algorithm is equivalent to assigning a unique color to every communication edge in the graph, such that source–destination pairs corresponding to *communication edges* with the same CDMA code can transmit simultaneously in a particular time slot.

4.1 PoE-LinkSchedule Algorithm

We use graph coloring approach for scheduling of CDMA codes among links. The term “colors” in graph coloring approach represents available CDMA codes. The proposed PoE-LinkSchedule algorithm is described in Algorithm 1. We use physical layer probability of error as metric in scheduling algorithm. Probability of error is formulated based only on first and second order statistics of the channel.

- Step-1: In first step of our PoE-LinkSchedule algorithm at Line 3, we label all the vertices (nodes) randomly using uniform distribution.
- Step-2: After the labeling step, communication graph $\mathcal{G}_c(\cdot)$ obtained in Sect. 4 (Eq. (25)) is decomposed into k number of out-oriented and in-oriented graphs $g_1, g_2, \dots, g_k$ in Line 4 [7]. This decomposition is achieved by partitioning graph $\mathcal{G}_c(\cdot)$, the undirected equivalent of $\mathcal{G}_c(\cdot)$ into undirected forests. To reduce intensive computations, a *successive breadth first searches* is used to decompose $\mathcal{G}_c(\cdot)$ into undirected forests. Each undirected forest is further mapped to two directed forests. In one forest, the edges (links) in every connected component point away from the root and every vertex has at most one incoming edge, thus producing an out-oriented graph. In the other forest, the edges in every connected component point toward the root and every vertex has at most one outgoing edge, thus producing an in-oriented graph. An in-oriented graph is also constructed by Algorithm 1 in [10] to determine a link schedule in a power-controlled STDMA network.
- Step-3: In last step Lines 5–14, the oriented graphs are considered sequentially. For each oriented graph, vertices are considered in increasing order by label and the unique edge associated with each vertex is colored using the PoE-LinkColor function. In essence, the edges are considered in a random order for scheduling, since labeling is random. In Line 8 and 10 *Label*($\cdot$) is the re-labeling function which assigns number j to nodes which are randomly considered for re-labeling. The PoE-LinkColor function is explained in Algorithm 2. In Algorithm 1 the *communication edge* between nodes s and d ($s \rightarrow d$) is denoted as $\lambda(s, d)$. We choose the first color such that the resulting probability of error at the receiver of λ and the receivers of all co-colored edges are below the threshold β . If no such color is found, we assign a new color to λ .

Algorithm 1 PoE-LinkSchedule

- 1: **input:** Given network $\Phi(\cdot)$ and its associated communication graph $\mathcal{G}_c(\cdot)$
 - 2: **output:** Set of colors $\mathbf{C}$ which ensures successful communication in all links, $\mathbf{C} : \mathcal{E}_c \rightarrow \{1, 2, \dots\}$
 - 3: Label the vertices of $\mathcal{G}_c$ randomly using uniform distribution
 - 4: Use *successive breadth first searches* algorithm to partition $\mathcal{G}_c$ into out and in oriented graphs g_i , $1 \leq i \leq k$
 - 5: **for** $i \leftarrow 1$ to k **do**
 - 6: **for** $j \leftarrow 1$ to n **do**
 - 7: **if** g_i is out-oriented **then**
 - 8: let $\lambda = (s, d)$ be such that $\text{Label}(d) = j$
 - 9: **else**
 - 10: let $\lambda = (s, d)$ be such that $\text{Label}(s) = j$
 - 11: **end if**
 - 12: $\mathbf{C}(\lambda) \leftarrow \text{PoE-LinkColor}(\lambda)$
 - 13: **end for**
 - 14: **end for**
-

Algorithm 2 PoE-LinkColor($\hat{\lambda}$)

```

1: input: Given network  $\Phi(\cdot)$  and its associated communication graph  $\mathcal{G}_c(\cdot)$ 
2: output: A color which ensures successful communication in a given link
3:  $\mathcal{C} \leftarrow$  set of existing colors
4:  $\mathcal{C}_c \leftarrow \{C(h) : h \in \mathcal{E}_c, h \text{ is colored, } \hat{\lambda} \text{ and } h \text{ interferes each other and hence } \text{PoE} \geq \beta \text{ in both } \hat{\lambda} \text{ and } h\}$ 
5:  $\mathcal{C}_{cf} = \mathcal{C} \setminus \mathcal{C}_c$  i.e.  $\mathcal{C}_{cf}$  is nothing but  $\mathcal{C}$  except  $\mathcal{C}_c$ 
6: for  $i \leftarrow 1$  to  $|\mathcal{C}_{cf}|$  do
7:    $r \leftarrow i^{\text{th}}$  color in  $\mathcal{C}_{cf}$ 
8:    $E_r \leftarrow \{h : h \in \mathcal{E}_c, \mathbf{C}(h) = r\}$ 
9:    $\mathbf{C}(\hat{\lambda}) \leftarrow r$ 
10:  if probability of error at all receivers of  $E_r \cup \{\hat{\lambda}\}$  lower than  $\beta$  then
11:    return  $r$ 
12:  end if
13: end for
14: return  $|\mathcal{C}| + 1$ 

```

5 Complexity Analysis

5.1 Complexity of the Probability of Error Scheduling Metric

As the $Q(\cdot)$ function value can be found from look-up table, the complexity in determining probability of error (24) essentially lies on determining the values of $\mu_{y_u^R|\mathbf{B}[i]}$ and $\sigma_{y_u^R|\mathbf{B}[i]}^2[i]$.

5.1.1 Complexity in Determining $\mu_{y_u^R|\mathbf{B}[i]}$

The received signal in (6) is expanded in the following equation with the assumption of perfect synchronization

$$y_u[i] = \sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u-1} h_u[m][j] (A_u b_u[j] \tilde{s}_u[iN+k-m]) + \sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \eta_u[iN+k] \quad (26)$$

Since $b_u[j]$ is either $+1$ or -1 and if we assume $A_u = 1, \forall u$ the computational complexity in determining $\mu_{y_u^R|\mathbf{B}[i]}$ of (26) is $2L_h$ flops. If we include the complexity in determining the index term j the total complexity will be $10L_h$ multiplications and $4L_h$ additions where we have assumed division takes 4 flops [19]. Therefore, the total complexity involved in determining $\mu_{y_u^R|\mathbf{B}[i]}$ is $O(L_h)$.

5.1.2 Complexity in Determining $\sigma_{y_u^R|\mathbf{B}[i]}^2[i]$

From (23)

$$\sigma_{y_u^R|\mathbf{B}[i]}^2[i] = \frac{1}{4} \left[\sum_{\substack{f_1, k_1, m_1, v_1, \\ f_2, k_2, m_2, v_2}} \tilde{C}_{v_1, v_2} \psi_1 \psi_2 + \tilde{C}_{v_1, v_2}^* \psi_1^* \psi_2^* + C_{v_1, v_2} \psi_1 \psi_2^* + C_{v_1, v_2}^* \psi_1^* \psi_2 \right] + N \times F \frac{\sigma^2}{2} \quad (27)$$

As $b_u[j]$, s_{uk} are either $+1$ or -1 and if we assume $A_u = 1$, $\forall u$ then ψ_1 and ψ_2 can be directly determined. Due to the above fact the summations $\sum_{k_1=0}^{N-1}$, $\sum_{k_2=0}^{N-1}$ in (27) will not constitute any additional complexity. Furthermore, the second order statistics of the channel coefficients C_u , C_u^* , $\tilde{C}_u$, $\tilde{C}_u^*$ are already available. Therefore, the terms which constitute complexity are $\sum_{m_1=0}^{L_h^u-1}$, $\sum_{m_2=0}^{L_h^u-1}$ as ψ_u s are functions of m and also the terms $\sum_{v_1=1}^U$, $\sum_{v_2=1}^U$ as C_u s are functions of u . Hence, the total number of operations involved in determining the $\sigma_{y_u^R|\mathbf{B}[i]}^2[i]$ is $O(U^2 L_h^2)$.

After analyzing the complexity involved in determining $\mu_{y_u^R|\mathbf{B}[i]}$ and $\sigma_{y_u^R|\mathbf{B}[i]}^2[i]$ we can conclude that the number of operations required to determine probability of error metric for a particular link is $O(U^2 L_h^2)$.

5.2 Complexity of the Proposed Scheduling Algorithm

In this section, we derive upper bounds on the running time complexity (computational complexity) of the PoE-LinkSchedule algorithm. Let us assume θ as thickness of the communication graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c)$ i.e., minimum number of graphs into which the undirected equivalent of $\mathcal{G}_c(\cdot)$ can be partitioned.

Lemma 1 *An oriented graph g can be colored with no more than $O(n)$ colors using PoE-LinkSchedule.*

Proof Since an oriented graph with n vertices has at most n edges, the edges of g can be colored with at most n colors. $\square$

Lemma 2 *For an oriented graph g , the running time of PoE-LinkSchedule is $O(n^2)$.*

Proof Assuming that an element can be chosen randomly and uniformly from a finite set in unit time (Chapter 1, [20]), the running time of Phase 1 can be shown to be $O(n)$. Since there is only one oriented graph, Phase 2 runs in time $O(1)$. In Phase 3, the unique edge associated with the vertex under consideration is assigned a color using PoE-LinkColor algorithm. From Lemma 1, the size of the set of colors to be examined $|\mathcal{C}_c \cup \mathcal{C}_{cf}|$ is $O(n)$. In PoE-LinkColor algorithm, the probability of error is checked only once for every colored edge in the set $\bigcup_{i=1}^{|\mathcal{C}_{cf}|} E_i$ and at most n times for the edge under consideration λ . With a careful implementation, PoE-LinkColor algorithm runs in time $O(n)$. So, the running time of Phase 3 is $O(n^2)$. Thus, the total running time is $O(n^2)$. $\square$

Theorem 1 *For an arbitrary graph $\mathcal{G}$, the running time of PoE-LinkSchedule is $O(Un \log n + Un\theta)$.*

Proof Assuming that an element can be chosen randomly and uniformly from a finite set in unit time [20], the running time of Phase 1 can be shown to be $O(n)$. For Phase 2, the optimal partitioning technique of [21] based on Matroids can be used to partition the communication graph $\mathcal{G}_c$ into at most 6θ oriented graphs in time $O(Un \log n)$. Thus, $k \leq 6\theta$ holds for Phase 3. From Lemma 2, it follows that the first oriented graph g_1 can be colored in time $O(n^2)$. However, consider the coloring of the j th oriented graph g_j , where $2 \leq j \leq k$. When coloring edge λ from g_j using PoE-LinkColor algorithm, conflicts can occur not only with the colored edges of g_j , but also with the edges of the previously colored oriented graphs $g_1, g_2, \dots, g_{j-1}$. Hence, the worst-case size of the set of colors to be examined $|\mathcal{C}_c \cup \mathcal{C}_{cf}|$ is $O(U)$. Note that in PoE-LinkColor algorithm, the probability of error is checked only once

for every colored edge in the set $\bigcup_{i=1}^{|C_{cf}|} E_i$ and at most U times for the edge under consideration λ . With a careful implementation, PoE-LinkColor algorithm runs in time $O(U)$. Hence, any subsequent oriented graph g_j can be colored in time $O(Un)$. Thus, the running time of Phase 3 is $O(Un\theta)$. Therefore, the overall running time of PoE-LinkSchedule is $O(Un \log n + Un\theta)$. $\square$

This complexity is comparable to that of ArboricalLinkSchedule algorithm of [7] and much lower than that of TruncatedGraphSchedule algorithm of [9]. Furthermore, this complexity is much lower than that of exhaustive search algorithm which has complexity of $O(U^U)$.

6 Effect of Channel on the PoE Metric

The numerator term of the $Q(\cdot)$ function of probability of error metric derived in (24) is

$$b_u[i] \mu_{y_u^R | \mathbf{B}[i]}[i] = b_u[i] \Re \left[\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U \gamma_v[m][j] A_v b_v[j] \tilde{s}_v[iN + k - m - l] \right] \quad (28)$$

By using the inequality that $\Re(a) \leq |a|$ for any complex number a (28) can be written as

$$b_u[i] \mu_{y_u^R | \mathbf{B}[i]}[i] \leq \left| b_u[i] \left[\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U \gamma_v[m][j] A_v b_v[j] \tilde{s}_v[iN + k - m - l] \right] \right| \quad (29)$$

By using the assumption of *i.i.d* distribution of channel and simple algebraic manipulation (29) can be further written as

$$b_u[i] \mu_{y_u^R | \mathbf{B}[i]}[i] \leq \sum_{f=1}^F \sum_{k=0}^{N-1} \sum_{m=0}^{L_h^u-1} \sum_{v=1}^U \left| s_{uk}^{(f)} \gamma_v[m][j] A_v b_v[j] \tilde{s}_v[iN + k - m - l] \right| \quad (30)$$

Since chip sequence is having unit energy, prefilter coefficients are normalized to unit energy and if we assume $A_u = 1 \forall u$ and also perfect synchronization and the chip waveforms matched with rake receiver fingers are independent then (30) will become

$$b_u[i] \mu_{y_u^R | \mathbf{B}[i]}[i] \leq F \sum_{m=0}^{L_h^u-1} |\gamma_u[m][j_1]| \quad (31)$$

From (13) and (14) the variance term (denominator) of the probability of error metric (24) can be written as

$$\begin{aligned} \sigma_{y_u^R | \mathbf{B}[i]}^2[i] &= \text{var} \left[\Re \left(\sum_{f=1}^F \sum_{k=0}^{N-1} s_{uk}^{(f)} \sum_{m=0}^{L_h^u-1} h_u[m][j_1] \sum_{v=1}^U A_v b_v[j] \tilde{s}_v[iN + k - m - l] \right) \right] \\ &\quad + N \times F \frac{\sigma^2}{2} \geq N \times F \frac{\sigma^2}{2} \end{aligned} \quad (32)$$

The above inequality is from the fact that $\text{var}(\cdot) \geq 0$. From (24), (31) and (32) the lower bound on individual probability of error can be written as

$$P_{E|\mathbf{B}[i]}^u \geq Q \left(\frac{\sqrt{F} \sum_{m=0}^{L_h^u-1} |\gamma_u[m][j_1]|}{\sqrt{(N\sigma^2/2)}} \right) \quad (33)$$

From (33) we can see that the lower bound on probability of error in a link increases when the channel mean, number of multipaths reduce and receiver noise increases. Therefore, we can conclude that the *spatial reuse* which is defined in (34) will increase when the mean of the channel, number of multipath increases and the receiver noise reduces.

7 Simulations and Results

7.1 Performance Metric-I: Spatial Reuse

The *spatial reuse* of the schedule is defined as the average number of successfully received packets per CDMA code in the SCDMA schedule. The transmission from node i using CDMA code c at t th time slot is successful at receiving node j only if $P_{E_{i,j}} \leq \beta$. Thus

$$\text{spatial reuse} = \tau = \frac{\sum_{c=1}^{|\mathbf{C}|} \sum_{k=1}^{|\mathcal{M}_c|} I(P_{E_{i,j}} < \beta)}{|\mathbf{C}|} \quad (34)$$

where $I(A)$ denotes the indicator function for event A , i.e., $I(A) = 1$ if event A occurs, $I(A) = 0$ if event A does not occur. From (34) we can see that large value of τ implies that there are many simultaneous successful communications.

7.2 Performance Metric-II: End-to-End Throughput

End-to-end throughput is defined as the probability of successful transmission from source node to destination node, which involves successful transmission at each and every intermediate hop. The closed form expression for the end-to-end throughput is derived as follows

- Case I: Number of hops > 1

Let us say $x_m = \{1, 2, 3, \dots, h\}$ is the path selected to relay the packets from source node 1 to the destination node h using some routing algorithms and number of hops in the routing path is $h - 1$. The probability $P(S_{1,h})$ that the message is successfully transmitted from source 1 to destination h is given by:

$$P(S_{1,h}) = P\left(\bigcap_{i=1}^{h-1} S_{i,i+1}\right) = 1 - P\left(\bigcup_{i=1}^{h-1} E_{i,i+1}\right) \geq 1 - \sum_{i=1}^{h-1} P(E_{i,i+1}) \quad (35)$$

where $P(E_{i,i+1})$ is the probability of error in the link $i, i + 1$ as derived in (24).

- Case II: Number of hops = 1, $P(S_{1,h}) = 1 - P(E_{1,h})$

For simulations a single circular cell of radius $R = 1,000$ m is considered with nodes distributed as stationary distribution derived from Random Waypoint (RWP) mobility model [22]. The radial distribution of nodes as a function of radial distance r from the center is given by

$$f(r) = \frac{12}{73} (27r - 35r^3 + 8r^5) \quad (36)$$

Fig. 2 Scatter plots of node distribution for uniform distribution and the corresponding RWP model

Fig. 3 Spatial reuse versus number of nodes

Figure 2 shows the steady state spatial distribution of nodes for RWP mobility model and its corresponding uniform distribution in the space of 1,000 m radius circular region. All the nodes are assumed to transmit with a constant power P . The value of R_c is considered to be 500 m. From the nodes' distribution we construct $\Phi(\cdot)$ and then map network $\Phi(\cdot)$ to the graph $\mathcal{G}_c(\cdot)$ using (25). The PoE-LinkSchedule is computed using the proposed algorithm. Once the link schedule is computed, the *spatial reuse* is calculated using (34). The *spatial reuse* is averaged over 100 nodes' distributions. In the underlined simulation environment with the SCDMA network the *spatial reuse* is calculated using the algorithms ArboricalLinkSchedule proposed in [7], TruncatedGraphSchedule algorithm of [9] and also ConflictFreeLinkSchedule of [18]. The *spatial reuse* is plotted against the varying number of nodes in Fig. 3. We can observe that the PoE-LinkSchedule has higher *spatial reuse* compared to all other algorithms. This is because PoE-LinkSchedule has probability of error as scheduling metric while all the

Fig. 4 Spatial reuse versus transmission range

other algorithms have SNR as schedule metric. Hence, the *spatial reuse* which is the function of probability of error is higher in the proposed PoE-LinkSchedule.

The *spatial reuse* is also plotted against the transmission range for a fixed node density. From Fig. 4, we can see that the *spatial reuse* increases for increasing transmission range (transmission power) until it reaches the maximum range. However, after it reaches the maximum, it starts decreasing. This is because at lower transmission ranges the received signal strength will be minimum and receiver noise dominates the received signal. Therefore, as we increase the transmission range received signal strength will also increase and so does the *spatial reuse*. However, after some level of transmission range, the interference dominates the received signal. Hence, the *spatial reuse* decreases as shown in Fig. 4.

We have also plotted the lower bound on end-to-end throughput by selecting a random source and random destination and using minimum hop count routing algorithm. The lower bound on end-to-end throughput is calculated using (35) and averaged over at least 100 different source–destination pairs. Thus obtained lower bound on end-to-end throughput is plotted in Fig. 5 for various SNRs. We can conclude from Fig. 5 that the proposed algorithm has better end-to-end throughput. This is because the end-to-end throughput metric is defined as a function of probability of error and the proposed algorithm has lower probability of error in all links due to probability of error constraint.

8 Conclusion

In this work, we have developed PoE-LinkSchedule algorithm for SCDMA MCNs. An empirical modeling shows that, on an average, our algorithm achieves higher *spatial reuse* and end-to-end throughput compared to the ConflictFreeLinkSchedule, ArboricalLinkSchedule and TruncatedGraphSchedule algorithms. Furthermore, PoE-LinkSchedule algorithm has complexity comparable to that of ArboricalLinkSchedule algorithm and much lower than that of TruncatedGraphSchedule algorithm. Since the statistics of the channel varies slower than the channel coefficients itself, computing PoE metric and scheduling the links offline in a centralized fashion is feasible. Thus, in cognizance of *spatial reuse* and end-to-end throughput

Fig. 5 End-to-end throughput versus SNR

PoE-LinkSchedule is a good candidate for efficient link scheduling algorithm. Once the scheduling information is available the source to destination path for multihop voice/data communication can be established through the scheduled links using some standard routing algorithms.

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