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ORIGINAL RESEARCH

# Counterpossibles in science: an experimental study

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## Abstract

A *counterpossible* is a counterfactual whose antecedent is impossible. The *vacuity thesis* says all counterpossibles are true solely because their antecedents are impossible. Recently, some have rejected the vacuity thesis by citing purported non-vacuous counterpossibles in science. One limitation of this work, however, is that it is not grounded in experimental data. Do scientists actually reason non-vacuously about counterpossibles? If so, what is their basis for doing so? We presented biologists ($N = 86$) with two counterfactual formulations of a well-known model in biology, the antecedents of which contain what many philosophers would characterize as a metaphysical impossibility. Participants consistently judged one counterfactual to be true, the other to be false, and they explained that they formed these judgments based on what they perceived to be the mathematical relationship between the antecedent and consequent. Moreover, we found no relationship between participants' judgments about the (im)possibility of the antecedent and whether they judged a counterfactual to be true or false. These are the first experimental results on counterpossibles in science with which we are familiar. We present a modal semantics that can capture these judgments, and we deal with a host of potential objections that a defender of the vacuity thesis might make.

**Keywords** Counterfactual reasoning · Counterpossibles · Model-based reasoning · The vacuity thesis · Impossible worlds

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## 1 Introduction

As we use the term, a *counterpossible* is a counterfactual conditional whose antecedent is logically, mathematically, or metaphysically impossible. Take the following example from Bjerring (2014, p. 328):

(1) If intuitionistic logic were correct, then the law of excluded middle would fail.

On the presumption that classical logic is necessarily true, (1) is a counterpossible because its antecedent describes a scenario that is logically impossible. Here's another example, from Dorr (2008, p. 37):

(2) If I were a dolphin, I would have arms and legs.

If it's metaphysically impossible for Dorr to be a dolphin, then (2) has a metaphysically impossible antecedent and is thereby a counterpossible too.

A common view in counterfactual semantics is the *vacuity thesis*, according to which all counterpossibles are vacuously true, that is, true solely because their antecedents are impossible. Lewis's (1973) influential account of counterfactuals implies the vacuity thesis, for instance. Lewis construes a counterfactual as a function $f$ that takes as arguments a sentence $A$ and the world $w$ at which $A$ is uttered, then returns as a value the set of closest possible worlds accessible to $w$ at which $A$ is true, given the context of the utterance. $A \mathrel{\Box\!\!\rightarrow} B$ is true at $w$ if $B$ is true at each world in the set given by $f(A, w)$. When $A$ is impossible, this condition is satisfied for the simple reason that there are no worlds given by $f(A, w)$. See also Kratzer (1979) and Stalnaker (1968).

The *conditional probability hypothesis* (Evans & Over, 2004), a prominent account of the psychology of counterfactual reasoning, raises a similar issue. Inspired by the approach of Ramsey (1929/1990), proponents of the conditional probability hypothesis maintain that the credence an individual assigns to "If $A$, then $B$" is given by the value they assign to $Pr(B|A)$. But if one accepts the "definition" of conditional probability, then $Pr(B|A)$ is undefined when $A$ is impossible, since $Pr(B|A) = Pr(A\&B)/Pr(A)$. One could obviate this issue by treating a conditional probability as a primitive, though this leaves unresolved how in such a case an agent assigns a value to $Pr(B|A)$. Stalnaker (1970), among others, adopted the vacuist convention that $Pr(B|A) = 1$ whenever $A$ is impossible.

While the vacuity thesis may be "orthodoxy," there is no dearth of heretics. It is not hard to see why. Take (1), the counterpossible about intuitionism. According to intuitionism, $A \vee \neg A$ is true just in case there is a proof of $A$ or a proof of $\neg A$. Now suppose $A$ is a theorem that has been neither proved nor disproved (e.g., Goldbach's Conjecture). Intuitionism holds that $A \vee \neg A$ is neither true nor false, which makes (1) look *non*-vacuously true. Likewise, (2) seems false: if Dorr were a dolphin, he'd have flippers and flukes, not arms and legs.

Recently, some have argued that there are non-vacuous counterpossibles in science (Jenkins & Nolan, 2012; Jenny, 2018; McLoone, 2021; Tan, 2019). One example comes from the use of differential equations to model the dynamics of a population of objects that are discrete as a matter of metaphysical necessity (McLoone, 2021; Tan, 2019).[1] For instance, the *logistic equation* is a differential equation from ecology that models the dynamics of population size ($N$) when a habitat has a "carrying capacity" ($K$), that is, a limit on how many individuals it can support. Because one can only differentiate a continuous function, the model assumes that $N$ is a continuous quantity. Suppose the model is used to characterize the growth of some population of organisms that are necessarily discrete, like rabbits. That means one of the assumptions of the model is impossible. But take the following two counterpossibles, from McLoone (2021, p. 12161):

(3) If some population of rabbits satisfied the assumptions of the logistic equation, then the size of the population ($N$) would eventually be equal to the carrying capacity ($K$).
(4) If some population of rabbits satisfied the assumptions of the logistic equation, then the population would eventually go extinct.

Because the logistic equation mathematically entails that the size of the population will eventually equal the carrying capacity, and that the population won't go extinct, it seems that (3) is non-vacuously true and that (4) is false.

Counterpossibles drawn from science are especially important to the debate about the vacuity thesis. If scientists reason non-vacuously about counterpossibles, this suggests that the vacuity thesis is wanting as a descriptive thesis. Moreover, if one takes scientific reasoning to be a model of exemplary reasoning, particularly when it is tethered to mathematical inference, then that suggests the vacuity thesis is wanting as a normative thesis too. This is precisely the argument that Jenny (2018) and Tan (2019) advance. However, we understand that the reader may not wish to infer how one *should* reason from how a group in fact *does* reason, in which case they can instead focus on the descriptive aspect of our project.

One limitation of work that claims scientists reason non-vacuously about counterpossibles is that it has not been supported by experimental data. What has occurred instead is that a case in which scientists appear to reason non-vacuously over an impossibility is construed counterfactually ex post facto, and on that basis it is claimed that scientists are committed to non-vacuous counterpossibles. This is the case, for instance, in Tan's and McLoone's discussion of the relationship between differential equations and counterpossibles. Would scientists *actually* judge counterpossibles like those in (3) and (4) to be non-vacuously true or false? And if so, what would be their basis for doing so?

In the following study, we attempt to answer these questions. We presented biologists with the logistic equation of population growth, described the difference between metaphysical and nomic (im)possibility, then asked whether, when applied to a population of rabbits, the assumption that rabbits can come in non-integer values was metaphysically impossible. We then asked whether sentences (3) and (4) were true or false, and why. We hypothesized that participants would judge (3) to be true because the antecedent entails the consequent, and that they would judge (4) to be false because the antecedent precludes the consequent. We further hypothesized that participants'

[1] Jenkins and Nolan (2012, pp. 745–746) make largely the same observation, though their discussion is couched in terms of impossible dispositions.

judgement about the (im)possibility of a non-integer numbers of rabbits would not be associated with their assessment of the truth value of (4); we believed participants would judge (4) to be false regardless of their beliefs about the (im)possibility of non-integer rabbits.

This is the first experimental study of counterpossibles in science with which we are familiar. There are in fact few experimental studies of counterpossibles in general, with notable and important exceptions [e.g., Bloom (1981)]. After describing our results, we present a modal semantics that can capture the judgment that not all counterpossibles are vacuously true, drawing in particular on the work of Berto and Jago (2019), Nolan (1997), and Priest (2016). We also address a host of potential objections that a defender of the vacuity thesis might make, with special attention paid to the arguments for vacuity that Williamson has made in various places, particularly his recent book, *Suppose and Tell* (Williamson, 2020).

## 2 Methods

### 2.1 Ethics statement

This research was approved by the Swiss National Science Foundation (grant number PZ00P1_179986). Participants were provided with an ethics brief, and participation was voluntary. Participants were given the option of entering a raffle to win a $100 USD voucher in exchange for their participation.

### 2.2 Participants

We used a convenience sampling method to recruit self-described biologists, as well as those who have attended or are currently enrolled in a graduate program in biology, for a within-sample study. A survey link was distributed directly to biologists working at North American and European universities, as well as on social media platforms, where participants were asked to distribute the survey to other appropriate participants. The entire text of the survey can be accessed on the website for the Center for Open Science.[2]

The number of participants who completed the survey was 87. We began data analysis approximately one week after the last participant completed the survey, and we kept the survey open for several months longer, though there were no additional participants. During data analysis, we excluded one participant who wrote in their response that they were not taking the survey seriously, since it seemed inappropriate to treat their answers as legitimate. No other participants were excluded from the study. The final sample therefore consisted of 86 participants (41.86% female).

We asked participants to provide information about their educational background and specialties. 43 (50.0%) of the participants reported that a Ph.D. (or equivalent) in biology was the highest degree earned, 18 (20.93%) said it was a Master's in biology, 21 (24.42%) said it was a Bachelor's in biology, and 4 (4.65%) didn't report their

2 https://osf.io/5zxh4.

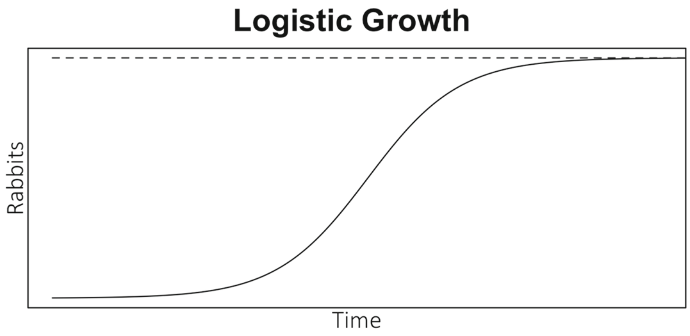



**Fig. 1** Plot of the logistic growth model that was provided to survey participants. The dashed line represents the habitat's carrying capacity ($K$)

highest degree earned. We also asked participants which subfield of biology they were most familiar with. The distribution was: cell and molecular biology (32, 37.21%); evolutionary biology (27.90%); organismal biology (9, 10.46%); ecology (6, 6.97%); and other (14, 16.27%).

### 2.3 Design and materials

To prepare participants, we presented the logistic equation:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right),$$

where $N$ is population size, $r$ is the intrinsic growth rate, and $K$ is the habitat's carrying capacity. We described how this equation, when used to model a population of rabbits, assumes that rabbits can come in non-integer values. We asked participants whether they judged it to be true or false that living rabbits could come in non-integer values, and to explain why.

We then introduced participants to the philosophical distinction between nomic and metaphysical possibility (for text, see pp. 4–5 of the survey). We asked participants to choose which of the following statements best captured their judgment about the possibility of rabbits coming in non-integer values: "it is nomically impossible but metaphysically possible"; "it is both nomically and metaphysically impossible"; "it is both nomically and metaphysically possible"; "it is nomically possible but metaphysically impossible"; "I don't know"; or "something else (please explain)."

Next, we informed participants that the below plot (Fig. 1) illustrates population growth in accordance with the logistic equation, where the dashed line represents the habitat's carrying capacity ($K$).

Participants were then asked to judge sentence (3) ["If some population of rabbits satisfied the assumptions of the logistic equation, then the size of the population ($N$) would eventually be equal to the carrying capacity ($K$)"] as true, false, or "I don't know." The same participants were then asked to judge sentence (4) ("If some population of rabbits satisfied the assumptions of the logistic equation, then the population would eventually go extinct") as true, false, or "I don't know." In both cases, participants were asked to explain their answers in text-entry boxes.

The order of all of the preceding questions was fixed. This is because we were interested in the true/false judgments of our target counterfactuals by scientists who were aware of modal distinctions and their own views about the (im)possibility of non-integer rabbits; we wanted our respondents to evaluate the counterfactuals right after they had said one of their assumptions was, or was not, impossible. This meant that we could only collect this data after we had first introduced our participants to the distinction between nomic and metaphysical possibility and solicited their judgments about the modal status of non-integer rabbits. We could have varied the order of (3) and (4), but we did not. We do not claim that our data is independent of the order in which we asked the questions.

Finally, we included a set of questions that addressed the role of visual imagination in one's ability to reason about counterfactual scenarios. We applied a standard check for aphantasia, which is a lessened capacity for sensorial imagination (Dawes, 2020). This is because we were curious whether those who scored higher on the aphantasia test were more likely to judge non-integer rabbits to be possible. Our reasoning was as follows. For some, judgments of (im)possibility seem to be in part based on one's (in)ability to visualize a given object or scenario. If aphantasics do not rely on visual imagination to make such judgments, or do so to a lesser extent, then their conception of what is possible may not be constrained by what they can visually imagine. [3]

## 3 Results

### 3.1 Judgments about metaphysical (im)possibility

A majority (57/85, 67.06%) of participants judged that it was false that living rabbits could come in non-integer values, and 10.59% (9/85) said they didn't know. Those participants who said that the claim was true (19/85, 22.35%) generally appealed to the nature of modelling. For instance, they explained that the claim is true for the following reasons: "Because it is an assumption"; "It's a model, it has to start somewhere!"; "In the real world, no, but an assumption for the purpose of the model is fine"; "This is only a model"; and "It is a mathematical property of the model and not a statement about biological census size."

Figure 2 displays the distribution of participants' answers about the sense in which they judged non-integer rabbits to be (im)possible. Many judged non-integer rabbits to be both nomically and metaphysically impossible (29/69, 42.03%). However, a slight majority of participants believed that it was metaphysically possible for rabbits to come in non-integer values (35/69, 50.07%), which contrasts with the relevant philosophical discussion of the issue (Jenkins & Nolan, 2012; McLoone, 2019, 2021; Williamson, 2017). One explanation of this divergence of opinions is that scientists have a wider conception of what is metaphysically possible than philosophers do. This would be

[3] We also asked participants whether they believe there are laws in biology. This question did not directly relate to the current study but was of interest for a future study. The question therefore does not play a role in any of the below analyses.

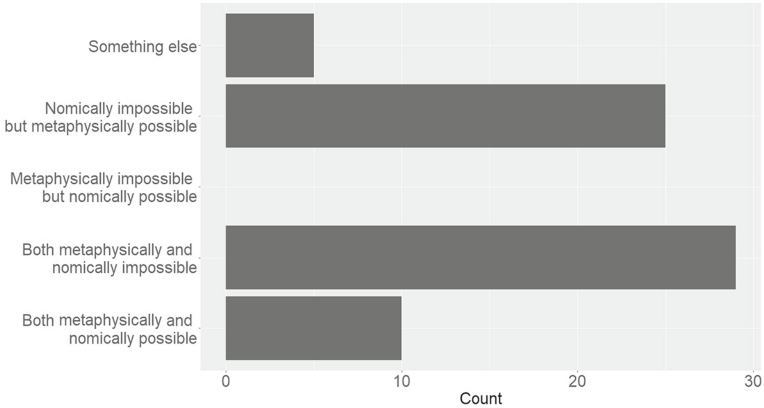



**Fig. 2** *Senses of (im)possibility.* A slight majority of participants (35/69, 50.72%) believed it was metaphysically possible for rabbits to come in non-integer values

consistent with the claim that what some philosophers characterize as metaphysically impossible is in fact consistent with well-confirmed physical theory (Norton, 2022; Putnam, 1968). Noteworthy is that not a single participant categorized non-integer rabbits as metaphysically impossible but nomically possible, though that option was available among the multiple choice options. This result follows naturally from our description of metaphysical and nomic possibility, so we take this to be a successful attention check. No participant responded to this question with "I don't know."

None of the demographic variables we looked at were significantly associated with how participants answered these questions about metaphysical (im)possibility.[4]

### 3.2 Judgments about counterfactuals' truth values

Of the 76 participants who evaluated the truth value of (3), 51 (67.11%) judged it to be true, and none responded "I don't know" (Fig. 3). To test whether this distribution was statistically significant, we ran a one sample binomial test with a null probability of 0.5 (the expected value of a random distribution between true and false), $\chi^2(1, N = 76) = 8.2237$, $p = .004$. This shows that the participants had a significant preference to judge (3) to be true. This is the correct answer insofar as the assumptions of the model do indeed entail the consequent.

There were 37 participants who judged (3) to be true and left a free text response explaining why. These participants quite often said that they formed their judgment based on the mathematical relationship between the assumptions of the model and the consequent of the conditional. They wrote, for example: "Because that is what

[4] Interestingly, there was a higher than expected prevalence of self-reported aphantasia in our sample: 16.28% of participants described themselves as fully aphantasic, and 32.56% of participants described themselves as aphantasic to some degree. It is currently believed that the general prevalence is around 2–3% (Faw, 2009; Zeman et al., 2020). However, we found no significant association between aphantasia scores and responses to any questions in our survey, including those about the (im)possibility of non-integer rabbits.

the equation predicts"; "This is how the equation is built"; "Given that it follows the assumptions, it must reach the carrying capacity"; "Because, in the absence of any other variables, after sufficient time that is what the equation predicts"; "Follows mathematically in the universe defined by the assumptions"; and so on. There was no participant who judged (3) to be true and then explained that they made this judgment because the antecedent was impossible. Based on these free text comments, we can conclude that at least 37/76 (48.68%) of respondents believed (3) is non-vacuously true. This is likely an underestimate, since presumably some among those who judged (3) to be true but left no free text response thought (3) was non-vacuously true.

The number of participants from our total sample who judged (3) to be false was 25/76 (32.89%) (Fig. 3). Of those, 23 left free text comments explaining why they answered this way. In many cases, it is clear that the participants had a slight misunderstanding of the mathematics of the model. For instance, they believed the model describes a system in which the population size approaches but never reaches the carrying capacity, or that the population reaches the carrying capacity but fluctuates around the carrying capacity without staying exactly at that value. The replies show that at least some who believed (3) was false did so based on what they (incorrectly) believed to be the mathematical relationship between its antecedent and consequent.

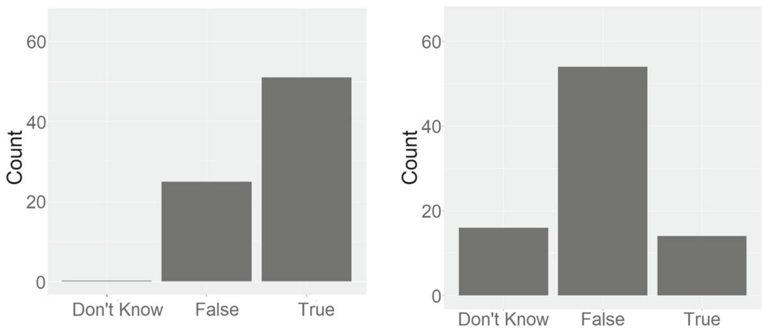
Participants with a Ph.D. were significantly more likely to believe that (3) is false ($\chi^2$ (1, $N = 77$) = 6.915, $p = .0315$; 18/38 believed it is false, while 20/38 believed it is true. In many of these cases, participants believed (3) was false because they had a misunderstanding of the mathematics (e.g., they believed population size would tend toward $K$ but not reach it). It is possible that those with a Ph.D. were "overthinking" the question.[5]

Of the 68 participants who evaluated the truth value of (4), 54 (79.41%) judged it to be false and 14 judged it to be true, while no one responded "I don't know" (Fig. 3). We ran a one sample binomial test with a null probability of 0.5, ($\chi^2(1, N = 68) = 22.368$, $p < .000$), showing a significant preference for participants to judge (4) to be false. That (4) is false is the correct answer insofar as the assumptions of the model mathematically preclude the consequent from being true.

67 of the 84 participants who evaluated (4) left a free-text response explaining why they evaluated (4) as they did. Among those who judged (4) to be false and explained why ($N = 42$), they generally said that (4) was false because of the mathematics of the model. For example, participants wrote: "if the population satisfied the assumptions then the population would never reach 0 unless N = 0 at t = 0"; "Because the logistic equation never reaches zero"; "That's not what the equation says"; "Assumptions of the equation are that at large time, the population is approximately K. K≠0, thus population will not eventually go extinct"; "The equation does not allow for the population to decrease in size"; "The population size does not decrease at any time point, there's no way it went extinct if the growth really satisfied the assumptions of the logistic equation"; and so on.

Our analysis of free text responses show that some who judged (4) to be true did so because of a misunderstanding of the mathematics. For instance, one participant

[5] There is one degree of freedom here because we collapsed Bachelor's and Master's students, so there were two educational categories, and no one answered "I don't know" for this question.




**(a)** Evaluation of (3) **(b)** Evaluation of (4)

**Fig. 3** Participants' truth value assignments to sentences (3) and (4)

wrote, "If growth rate is less than one then sure the population could go extinct." This is incorrect; all that is required for the population not to go extinct is that the growth rate be positive. Another participant wrote, "It could happen if birth rate is too low and death rate high or if birth rate is too high that the resources run out." This too is incorrect, since the logistic equation has no death rate, and depletion of resources means that the population will stabilize at $K$, not go extinct. The participants here are basing their judgment that (4) is true on what they (incorrectly) believed to be the mathematical relationship between its antecedent and consequent.

Among those who replied "I don't know" (16) or "true" (14) when asked to judge the truth value of (4), some did so because they were not sure whether the statement referred to an actual population of rabbits or to the hypothetical population of rabbits described by the model. For example, one participant who responded "I don't know" explained, "That sounds false, because all else being equal, the logistic equation by itself does not offer a route to population decrease, much less extinction. On the other hand, in the real world, all populations go extinct." A participant who answered "true" wrote, "All species eventually go extinct. As far as we know," and another who answered "true" wrote, "All populations and species will eventually go extinct." Notice that in these responses the participants are focused on the truth of the consequent. McCloy and Byrne (2002) show that, when presented with a conditional with a consequent that is true, this weakens the perceived connection between the antecedent and consequent. A possible explanation as to why some participants judged (4) to be true is that their belief that all populations go extinct prevented them from fully working through the consequences of the antecedent being true.

As with (3), no participant judged (4) to be true and explained that they did so because its antecedent is impossible.

### 3.3 Influence of modal judgments on beliefs about counterfactuals' truth values

Finally, we used a chi-squared test to assess whether participants' responses to the claim that it is metaphysically (im)possible for rabbits to come in non-integer values was associated with the truth values they assigned to (4). If the vacuity thesis were correct as a descriptive account of how individuals process counterpossibles, then participants who believed the antecedent of (4) is metaphysically impossible should have evaluated (4) differently than those who believed the antecedent is metaphysically possible.

To carry out this analysis, we grouped together all individuals who believed it was metaphysically possible for rabbits to come in non-integer values, regardless of their beliefs about the nomic possibility of non-integer rabbits. (It was unnecessary to group all individuals who believed it was metaphysically impossible for rabbits to come in non-integer values, regardless of what they believed about the nomic possibility of non-integer rabbits, because, recall, there were no individuals who believed it was metaphysically impossible for rabbits to come in non-integer values but nomically possible.) Combining respondents in this manner is legitimate, since, in terms of assessing the vacuity thesis, it is metaphysical (im)possibility that matters, not nomic. We excluded from this analysis those who answered "Something else" to our question about the (im)possibility of non-integer rabbits, since it was unclear how to integrate these answers into the analysis.

We found no significant association between participants' judgments about the modal status of a non-integer number of rabbits and their evaluation of (4) [$\chi^2$ (2, $N$ = 68) = 2.21, $p$ = .33]. Participants who believed (4) was an "ordinary" counterfactual evaluated it just like those who believed (4) was a counterpossible. Namely, participants generally relied on what they believed to be the relationship between the antecedent and consequent to determine whether (4) was true or false, regardless of what they believed about the (im)possibility of their shared antecedent.

## 4 Discussion

A significant majority of our participants judged (3) to be true and (4) to be false. These are the correct responses insofar as the shared antecedent of (3) and (4) entails the consequent in (3) and precludes the consequent in (4). Moreover, as we just saw, those who believed the antecedent of (4) is impossible were just as likely to judge (4) to be false as those who believed the antecedent of (4) is possible. Here we will discuss how these results bear on traditional accounts of counterfactual semantics (4.1) and on Williamson's (2018, 2020) theory of how we assign truth values to counterpossibles (4.2).[6]

[6] It would also be interesting to discuss how our results interact with other accounts of counterpossibles, including those of Emery and Hill (2017), Kim and Maslen (2006), Kment (2014), and Wilson (2021). But as Williamson is the main defender of the vacuity thesis, and since much of the debate turns on the truth of this thesis, we here focus on Williamson.

### 4.1 Capturing non-vacuity

One aspect of our results that poses a challenge for standard accounts of counterfactual semantics is that many participants implicitly believed both that (5) is true and that (6) could be non-vacuously true, or false, depending on what goes in its consequent:

(5) Necessarily, rabbits are discrete.
(6) If rabbits were continuous (etc.), then [...].[7]

According to standard accounts, the truth of (5) rules out that (6) could be non-vacuously true, or false (Lewis, 1973; Kratzer, 1979; Stalnaker, 1968). To the extent that those accounts are partly formulated to capture linguistic usage, our results suggest they are in error.

We stress that (5) and (6) are not idiosyncratic; an analogous pair of sentences would appear to arise whenever calculus is used to describe the dynamics of objects that one believes are necessarily discrete:

(7) Necessarily, [rabbits, $H_2O$, firms, SARS-CoV-2 infections, ...] are discrete.
(8) If [rabbits, $H_2O$, firms, SARS-CoV-2 infections, ...] were continuous (etc.), then [...].

Purely as a descriptive exercise, if not also a normative one, it seems we should want a modal logic that allows for sentences like (5)/(7) to be true but doesn't require sentences like (6)/(8) to be vacuously true. How can this be done?

One strategy is to supplement standard accounts of counterfactual semantics with impossible worlds. Following roughly the framework one finds in Berto and Jago (2019, Nolan (1997), and Priest (2016), we can employ a structure $\langle P, I, R, v, @\rangle$, where $P$ and $I$ are sets of possible and impossible worlds, respectively, $R$ is a binary accessibility relation between worlds, $v$ is a function that assigns "true" or "false" to a proposition at a world, and @ is the actual world. We assume $P \cap I = \emptyset$, and that both possible and impossible worlds are consistent sets of propositions closed under mathematical and logical entailment.[8]

We can further assume that the necessity operator ($\Box$) and the possibility operator ($\Diamond$) are defined so that they range exclusively over the worlds in $P$. In particular, where $wRw'$ means world $w$ accesses world $w'$, $\Box A$ is true at $w$ just in case $A$ is true at each $w' \in P$ where $wRw'$. Likewise, $\Diamond A$ is true just in case $A$ is true at some $w' \in P$ where $wRw'$. We can use $\blacklozenge$ and $\blacksquare$ to talk about possibility and necessity in a different, quantificational sense. To say $\blacklozenge A$ is true at a world is to say $A$ is true at some world, whether possible or impossible, and to say $\blacksquare A$ is true at a world is to say $A$ is true at all worlds, both possible and impossible (French et al., 2020).

[7] Here the "etc." is meant to indicate whatever other assumptions are specified by the model.

[8] We can get by with the assumption that impossible worlds are closed under mathematical and logical entailment because we're dealing with counterpossibles whose antecedents violate metaphysics, not logic or math. Whether this is the appropriate way to construe impossible worlds when evaluating counterpossibles with mathematically or logically impossible antecedents raises questions about the nature of impossible worlds that are outside the scope of this paper. For discussion and varied proposals concerning the nature of impossible worlds, see Berto and Jago (2019), Bjerring (2014), Brogaard and Salerno (2013), Sandgren and Tanaka (2019). We do not take our approach to provide guidance on how to evaluate counterpossibles with mathematically or logically impossible antecedents.

The introduction of impossible worlds still allows us to say (5) is true, so long as we identify the word "necessarily" in (5) with $\Box$. Since $\Box$ ranges only over possible worlds, that "rabbits are continuous" is true at some (impossible) world doesn't preclude its being the case that "Necessarily, rabbits are discrete" is true.[9]

We need to supplement the above account to get the judgment that (6) can be non-vacuously true, or false, depending on the relationship between the antecedent and consequent. Our approach is similar in spirit to what one finds in McLoone (2021), though considerably simpler. We'll adopt a modification of what Priest (2016, p. 2655) calls the *primary directive* (PD), which we'll formulate as follows:

> (**PD**): For any proposition $A$, there is some world at which $A$ is true, so long as $A$ is logically and mathematically consistent.

(Note that the "world" referenced in PD could be an impossible world.) We'll also adopt Bjerring's extension of Stalnaker–Lewis semantics (ESL) (2014, p. 331):

> (**ESL**): $A \mathrel{\Box\!\!\rightarrow} B$ is true at @ if some world in which $A$ and $B$ are true is closer to @ than any world in which $A$ is true and $B$ is false.[10]

ESL is "extended" from what one finds in Stalnaker–Lewis in that it ranges over both possible and impossible worlds.

Let's return to (6). PD ensures that there is some world where the antecedent is true. If the antecedent mathematically entails the consequent, then the world at which the antecedent and consequent are both true is closer to @ than is the world where the antecedent is true and the consequent is false, for the consistency of impossible worlds ensures that there's no world of that second sort. Given ESL, that means the counterpossible will be true. Now suppose the antecedent of (6) precludes the truth of the consequent. Because impossible worlds are consistent, that means there is no world at which both the antecedent and consequent are true, from which it follows that the world where the antecedent is true and the consequent is false is closer to @. Given ESL, that means the counterpossible is false.

Note that by "filling in" the antecedent and consequent of (6) in the right way, one can recover the counterpossibles that were the focus of our study, (3) and (4). That is, the above framework allows one to say that (3) is non-vacuously true, and that (4) is false. This approach can of course be applied to other counterpossible formulations of differential equation models, like those suggested by (8).

The attitudes of those participants who believed that (5) is true, and that (6) could be true or false, match our own. There *is* a sense in which a population of continuous rabbits is impossible, but there is also a sense in which, if there *were* such a population (and other conditions were met), certain dynamics would and would not follow. Our

[9] We should note here that we intend this account of (im)possibility to be compatible with – though not to require – a deflationary account of metaphysics, according to which some claims of metaphysical (im)possibility can be reduced to claims of conceptual (im)possibility (see, e.g., Norton 2022) Integrating this with the formal account just presented, we can let the content of one's concepts determine which propositions belong in $P$ and which belong in $I$. For instance, a proposition that expresses the sentence "rabbits are continuous" is placed only among the impossible worlds if the proposition's truth is ruled out by one's concept RABBIT.

[10] This notation varies slightly from Bjerring's, and the label "ESL" is not his.

data suggest that one expands their notion of possibility when evaluating counterpossibles [like (6)], relative to a scenario in which they are evaluating indicatives that don't have a conditional form [like (5)]. While this is a non-standard position, we don't find it to be particularly extravagant. After all, it is uncontroversial that, when engaged in counterfactual reasoning, we often assign the truth value "true" to propositions that we know to be false ("If Churchill had been born in Moscow, ..."). We think it is no great leap to maintain that, when reasoning counterfactually, one can also assign the truth value "true" to propositions that in other contexts (i.e., not counterfactual ones) they would judge to be impossible. Indeed, it is not clear how to make sense of our data otherwise.

### 4.2 Williamson's skepticism of non-vacuity

Timothy Williamson has argued in numerous places that counterpossibles that seem non-vacuously true or false do not undermine the view that all counterpossibles are in fact vacuously true. Williamson has also argued that a non-vacuist logic will require the untoward rejection of fundamental modal axioms, and that counterpossible reasoning does not obviously require one to reason about impossibilities. We will deal with all three of these claims in this section.

To situate ourselves, let's begin with Williamson's (2018, p. 359) discussion of two counterpossibles from Nolan (1997, p. 543):

(9) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would have cared.

(10) If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would not have cared.

Since squaring the circle is (mathematically) impossible, these are counterpossibles. But Nolan claimed that (9) is false and that (10) is non-vacuously true.

Williamson is a vacuist, so he denies Nolan's claim that (9) and (10) are anything other than vacuously true. However, Williamson admits that (9) is "seemingly false" (2018, p. 364). To this extent, Williamson, like us, acknowledges that the vacuity thesis is descriptively inadequate; it doesn't entirely capture linguistic usage. Williamson's defense of vacuism is instead oriented around the idea that counterpossibles are, despite their superficial appearance, in fact all vacuously true.

To explain why counterpossibles may *seem* non-vacuously true or false, Williamson has presented an error theory. The error theory is based around Williamson's claim that we use "fallible heuristics" to evaluate counterfactuals, and these heuristics lead one (mistakenly) to believe that counterpossibles can be non-vacuously true, or false. Williamson (2018, p. 364) says "our unreflective assessment of counterfactual conditionals" employs the following heuristic:

**HCC**: Given that $\beta$ is inconsistent with $\gamma$, treat $\alpha \mathrel{\Box\!\!\rightarrow} \beta$ as inconsistent with $\alpha \mathrel{\Box\!\!\rightarrow} \gamma$.

To see an application of HCC, note that the consequent in (9) is inconsistent with the consequent in (10), and so HCC rules that (9) is inconsistent with (10). This is why we judge (9) to be false: "Thus, having verified (10), we treat ourselves as having falsified (9)" (p. 364). In fact, Williamson believes we might often rely on a "simpler heuristic," HCC* (Ibid.):

**HCC***: If you accept one of $\alpha \Box\!\!\rightarrow \beta$ and $\alpha \Box\!\!\rightarrow \neg\beta$, reject the other.

HCC* too could explain our judgment that (9) is false. For if we accept (10), then HCC* tells us to reject (9).

HCC and HCC* appear to require one to be able to determine that some counterpossible is true. As we just saw, in explaining why we judge (9) to be false, Williamson explains that this is because we first "verified" (10) (Ibid., p. 364). We think it is clear that, when Williamson says we verify (10), he means that we judge (10) to be *non*-vacuously true, for, if one judged (10) to be vacuously true, then one would presumably arrive at the same judgment about (9), since those counterfactuals' antecedents are identical. Williamson does not provide an explanation as to how or why we judge (10) to be true.

HCC and HCC* do not appear in Williamson's recent book, where he presents a modified heuristic that explains why it seems some counterpossibles are non-vacuously true or false (Williamson, 2020). Williamson claims that, when evaluating counterfactuals, we often employ what he calls the "modal conditional suppositional rule" (MCSR) (Ibid., p. 201):

**MCSR**: For any attitude $\|$, sentences $A$, $C$, and set of sentences $BB$:
$BB \;\|$ 'would(if $A$, $C$)' just in case $BB$, $\underline{R} \;\|$ $\underline{\text{'if } A, C'}$
just in case $BB$, $\underline{R, A} \;\|\; \underline{C}$.

where $BB$ are one's background beliefs, $\underline{R}$ specifies the contextually-relevant world(s) we're considering, and the would(*) operator indicates that we treat the embedded conditional 'If $A$, $C$' as a counterfactual. (The underlined text in MCSR is Williamson's notation to indicate that we are evaluating the underlined propositions at contextually-relevant worlds.) For example, "$BB \;\|$ 'would(if $A$, $C$)'" can be read as "Given $BB$, believe that 'would(if $A$, $C$)' is true." As we understand it, a main difference between HCC/HCC* and MCSR is that MCSR allows one to directly judge a counterpossible to be false, whereas HCC and HCC* only allows one to judge a counterpossible to be false after first judging some other counterpossible to be non-vacuously true.

Returning to our counterpossibles about Hobbes, Williamson believes those sentences have the following, underlying form (2020, p. 256):

(11) Would (if Hobbes secretly squared the circle, sick children in the mountains of South America at the time cared).
(12) Would (if Hobbes secretly squared the circle, sick children in the mountains of South America at the time did not care).

To assign a truth value to (11) and (12), MCSR says that we ask whether, given our background beliefs, the world at which Hobbes squared the circle is also one at which sick children in the mountains of South America care about his doing so. Since "the obvious answer is 'No'" (Ibid, p. 256), we accept (12)/(10) and reject (11)/(9).

Are HCC/HCC* and MCSR compatible with our experimental results? They may be. Williamson says very little about how one judges counterpossibles to be true in the case of HCC/HCC*, or true or false in the case of MCSR. It is possible that our respondents' use of mathematical reasoning to evaluate (3) and (4) is consistent with their using HCC/HCC* or MCSR to arrive at that judgment. Indeed, some of our respondents explained their reasoning in a way that sounds much like HCC/HCC*. For instance, as quoted above, one respondent explained their reasoning that (4) was false as follows: "Assumptions of the equation are that at large time, the population is approximately K. K≠0, thus population will not eventually go extinct." If we let $\alpha \;\Box\!\!\rightarrow\; \beta$ represent the counterpossible in (4), the respondent reasoned that in fact $\alpha \;\Box\!\!\rightarrow\; \gamma$ is true, where $\gamma$ is that the population size will be approximately equal to $K$ in the long run. Because $\beta$ and $\gamma$ are inconsistent, the respondent reasoned that $\alpha \;\Box\!\!\rightarrow\; \beta$ is false. This is the procedure HCC describes. That is why we want to be clear that we are not claiming that HCC/HCC* and MCSR are incorrect models of counterfactual reasoning.

Rather, we disagree with Williamson about a different point. Williamson claims that HCC/HCC* and MCSR undermine the import of any examples of counterpossibles that appear to be non-vacuously true or false; he does not believe such examples compel us to hold that counterpossibles should be judged to be non-vacuously true or false. As Williamson explains, "...it is methodologically naive to take the debate over counterpossibles to be settled by some supposed examples of clearly false counterpossibles. As we have seen, a simple and mostly reliable heuristic would lead us to judge them false even if they were true" (2018, p. 367). The "simple and mostly reliable heuristic" Williamson has in mind here is HCC/HCC*, though presumably he would make the same point about his newer MCSR.

We do not believe this view is defensible when the counterpossibles in question are based around mathematical inference. If Williamson believes we employ HCC/HCC* or MCSR to evaluate counterpossibles like (3) and (4), then that means he believes HCC/HCC* or MCSR applies in those cases in which the antecedent of a counterpossible mathematically entails or mathematically precludes its consequent. As we have said, we have no issue with the view that HCC/HCC* or MCSR are employed to evaluate mathematical counterpossibles of this sort. However, what we do have an issue with is saying that those are among the cases in which HCC/HCC* or MCSR would be in error. Correct mathematical inference is not wrong, ex hypothesi. Or, another way to put it: We don't think our respondents said something incorrect when they judged (3) to be non-vacuously true and (4) to be false, even if they used HCC/HCC* or MCSR to arrive at these judgments. Indeed, we wonder if Williamson may agree with us on this point. He takes it to be a feature of his error theory that no part of it "impugns the reliability of counterfactual judgments made on the basis of mathematical reasoning" (2018, p. 367). If that's true, then the error theory shouldn't impugn the reliability of our respondents' judgments about (3) and (4), anchored as they are to mathematical inference.

In general, we think counterfactual formulations of mathematical models with impossible assumptions highlight a tension in Williamson's views about counterfactual reasoning. The tension emerges because Williamson believes *both* that counterfactual inference based in mathematics is good inference *and* that some mathematical models posit impossible scenarios. The below passage illustrates the matter vividly. Discussing a differential equation model of population dynamics, Williamson writes (2017, p. 161, emphasis his):

> ...evolutionary biology typically uses differential equations for population change, even though they treat the change in the number of group members as continuous whereas really it must be discrete...Strictly speaking, such a model is *impossible*; it is a type metaphysically incapable of having instances. But that does not mean that the model collapses. The differential equations are mathematically consistent; we can still make a stable tripartite distinction between what follows from them, what is inconsistent with them, and what is neither...In advance, we might not have expected impossible models to have such cognitive value, but it has become clear that they can.

We don't know what Williamson means by "cognitive value," which he doesn't define. But, as we read him, Williamson here acknowledges that we can reason non-vacuously about impossible scenarios, similar to those described in sentences (3) and (4), and that such reasoning is non-vacuous. We agree! We also believe the above passage is incompatible with Williamson's view that counterpossibles are all vacuously true. It seems something has to give. We believe that what should be jettisoned is the view that counterpossibles anchored to logical or mathematical inference are all vacuously true.

Now, Williamson might respond by saying that, compelling though the preceding discussion may be, the benefits of a counterfactual logic that allows for non-vacuity would not be worth its costs. Williamson writes that a counterfactual logic that accommodates non-vacuously true counterpossibles would require us to reject "elementary principles of the pure logic of counterfactual conditionals," principles that he appears to believe must be kept (2007, p. 174). Williamson doesn't say what these principles are, but we can surmise that they include the modal axioms he himself endorses (Ibid., p. 293). Of those, the axioms that a non-vacuist logic would need to reject are CLOSURE (CLOS) and EQUIVALENCE (EQUI)[11]:

[11] Berto et al. (2018, p. 699) claim that non-vacuists will also reject Williamson's axiom VACUITY. McLoone (2021, p. 12164, Fn. 9) has argued that this is not so.

**CLOS**: If $\vdash (B_1 \wedge ... \wedge B_n) \rightarrow C$ then
$\vdash ((A \mathrel{\Box\!\!\rightarrow} B_1) \wedge ... \wedge (A \mathrel{\Box\!\!\rightarrow} B_n)) \rightarrow (A \mathrel{\Box\!\!\rightarrow} C)$
**EQUI**: If $\vdash A \equiv A^*$ then
$\vdash (A \mathrel{\Box\!\!\rightarrow} B) \equiv (A^* \mathrel{\Box\!\!\rightarrow} B)$

But CLOS and EQUI are defensible only if one assumes that counterpossibles are all vacuously true. As Berto et al. (2018, p. 699) have shown, CLOS, in conjunction with axioms that vacuists and non-vacuists both accept, entails that the truth of a contradiction makes every other contradiction true. But a non-vacuist will not want to say that 2's being both equal and not equal to 3 entails that it is both raining and not raining (Ibid.). Likewise, as McLoone has observed (2021, p. 12164), EQUI implies that the antecedent position of a counterfactual is never hyperintensional. But take the following two counterpossibles:

(13) If Hobbes had squared the circle, he would have been a famous mathematician.
(14) If Hobbes had squared the circle *in secret*, he would have been a famous mathematician.

If we identify the intension of a proposition with the set of possible worlds at which the proposition is true, then the antecedents of (13) and (14) have the same intension (i.e., the empty set); the antecedent of (13) is true precisely when the antecedent of (14) is. EQUI says that (13) must thereby be true if (14) is. But it seems (13) is non-vacuously true and that (14) is false. At least, that is what a non-vacuist will maintain. In sum, then, non-vacuists will have no problem rejecting CLOS and EQUI, so this component of Williamson's defense of the vacuity thesis will be compelling only to one who already endorses vacuism.

Finally, in defense of the vacuity thesis, Williamson could also deny that we in fact reason non-vacuously about impossibilities. In places, Williamson appears to do precisely this. When we use HCC/HCC* or MCSR to evaluate a counterpossible, we don't *actually* suppose the antecedent is true. We instead suppose some proxy scenario that is possible is true and reason from there. For instance, when we consider sentences like (9)/(11) and (10)/(12), we don't suppose in a fine-grained manner that Hobbes secretly squared the circle; we don't observe in our mind's eye what that proof would look like-for there could be no such proof. Rather, we suppose something like "Hobbes secretly proved some important mathematical theorem" is true and then use common sense to reason about how sick children in a geographically-removed location would have responded. As Williamson correctly notes (2018, p. 364), to form judgments about the truth values of (9)/(11) and (10)/(12), we don't even need to consider the impossibility of the antecedent. Hobbes could have written his grocery list in secret and the sick children wouldn't have cared about that either. An analogous point can be made about the following alleged counterpossible from Vetter (2016, p. 781), uttered after a gazelle leaps from a bush and scares two hikers:

(15) If that had been a tiger, we would be dead now.

Williamson notes (2020, pp. 252–253) that the “that” in (15) is not obviously a rigid designator, so the sentence could be read as saying something like, “If a tiger, rather than a gazelle, had jumped from that bush, we would be dead now,” which is not a counterpossible and so its non-vacuous truth poses no problem for a proponent of the vacuity thesis.

While we agree that judging counterpossibles like (9)–(15) to be non-vacuously true, or false, doesn’t require one to reason about an impossible scenario, we believe the situation changes when considering counterpossibles like those in our study, sentences (3) and (4). The impossibility in those counterpossibles is that rabbits can come in non-integer values, and that assumption is necessary to get the dynamics that result in the population staying at the size of the carrying capacity and not ever going extinct—i.e., the consequents of (3) and (4), respectively. To reason about whether the consequent follows from the antecedent requires one to suppose that the impossible assumption in the antecedent is true. We think the same can be said of the counterpossible about intuitionistic logic in (1), along with the counterpossibles discussed by Jenny (2018) and at least many of those in Tan (2019). In these cases, the impossibilities cannot be waved away as unnecessary.

## 5 Conclusion

False assumptions are ubiquitous in model-based science: one may assume that planes are frictionless, markets are composed of perfectly rational agents, populations are infinitely large, and so on. Do models sometimes assume not just what is false but what is *impossible*? Many of the participants in our study appeared to believe so; they thought the logistic equation’s assumption that rabbits can come in non-integer values was metaphysically impossible. However, this did not stop these participants from reasoning about what would or would not follow, were that assumption true. When the logistic equation was construed counterfactually, these participants relied on what they perceived to be the mathematical relationship between the antecedent and consequent to determine whether the counterfactual was true or false.

We believe our results have important implications for how we represent the logic of counterfactual reasoning. At the very least, they demonstrate that the vacuity thesis is inadequate as a purely descriptive account of counterfactual reasoning in science. This itself should serve as a motivation to consider how to extend standard modal logic to allow for counterpossibles that are non-vacuously true. We take the framework we presented in Sect. 4.1 to be a step in that direction. Moreover, if one believes that scientific reasoning should serve as a model of good reasoning, especially when scientific reasoning is based on mathematical inference, then we believe our results also present a challenge to the view that the vacuity thesis is defensible on normative grounds. The application of calculus to model the dynamics of necessarily discrete objects can result in metaphysically impossible scenarios, but we do not believe it is appropriate to maintain the orthodox position that all that can be said about those scenarios is vacuously true. Sometimes heretics are right.

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