



# Mathematical practice and epistemic virtue and vice

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## Abstract

What sorts of epistemic virtues are required for effective mathematical practice? Should these be virtues of individual or collective agents? What sorts of corresponding epistemic vices might interfere with mathematical practice? How do these virtues and vices of mathematics relate to the virtue-theoretic terminology used by philosophers? We engage in these foundational questions, and explore how the richness of mathematical practices is enhanced by thinking in terms of virtues and vices, and how the philosophical picture is challenged by the complexity of the case of mathematics. For example, within different social and interpersonal conditions, a trait often classified as a vice might be epistemically productive and vice versa. We illustrate that this occurs in mathematics by discussing Gerovitch's historical study of the aggressive adversarialism of the Gelfand seminar in post-war Moscow. From this we conclude that virtue epistemologies of mathematics should avoid pre-emptive judgments about the sorts of epistemic character traits that ought to be promoted and criticised.

**Keywords** Virtue epistemology · Mathematical practice · Vice epistemology · Character epistemology · Productive vice · Gelfand seminar · Mathematical virtue

## 1 Introduction

This paper offers a preliminary investigation of the sorts of epistemic virtues and epistemic vices that are especially pertinent to the case of mathematics and its practices. There is much study of epistemic virtues and epistemic vices these days, although most of it remains focused at the generic level of epistemic virtues and vices, rather than at the level of specific domains of enquiry. We think

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that epistemic virtues and vices will typically be domain-sensitive in ways that encourage the idea that there are domain-specific virtue and vice epistemologies—including *character epistemologies of mathematics*. This raises interesting questions: what would justify talk of ‘mathematical virtues’ and ‘mathematical vices’? There are many options—certain epistemic features of mathematical practice, certain ontological features of the objects of mathematical enquiry, certain aspects of the social organisation of mathematics, whether one focuses on pure or applied mathematics, and so on. (In this paper, by the way, our focus is on pure mathematics).

Thinking about specific disciplines, such as mathematics, is useful to the development of virtue and vice epistemology for a simple reason: ‘enquiry’ is extremely heterogeneous. The philosophy of science a long time ago embraced pluralism and disunity, the recognition that the singular term, ‘science’, encompasses an enormous range of different integrated and often changing material, epistemic, and social components. Unfortunately, contemporary virtue epistemologists tend to speak generally of ‘enquiry’ and focus on disciplinary variations only when appealing to case studies (e.g. Baehr 2011, p. 1; Roberts and Wood 2007). We take seriously the possibility that the range and forms of epistemic virtues and vices tend to vary across and within different disciplines and projects of enquiry and want to use it to float the possibility of a tripartite distinction:

- (a) *Generic epistemic virtues* pertinent to all types of enquiry in a domain-neutral way.
- (b) *Specific epistemic virtues* generic ones that take specific forms, inflected by some specific features of specific domains.
- (c) *Local virtues* confined or ‘local’ to a certain subject—e.g. mathematics.

We use this distinction to explore the idea that there are distinctively mathematical virtues. If (a), then talk of *mathematical virtues* can only really be idiomatic; there are, in practice, only *virtues*, some of which just happen to be exercised in mathematics. If (b) then we owe an account of how mathematics inflects a certain virtue in a way that would justify talk of that virtue having a *mathematical* form—for instance, of a specific form of *mathematical precision*. Things get trickier with (c), the local virtues, which are confined to specific epistemic domains, such as mathematical enquiry. Two obvious worries about this category are (i) that virtues ought to have a reasonably cross-situational character and (ii) the distinction between specific and local virtues might collapse. If the virtue of rigorousness can take specific forms in mathematical practice, then why not choose parsimony and have two categories—*generic* and *specific* epistemic virtues?

Since one of our motivations for looking at mathematics is to see if these possibilities can be made out, we only want to propose the idea of *virtue-locality*. We suspect that the difference between (b) and (c) might come down to *identity conditions for virtues*. Tanswell (2016) argues that *rigour* is a mathematical virtue, for example: but is this a specifically local form that carefulness and

meticulousness take when enacted within mathematical practice, and therefore a specific inflection of a general form as in (b), or is it perhaps a disposition or excellence that is only exercisable in relation to mathematical objects, and therefore a local virtue as in (c)? To make further progress, what's needed are in-depth case studies of the sort we hope might be inspired by what we're going to offer. If the concept of virtue-locality does not bear out, that's a theoretical contribution from character epistemology of mathematics.

The possibility of specific and local virtues has a wider significance beyond philosophy and epistemology of mathematics. One of the newly emerging areas of vice epistemology, for instance, is vice ontology, which asks: *what sorts of things are epistemic vices?* Typically, most virtue theorists embrace a sort of monism, by arguing that virtues are *character traits*. Subsequent work has moved towards virtue-pluralism, admitting *attitudes*, *sensibilities*, and *ways of thinking* as other things that can be virtues. Such ontological debates have, recently, been focused within vice epistemology. A main question pushed by Quassim Cassam (2020: §3) is the *Individuation Question*, which concerns ways of distinguishing between putatively distinct epistemic vices. Where Cassam explores this abstractly, without looking at specific domains, what we offer is a discipline-specific approach to the Individuation Question.

There are, *prima facie*, some general features of mathematics *qua* discipline that necessitate more careful attention to the ways that epistemic virtues and vices play out. At least three stand out:

1. Mathematics is a very epistemically conservative discipline with respect to its established body of knowledge—you trust your inheritance and do not easily admit new things (for instance, when new, peculiar methods are proposed, or, crossing between subdisciplines in maths). Such conservatism is typically principled rather than reactionary, usually justified by confidence in rigorous processes of proof.
2. Modern mathematics as a discipline has striking confidence in the stability of its foundations, unlike the physical sciences, on which virtue epistemologists tend to focus, which are typically animated by a sense of the possibility of periodic substantive revisions in their underlying ontological commitments.
3. Certain features of mathematics might also reduce some of the temptations and other incentives to vicious conduct. Consider, for instance, the abstractness of mathematics, in the sense of the minimal roles for empirical data, a happy consequence of which is that there are fewer temptations to distort, misrepresent, omit, or otherwise abuse data (e.g. p-hacking, selective reporting).

An additional complication that arises in practice, and in the case of mathematics, is that there are plenty of examples where putatively vicious behaviour seemed to be systematically productive, and putatively virtuous behaviour seemed systematically

to be unsuccessful. We are broadly following Paternotte and Ivanova's (2017) work on epistemic virtues in science, where they use the history of science to describe two types of puzzle case<sup>1</sup>:

- (a) *Productive vices* where arrogance, dogmatism, etc., were epistemically productive in the sense of reliably attaining certain epistemic goals, usually in ways integral to the success of a given project of enquiry.
- (b) *Obstructive virtues* where diligence, open-mindedness, etc., were obstructive, such that their exercise thwarted the attainment of certain epistemic goals.

In Sect. 4, we will return to this point and discuss the extensive study by Slava Gerovitch on the Gelfand seminar (2016), which we argue demonstrates a range of vices that appear to have been systematically productive for mathematics. For now, we suggest that the concepts of productive vices and obstructive virtues is easier to articulate if one endorses the consequentialist conception of epistemic character traits (of which more in the next section). Moreover, we think that the converse cases, of obstructive vices, do not appear as obviously in mathematics, and will speculate on the reasons that might be behind this.

The upshot for character epistemology (other than new contacts with mathematics) are new ways of thinking about the central normative question: what is the basis for assessing the normative status of epistemic character traits? Before we explore the contributions that reflection on mathematics can make to this question, we first need to sketch out some of the main features of contemporary virtue and vice epistemology, which we will do in Sect. 2. In Sect. 3, we will return to the mathematical case, and discuss mathematical virtues. In Sect. 4 we will address the complications brought by the fact that virtues and vices can be ascribed to individuals, groups and institutions, and how this interacts with the possibility of productive vices and obstructive virtues.

## 2 Character epistemology and enquiry

Character epistemology in its modern form emerged over the last forty years as an offshoot of analytic epistemology. Initially the focus was upon its happier branch, virtue epistemology, spurred by the work of Linda Zagzebski (1996). Roberts and Wood (2007) encouraged interest in sustained studies of specific epistemic virtues and called attention to earlier precursor work in virtue epistemology. Shortly after, Jason Baehr (2011) distinguishes 'conservative' and 'autonomous' conceptions of virtue epistemology, the former construing virtue epistemic resources as ways of contributing to established debates (about the justification of belief, say) and the latter seeing a more independent agenda for virtue epistemology,

<sup>1</sup> Paternotte and Ivanova talk of "beneficial vices", and conclude that virtue is neither necessary nor sufficient for scientific success. However, they argue that virtue is a kind of catalyst in accelerating the development of successful theories amongst already available theories.

one going beyond issues about justification, warrant, and the like. Most virtue epistemologists subscribe to what is usually called a ‘regulative’ conception of epistemology: the epistemic virtues are excellent traits or dispositions that enable agents to better regulate their epistemic activities and their projects of enquiry (see Hookway 1994; Wolterstorff 1996). We set aside the parallel track of virtue reliabilism which characterises virtues as abilities, competences, skills or the like. The guiding idea that puts virtue into the picture is that enquiry is *active* and therefore affected substantially by the epistemic character of the relevant agents. Within any project of enquiry, there are actions to perform, and standards that determine how well they were performed. We might talk, for instance, of an agent’s *diligence* in performing epistemic actions, their *imaginativeness* in generating hypotheses, their *fairmindedness* in assessing criticisms, and so on. As an example, Zagzebski (1996) considers intellectual virtues including: adaptability of intellect; autonomy; creativity; intellectual courage; intellectual humility; intellectual perseverance, diligence, care and thoroughness; open-mindedness; etc. Within the current literature, there are two main normative conceptions of the nature of epistemic virtue, as well as a hybrid position (cf. Battaly 2014; Cassam 2016):

- (A) *Consequentialism* epistemic virtues are character traits that tend to systematically create epistemically good effects. For example, *attentiveness* is epistemically virtuous since an attentive agent is able and disposed to effectively identify epistemically salient features of an object or situation.
- (B) *Motivationalism* epistemic virtues are character traits that express or manifest one or more epistemically good motives, desires, values, such as what Zagzebski (1996, p. 121) famously called the desire for ‘cognitive contact with reality’. For example, truthfulness is a respect for the demands of truth that consists of an abiding desire to ensure that one’s beliefs and utterances are accurate and sincere.
- (C) *Pluralism*: virtues are epistemic character traits that (a) tend systematically to cause a preponderance of epistemically good effects *or* (b) express epistemically good motives (etc.) *or* (c) both.

For our purposes, we endorse pluralism.

There are some other comments needed about epistemic virtues and vices which will help to set up our later remarks about mathematical virtues.

First, the bearers of virtues and vices. Traditional Aristotelian character ethics focused on individual agents as the primary or exclusive bearers of virtues and vices. But modern character epistemology has proposed two further possibilities. One is *collective epistemic virtues and vices*, the idea that groups can be epistemic agents—juries, scientific research teams, etc.—with virtues and vices of their own (cf. Lahroodi 2019). A collectivist notes that we talk about a *fair-minded jury* or a *dogmatic research team* and urges that such talk is not merely figurative.

The big question here is how to fill out the idea of collective virtues and vices. The *reductionist or summative* position is that the group level is just reducible to

individual members' virtues and vices—a fair-minded jury is just a jury whose individual members have the virtue of fair-mindedness. The *joint commitment model* adds the requirement that the group share a joint commitment to a certain set of goals or convictions—for instance, the fair-minded jury can only be described in that way if its individual jurors share a joint commitment to be fair-minded in their deliberations (Gilbert 2014; Lahroodi 2007). *Collectivism* holds that groups can be distinctive, genuine bearers of virtues and vices, which can exist at the group level only as a result of the interaction of the individual members or as an emergent group-level disposition (de Ridder 2014).

Beyond the individual and collective bearers of virtues and vices, there is also the possibility of *institutional epistemic agents*. The idea is that institutions and other social organisations could be epistemically virtuous or vicious. For example, the norms and structures of a scientific institution could be tolerant or intolerant, just or unjust (cf. Anderson 2012; Fricker 2007, 2012). As with the collective epistemic virtues, there are many ways to cash out the idea of institutional virtues and vices, and how they relate to the individuals, society and history of the institution. For our purposes it is worth noting that institutional pressures can shape the sorts of virtues and vices expected of individual and collective agents, and not always in productive ways (e.g. the acute pressure to publish in academia). The concept of epistemic injustice for example, is often closely linked to institutional-level systematic injustice, which has been discussed in the case of mathematics in (Rittberg et al. 2018).

A second set of issues concerns the nowadays neglected idea of *professional epistemic virtues and vices*. Intuitively, even if there is a very general need for any epistemic agents to have certain virtues and avoid certain vices, certain virtues and vices could have a special professional significance to certain agents. Everyone ought to be attentive to some degree, but attentiveness has a special significance to those whose professional activities are focused on especially epistemically complex objects where attention to specific, subtle details are paramount (e.g. forensic investigators, radiographers). There is hardly any literature on the idea of mathematicians having professional virtues (with notable contributions such as Harris (2017) and Su (2017)), though there is a small literature on professional *skills*. Löwe and Müller (2010: §5.1), for instance, argue that there are distinctive professional skills of a research mathematician. If virtues are related to skills, then this may be one way to motivate an argument for the existence of professional virtues of mathematicians.

One line of argument for professional epistemic virtues is that the sorts of activities, demands, and experiences that are integral to the epistemic life of the member of a certain profession will render especially salient particular virtues and vices. These are consistent with what we called *specific* and *local* virtues, depending on how strongly one wishes to define 'salience'. There is a very small literature defending idea of *professional epistemic virtues*, concerning the virtue of truthfulness for archaeologists (Cooper 2006) and an historical rhetoric of virtues throughout history of science (cf. Paul 2019).

All of this makes a complex framework for thinking about epistemic virtues and vices that includes at least the following framing possibilities:

- *Normative pluralism* (consequentialist, motivationalist, hybrid accounts)

- *Ontological pluralism* about virtue- and vice-bearers (individuals, collectives, institutions)
- *Possibility of professional epistemic virtues and vices*
- *Generic, specific and local virtues and vices*

We see certain natural ‘pairings’ here. If there are professional virtues, that’s one way to think about specific and local virtues (a professional virtue might be locally confined to a specific profession). Similarly, collective virtues might work better with consequentialist accounts, since the idea of collective motivations can be difficult to articulate. Ontological pluralism about virtues-bearers seems to fit better with specific and local virtues, since virtue-locality could be defined in terms of collective and institutional bearers.

One strategy for exploring these possibilities and pairings is to look at how they might play out in the specific domain of mathematics, such as by using historical case studies. Our case study below also shows something else interesting: a case where individual and collective virtue/vice come apart. This is often a clash between consequentialist and motivationalist accounts, in which character vices seem to be helpful in attaining epistemically good effects and character virtues fail to reliably generate beneficial ends. Before we move on to that, though, let us get deeper into the virtues and vices associated with mathematics.

### 3 Mathematical virtues and vices

In this section we will set out some key insights into deploying virtue theory in philosophising about mathematical practice. We argue that there is a naturally arising vocabulary of virtues and vices for talking about mathematics, and existent within the mathematics community.

Here are some brisk opening considerations for thinking there are *virtues of mathematical practice*. If there are *agents*, there are epistemic characters, hence, virtues and vices. If there are *practices*, there are better and worse forms of practice, and we can articulate those forms in terms of *virtuous* and *vicious* forms of practice. Within mathematics, not least maths education, there are clear normative expectations and stipulations that could be articulated in character-epistemic terms—for example, they *train* junior practitioners in a set of virtues and *praise* certain virtues in their peers and *warn* against and *criticise* certain vices.

How do we identify relevant virtues in mathematics? One source is to examine the generic epistemic virtues found in philosophy, and consider which are relevant to mathematics. It is not hard to make specific arguments for why most of these would be broadly beneficial (in the consequentialist sense) to the working mathematician. For example, good mathematics relies on getting the details right, so mathematicians should be sensitive to those details, and a failure to be so can potentially lead to snowballing and irreparable errors. Creativity, boldness, inventiveness, perseverance, diligence, thoroughness and adaptability of intellect all seem to be key character traits involved in proving.

We might also consider relevant virtues that have been identified in relation to other specific domains that might carry across to mathematics. For example, Aberdein has compiled lists of virtues and vices in argumentation (2010, 2016), of which some have clear relevance to mathematics. For instance, Aberdein lists “obsessiveness” as a virtue of argumentation, but we can more easily imagine this as a virtue of mathematics, as the relentless pursuit of answers to mathematical problems.

Virtue talk can also be found in historical studies of mathematics and mathematicians. For example, Henrik Kragh Sørensen (2016) uses meta-biographical study (that is, the study of mathematical biographies) to identify which virtues were held to be important by the mathematicians who were writing the biographies. Drawing on mathematical biographies published in the *American Mathematical Monthly* around 1900, the following are the “virtues” that appear:

- Ability to keep “pace with the most rapid advances in mathematical learning” (p. 93)
- Actively engaging with these “for the benefit of the readers of the *American Journal of Mathematics*” (p. 93)
- An “immense production in mathematics” (p. 95)
- A willingness (on the part of Euler, for instance) to devote “all [one’s] time to science” (p. 95)
- The “joy at the discoveries of science whether made by himself or others” (p. 95)
- A “single and unselfish devotion to the truth” (p. 95)
- “His temperament was essentially poetic, and it would have been as impossible for him to concentrate the powers of his mind on one subject when the current of his thought was setting toward another.” (p. 101, citing Franklin (1897), p. 300)
- Enthusiasm
- Hard work
- Efforts in teaching and mentoring
- Great skill at mathematical manipulation
- The ability to create entire new theories out of nothing
- “a strong mathematical mind bordering on the stubborn or absent-minded could overcome external adversary” (p. 103)

It is worth noting that Sørensen’s point is that the biographers were trying to frame certain “virtues” in a particular way to encourage a certain kind of professionalization within their own discipline, so some of the items on the list would no longer be considered virtues now and maybe only tentatively have been considered so at the time.

Some further comments on this list. First of all, there is enormous variety in the scope, from the general to the particular, of the cited traits—from general dispositions like a willingness to do ‘hard work’, presumably referring to virtues of perseverance, to something very specific like ‘great skill at mathematical manipulation’. There are also cases where a general virtue is given a highly specific definition—like ‘keeping pace with the state-of-the-art in one’s discipline’, defined in terms of keeping up with the *American Journal of Mathematics*, which is surely a contingent

specific expression of a more general virtue, perhaps diligence. Second, there are contestable or unusual claims about the virtuousness of a defined character trait. Consider the idea of a ‘poetic’ temperament, and an inability to ‘concentrate the powers of [one’s] mind on one subject when the current of [one’s] thought was setting toward another’. These hardly seem virtuous at all, but within the specific context and setting are considered by the biographer to be part of a virtuous approach to mathematics. Thirdly, the positive framing of absent-mindedness and stubbornness falls into what we think of as productive vices, rather than virtues. Finally, some things that are cited as virtues may really be *components* of traits, rather than traits themselves: for instance, ‘joy at the discoveries of science’ may be the affective component of virtues such as curiosity and inquisitiveness, which motivate enquiry partly by ensuring that the attainment of epistemic goods affords pleasure, delight, and joy (Morton 2010).

These critical comments should be balanced against the well-established historical variations in conceptions of virtue, including the specificity of that term. First, not everyone uses the term ‘virtue’ in the strict sense typical of philosophers. Second, the term ‘virtue’ has altered its sense over time, for instance, to refer to admirable character traits to any general positive trait to a more narrowly defined range of moral virtues.

A final source we consider for formulating a tentative list of potential mathematical virtues is in more contemporary accounts of mathematics, both by mathematician and philosophers. To give a small selection:

- Francis Su (2017) talks of mathematics needing the virtues of *hopefulness*, *perseverance*, *joy* (“the wonder or awe or delight in the beauty of the created order”, p. 486), *transcendence* (“the ability to embrace the mystery of it all”, p. 486), *rigorous thinking*, *humility*, and *circumspection* (“know the limits of our arguments, and we don’t overgeneralize”, p. 488).
- The autobiographical essays by mathematicians in the American Mathematical Association’s volume, *Living Proof* (2019)—whose subtitle is *Stories of Resilience along the Mathematical Journey*—mention *generosity*, *humility*, *kindness*, and *persistence*.
- Tanswell (2016) argues that *mathematical rigour* is an epistemic virtue.
- Tanswell also presents a number of quotes from the mathematician Shinichi Mochizuki using virtue terminology in discussing the ongoing controversy surrounding his work on the *abc* conjecture. He mentions *thoroughness*, *meticulousness*, *rigorousness*, *appropriate understanding*, and “to maintain a humble stance dedicated to uncovering the ultimate truth of things” (Mochizuki 2014, p. 14, cited in Tanswell 2016, pp. 187–193).

Just because these discussions are more recent than Sørensen’s biographies does not mean that these proposals for mathematical virtues are not contextually loaded. For example, Mochizuki was writing in response to critics of the slow dissemination and impenetrability of his work.

Still, these examples and lists of virtues confirm our main claim, for now, about a naturally arising vocabulary of virtues and vices used within and about

the mathematical community. If so, our framework is not imposing on mathematicians an alien framework of virtues and vices. Moreover, it hopefully shows that there is real philosophical work to do in identifying and organising these virtues and tracing the ways they play out in mathematical practice.

A critic might protest that substantive conclusions should not be drawn from the lists, for the reason that mathematicians are not engaged in philosophical virtue theory. We offer two qualifications. First, some virtues and vice talk may be *merely* figurative, such that it does not indicate or register any substantive views about the sorts of epistemic characters appropriate to maths. A vocabulary of virtues of vices is part of our everyday language and is not always expressive of convictions about character (consider talk of ‘a reliable hammer’, the ‘cruel sea’, and so on). Second, mathematicians also use virtue and vice terms to describe the products of their work—the proofs, theorems, concepts, theories etc.—which should not be confused with descriptions of the practitioners who did the work. Some of these terms only make sense as applying to these objects, but some of them are implicitly virtues and vices of the mathematician responsible—for example, ‘creative proof’ would be a category mistake unless it is actually praise for the agent who created the proof, while a ‘lazy method’ could be a criticism of the mathematician. Of course, these don’t always align with the generic level of virtue talk, since people say things like ‘the best mathematician is a lazy mathematician’, suggesting that a lazy mathematician will be more creative in finding a quicker solution (sometimes, laziness is a spur to innovation). This last point directs us towards the challenge raised by productive vices, something we will return to in the next section.

Consideration of how mathematicians tend to use a vocabulary of virtue and vices should therefore be discerning. Sometimes it is figurative, sometimes it is focused on products rather than practitioners, and sometimes it might be ambivalent. Granted, too, it is often difficult to distinguish figurative from non-figurative uses, and similarly difficult to determine if and when *virtue-attributions* should ‘go all the way’ from *product* to the *producer*. But there are clearly cases where the virtues and vices of a product will reflect the virtues or vices of its producer—creativity, carelessness, sloppiness, and so on. It might be, though, that this is confined to only certain clusters of character traits, such as the performative virtues, like carefulness, diligence, thoroughness.

We can therefore think about the challenges specific to mathematical practice, those that might be especially salient in maths that would yield *specific virtues* or that are exclusive to maths that could yield *local virtues*.

Start with *specific virtues*, where a generic virtue that takes specifically inflected forms when exercised in the context of specific sorts of *activity* or in relation to *specific* sorts of object. Take the virtue of *attentiveness*: this look different if one is dealing with abstract objects, like numbers, sets, triangles. Or the virtue of *diligence*, which will look different in domains of enquiry that make less use of experimentation or empirical observation. It also seems true of the vices. *Carelessness* looks different in contexts of enquiry with little or no prospect for empirical confirmation, where there can be no independent evidence of one’s correctness other than properly performed epistemic practices.

Moving onto *local virtues*, those that are specific to a *local* domain, like maths, and so are, in effect, virtues only pertinent to mathematics, i.e. exclusively mathematical virtues. We think there are tougher cases, not least since mathematics is so entangled within so many other disciplines. Disciplinary boundaries are fluid, and it is often hard to say where mathematics ends and physics, computer science, linguistics, chemistry, sports science etc. begin. To deal with these, we should ask what would make some virtue an exclusively mathematical virtue, by looking for distinctive features of that domain—certain exclusively mathematical practices, perhaps, or unique features of the objects of mathematical enquiry, or perhaps of the sociological organisation of the discipline of maths, and so on. MacIntyre's (1981) famous idea of the virtues 'internal to a practice' offers one example, since if there are unique mathematical practices, the they will incorporate their own virtues, which would then be local to those practices and to the domain of mathematics.

Some mathematicians have pointed to certain candidate local virtues, as described above. One is Su's (2018) proposed capacity for 'free play' among mathematical concepts, a sort of *imaginativeness*, necessary to epistemic practices in contexts of enquiry focused solely on abstract objects:

We play with patterns, and within the structure of certain axioms, we exercise freedom in exploring their consequences, joyful at any truths we find. (Su 2017, p. 485)

This is local to mathematics since all of its objects of investigation are abstract. Another is what Emily Grosholz (2007) nicely labels *productive ambiguity*, the idea that a certain degree of deliberate sloppiness about the identity conditions between objects lets one swap freely between identify and distinctness that enables you to make mathematical progress.<sup>2</sup>

In this section, we sketched out some strategies for identifying specific and local virtues of mathematics and indicated some potential candidates. However, we have not yet set out whether these are best understood in terms of consequentialist or motivationalist accounts of virtue and vices. In the next section, we turn to a challenge for such a simple characterisation: productive vices and obstructive virtues.

## 4 The productive vices of the Gelfand seminar

In Sect. 1 we introduced the notions of productive vices and obstructive virtues, following a similar approach taken by Paternotte and Ivanova (2017) for science. In this section we discuss a case of the same effect in mathematics: the potentially productive effects of vicious behaviour. To recap, consideration of productive vices and obstructive virtues fundamentally turns on which normative conception of epistemic character traits one adopts. An *epistemic consequentialist* appraises the virtuous or

<sup>2</sup> A similar idea was labelled by Abramsky as 'creative ambiguity' (Abramsky 2008, p. 494), discussed in Tanswell (2016, p. 137). Colin Foster also explores productive ambiguity in mathematics education in Foster (2011).

vicious status of character traits in terms of their good or bad epistemic effects; in practice, this allows for greater flexibility and context-sensitivity about whether a trait counts as virtue or as vice (see Cassam 2019). If a trait reliably produces epistemically good effects, then it is an epistemic virtue, but—crucially—contextual factors will shape the connection of traits to effects. By contrast, an *epistemic motivationalist* appraises the virtuous or vicious status of character traits in terms of the constituent epistemic motives, values, or desires; this allows for less flexibility—the vice of epistemic malevolence involves a opposition to another person’s epistemic well-being, for instance, by deliberately concealing from them vital truths (Baehr 2010).

Since we endorsed normative pluralism about epistemic virtues and vices, we remain open to consequentialist and motivationalist analyses of the Gelfand seminar case study below. Consistent with the remarks earlier, thinking like a consequentialist requires attending to the ways contextual factors shape the connection of traits to actions to effects; thinking like a motivationalist, we also attend to the epistemic motives, desires, and values of those involved, as best as they can be discerned. We also want to note that, similar to Paternotte and Ivanova’s discussion of science, it is easier to observe productive vice than obstructive virtue. In keeping with this, there seem to be fewer cases of obstructive virtues than productive vices in mathematics. This may simply be because productive vices are made salient by their successes, while the failures caused by obstructive virtue will draw less attention.

Part of the complexity of studying the virtues and vices of mathematics is the splitting of virtue bearers into individual, collective and institutional level, all of which are relevant in mathematics. Concentrating on the consequentialist picture of virtues sketched above, where virtues are defined in terms of systematically leading to good effects, we see that this is complicated in practice by the interplay of the different virtue- and vice-bearers. Individual characters affect the groups and institutions they are part of, and vice versa. This means that apparently vicious behaviour at the individual level can, in certain cases, systematically lead to group-level benefits, and bad institutional policies might inadvertently lead to successful individuals. Similarly, virtuous behaviours at one level might systematically stymie the success at other levels.<sup>3</sup>

This complication caused by the interplay of levels of virtue and vice bearers does occur in mathematics. In this section we will examine a historical case of productive vice in mathematics: the Gelfand seminar at Moscow University. We build our understanding of the Gelfand seminar on the excellent paper by Gerovitch (2016) which provides a detailed historical account of the seminar and its broader context.

The Gelfand seminar was run by the mathematician, Israel Gelfand, at Moscow State University between 1943 and 1989, covering any mathematics of interest to

<sup>3</sup> An epistemic consequentialist about vices can still insist on certain constraints—for instance, by proscribing acts of epistemic arrogance that tend to destroy the self-esteem of other epistemic agents, even if doing so also produces good epistemic affects along the way. But this is an under-theorised aspect of contemporary consequentialist analyses of epistemic vice (cf Cassam 2019: pp. 17–18). A good starting point would be Baehr (2011: Appendix).

its organiser. What is remarkable about the description of the seminar provided by Gerovitch is the extreme contrast between, on the one hand, the many recollections from seminar participants describing the seminar as one of the greatest seminars in the history of mathematics, with a formative impact on a whole generation of mathematicians, and, on the other hand, the extensive descriptions of a toxic culture of abuse, sniping, bullying, arrogance, and personal humiliation. These two facets of the seminar are not taken to have been simply simultaneous: the culture is attributed as directly and systematically responsible for the high regard for and productivity of the seminar. In essence, Gelfand cultivated conditions of extreme mathematical adversity, where the demands on the participants were far beyond reasonable, but thereby also forced everyone to get significantly more involved in the seminar proceedings than normal and learn a great deal more mathematics.

We have identified five examples in Gerovitch's paper where vices systematically produce epistemically beneficial outcomes. Let us take these in turn.

Gerovitch describes how seminar participants' recollections of the Gelfand seminar seemingly cannot agree on when it *started* (fn. 25). This was caused by the fact that whatever the "official" start time was, the seminar did not begin in practice until Gelfand arrived, which was always late, but as soon as he did the door was closed and nobody further could join the seminar (p. 56). This kind of behaviour shows a disrespect for others' time and places himself as the most important person in the seminar (a theme that will appear repeatedly), where he is extremely flexible for himself, but entirely inflexible for others.

These apparent vices of Gelfand in establishing the social dynamic of his seminar had an important side effect, though. This behaviour forced all of the other attendees to arrive early, and would then spend time discussing mathematics; such that, as one participant recalled, this 'delay tactic, whether consciously employed or resulting from Gelfand's idiosyncratic ways, effectively turned the seminar into a major communication hub.' (p. 56). The large delay in the start of the seminar had the effect of fostering the community of the large number of mathematicians other than Gelfand who attended the seminar, giving them frequent and ample time to discuss new results. The somewhat hypocritical inflexibility was highly productive in bolstering the mathematical productivity of the seminar participants and creating a hub for the discussion of new ideas.

A second major feature of the seminar is that its intellectual content and agenda were unpredictable in a way that Gerovitch describes as "chaotic" (p. 57). While Gelfand would usually announce a seminar speaker for the next seminar, the schedule would regularly change at his whim based on new mathematics that caught his interest or a new mathematician he met in the intervening week. Chaos and unpredictability in seminar organisation is not a virtue but a vice, with many obvious potential downsides. Nonetheless, the chaos with which Gelfand approached the organisation is described by Gerovitch as causing the seminar to focus on the newest and most exciting areas of mathematics, and to keep the seminar contents spanning many of the newest ideas of the day:

The unpredictable, chaotic trajectory of seminar proceedings might in effect be seen as a strategy to cover, perhaps, haphazardly, as wide an area of mathemat-

ics as possible, achieving some kind of universality, if not through a systematic approach, then perhaps by random walk. (p. 57)

Here we see the vices of disorganisation, inconsistency, and unpredictability actually being used in service of achieving the best coverage of mathematics, and to make the seminar as cutting-edge as possible. The vices were thus productive in maximising the exposure of the epistemic community to the vast range of modern mathematics that existed at the time.<sup>4</sup>

The first two examples of productive vices here might only be described as social or moral vices to do with the organisation of the seminar, rather than properly epistemic vices. However, the descriptions of the behaviour of Gelfand during the seminars shows several epistemic vices at work, including intellectual arrogance, aggression and uncharitability. Even with a speaker nominally presenting, Gelfand would dominate the proceedings:

Gelfand's hallmark behavior was to interrupt the speaker constantly. [...] In the end, Gelfand usually completely took over and replaced the speaker at the blackboard, explaining to the audience and to the speaker what was the "correct way" of speaking about the discussed topic. [...] He listened only as long as was needed for him to grasp the essence of the talk. Once he did that, the speaker became superfluous. Gelfand picked up the train of thought and directed it where he, not the speaker, wanted to go. (p. 58)

This is a rather extreme description of intellectual arrogance, especially for a social setting like a seminar. The belief that he only needed the speaker as a prompt for himself to find the path to take, rather than as an epistemic peer, does not give due credit to any of the interlocutors. Not only was the speaker subjected to this behaviour, but members of the audience were kept constantly on edge with the very-real threat that they may be called upon to continue giving the presentation at any moment, with all of the public ridicule this would involve. This acts as a threat to both the speaker and audience that the mathematics being presented is seen by Gelfand as sufficiently easy to be taken over by anyone at a moment's notice.

Nonetheless, Gerovitch argues that this brash and aggressive style displaying multiple epistemic vices was highly productive:

Unlike other seminars, in which the speakers were allowed to drone on indefinitely and without interruption, putting the audience into a half-asleep mode, the Gelfand seminar captivated the audience and kept everyone on edge all the time, forcing seminar participants to focus and grasp the meaning of discussion. (p. 59)

Rather than *attending* the seminar to *watch* mathematics being done, the vices that Gelfand manifested actually insisted that all the participants were actively *doing* mathematics constantly throughout the seminar. The high participatory demands of

<sup>4</sup> This was especially important, according the broader theme of Gerovitch's paper, in the particular setting of post-war soviet Russia.

the seminar, caused by aggressive interruptions and questioning from Gelfand, are examples of productive vice because of the mathematical gains made by the seminar participants being forced to try to keep up. Gerovitch proposes that “Gelfand deliberately sacrificed the comfort and sometimes the dignity of the speaker to the task of achieving collective understanding” (p. 61).

Finally, the aggressive and arrogant style of questioning in the seminars also displayed the vice of intellectual dishonesty. Rather than asking the questions in earnest, Gelfand’s style was to feign a lack of understanding as a rhetorical tactic, including the technique—familiar in the West since Socrates—of “feigned incomprehension”:

There are mathematicians famous for their ability to quickly understand the most complex mathematical argument. Gelfand was famous for his incredible skill of non-understanding. He frequently declared his lack of comprehension of the speaker’s argument (p. 62)

This feigned lack of understanding was part of making sure that he came to understand the many different topics covered by the chaotic style of the seminar, but it also seemed to be an ongoing technique Gelfand used to show his style of doing mathematics. Eventually, he would reveal some insight that he was aiming at, that would have been overlooked by the speaker. In a sense, Gelfand used the dishonesty to cultivate mathematical discernment in the seminar participants, for them to be able to tell the important mathematics from the trivial. Of course, from the outside, it looks as if Gelfand simultaneously gets impatient and aggressive when presented with too many details, and feigns incomprehension if they are left out. For the seminar presenters, this sounds like there was no way to win; yet, the systematic upshot of this impossible game is that the community of mathematicians gained a great deal of illumination from the seminars.

Overall, then, we have seen how Gerovitch’s account of the Gelfand seminar shows a case of systematically productive vices in mathematics.<sup>5</sup> These vices range from moral vices concerning poor character in Gelfand’s treatment of others to clear-cut cases of epistemic vice—notably intellectual arrogance, intellectual dishonesty, uncharitability. Within the adverse, demanding conditions thus created for seminar participants and speakers, these vices pushed the mathematical community built around the seminar to an unusually high standard of mathematics, one which propelled many of its participants to great future success.

We must be careful not to glorify this behaviour. We cannot know whether the same level of mathematics might have been achieved, for instance, via some particularly virtuous set of practices and behaviours. Nor can we know whether the behaviour would succeed under different social contexts or if enacted by someone

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<sup>5</sup> One potential objection is that the systematic linking of vices to the productivity in this case is built just on Gerovitch’s *interpretation* of the seminar. However, this interpretation seems accurate, as the Gelfand seminar is contrasted by Gerovitch with other contemporaneous seminars that were run in Moscow without similar vices or the productive upshots. Gerovitch makes a compelling case for the causal link based on detailed historical research.

other than the charismatic Gelfand. We can, however, point out the clear costs associated with those vices. For example, speed in doing mathematics and understanding ideas is not the only way to be good at it, but the seminar style will not highlight the slower, more ponderous approach to mathematics. Indeed, the aggression and uncharitability will put people off of mathematics, as is seen in the blogpost of Tanya Khovanova where she recalls the Gelfand seminar:

Ironically, I admired Gelfand for the way he conducted his seminars. I went to so many seminars where it was clear that no one understood anything. He was the only professor I knew who made sure that at least one person at his seminar — himself — understood everything. The problem was that he convinced me that I really was a fool. I dreaded Mondays and I considered quitting mathematics.<sup>6</sup>

Khovanova's recollection also points to a further problem, namely, the highly gendered character of the epistemic and interpersonal attitudes built into the Gelfand seminar. Though such aggressive, dominative behaviours are not confined to men, nor the inverse behaviours to women, there are clear dependencies between the extent to which Gelfand's behaviours were tolerated and able to achieve the effects that they did, and the gendered power dynamics of the seminar—a point that underscores the need for further work in feminist philosophy and sociology of mathematics (a good starting point is Burton 1995), especially in conjunction with contemporary calls for a feminist character epistemology (Dillon 2012; Daukas 2019). We can only speculate how many young mathematicians were broken rather than built up in Gelfand's seminar, and how many did quit mathematics altogether as a result.

Careful consideration of the Gelfand seminar in terms of virtue and vice reveals to us a number of challenges in applying the distinctions from the earlier parts of this paper to mathematical practices. Let us consider these distinctions in turn.

First of all, we have seen the challenge we described at the start of this section about the interplay of different kinds of virtue- and vice-bearers. In particular, the Gelfand seminar illustrates a number of individual epistemic and moral vices of Gelfand himself, through aggressive, arrogant and chaotic behaviours. Nonetheless, these resulted in the community of mathematicians around the seminar flourishing, and displaying many collective virtues of enquiry. Epistemic vice at the individual level can lead to virtue at the collective level. Complicating this picture even further, Gerovitch describes how some of the features of the seminar resulted from institutional restrictions:

In the postwar period, the Soviet mathematics community was increasingly subjected to serious administrative constraints and pressures, including discriminatory policies in university admissions, hiring, and publishing toward the “undesirables,” such as Jews and political dissidents (Gerovitch 2016, p. 52)

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<sup>6</sup> Tanya Khovanova, “The Designated Listener,” 19 November (2008) (<http://blog.tanyakhovanova.com/?p=76>). Cited by Gerovitch (fn. 61).

As such, institutional vice in the form of vicious and discriminatory policies, in this case impacted the productivity of the individual and collective levels of the Gelfand seminar. By creating the parallel mathematical infrastructure, the Gelfand seminar escaped some of the institutional problems and build a community of students and researchers to collaborate and communicate new ideas. Individual, collective and institutional virtues and vices all form part of the intellectual environment and thereby have effects on one another.

Secondly, then, we can reflect on the motivational and consequential approaches to virtues in mathematics. The Gelfand seminar gives us a case where the individual vices of Gelfand do appear to have been systematically epistemically productive for both Gelfand and the wider community. On a consequentialist picture, this would suggest that these supposed vices may be epistemic virtues after all. For a motivationalist, we would need to say what was motivating Gelfand's behaviour, something not easily accessible to us. However, it would not be unfair to speculate that his behaviour had a selection of motivations, amongst which there must've been a desire to do good, interesting, exciting mathematics. This would potentially leave the motivational account of virtues endorsing these, too.

This puzzling consequence should not be endorsed immediately, though. For the motivational account especially, the motivation to engage in mathematics well is too coarse-grained a motivation to justify the full range of seminar behaviours. For example, Gelfand might well have engaged in better mathematics by listening properly to the seminar speakers, such that the intellectual arrogance of constant interruptions is properly classified as a vice. It is also unclear whether he consciously selected these putatively vicious styles after careful and considered appraisal of their merits relative to other, more 'tender-hearted' alternatives. Any good teacher knows the importance of being able to intelligently toggle between softness and harshness when giving critical feedback. The question is whether one is actually choosing how one interacts with others in a careful, considered way—sometimes harsh, sometimes soft, and sometimes interrupting, sometimes listening. This sort of intelligent management can be articulated in various ways: Aristotelian virtue theory would call it *phronesis*, while philosophy of science could think of it in terms of the adaptable, non-dogmatic methodological pluralism endorsed by Paul Feyerabend (1975; cf. Oberheim 2006).

The difficulty of identifying epistemic virtue and vice in mathematics in the face of productive vices (and obstructive virtues) takes us back to our third distinction: between generic, specific and local virtues and vices. The epistemic vices of Gelfand including intellectual arrogance, dishonesty and uncharitability, are generic vices, found across a range of contexts of epistemic investigation. Here they have been inflected to the mathematics case as specific virtues, taking the generic virtues into the mathematical context by guiding seminar behaviours, mathematical interactions and collaborations. However, they are certainly not local virtues found exclusively in mathematics. As such, even though they may be systematically productive in the very narrow context of one seminar in post-War, Soviet mathematics, the same character traits will generally lead to poorer epistemic outcomes in other situations. Indeed, on the motivational account, traits like intellectual arrogance, dishonesty and uncharitability are vices in the mathematical context more broadly too, since

across mathematical contexts they would lead to worse epistemic positions for the vice-bearers, their interlocutors and their epistemic community. The Gelfand seminar has thus brought out several tensions: between the individual and their socio-epistemic environment; between the motivational and the consequential accounts of virtues and vices; and how to characterise epistemic virtue and vice at all.

## 5 Conclusion

This paper offered a preliminary effort to draw connections between philosophy of mathematics and character epistemology. We think that there are distinctive features of mathematical practice which mean that it ought to be investigated in its own right by those interested in the role of epistemic virtues and vices in epistemic practice and our systems of enquiry. A focus on mathematics can also offer new ways to think about emerging questions in character epistemology, such as the possibility of virtues and vices that are local and specific to a particular domain of enquiry, such as mathematics. Such a focus also offers additional ways to think about foundational issues in character epistemology, such as the debate about the normative foundations, of the sort we see in the consequentialist-motivationalist conceptions of epistemic character traits. The history of mathematics affords many rich case studies for those interested in character epistemologies of mathematics, too, with the Gelfand seminar being just one of many. We hope that this paper might inspire further work on epistemic virtues and vices and mathematical practice.

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