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Confirmation and justification. A commentary on Shogenji's measure

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Abstract So far no known measure of confirmation of a hypothesis by evidence has satisfied a minimal requirement concerning thresholds of acceptance. In contrast, Shogenji's new measure of justification (Shogenji, Synthese, this number 2009) does the trick. As we show, it is ordinally equivalent to the most general measure which satisfies this requirement. We further demonstrate that this general measure resolves the problem of the irrelevant conjunction. Finally, we spell out some implications of the general measure for the Conjunction Effect; in particular we give an example in which the effect occurs in a larger domain, according to Shogenji justification, than Carnap's measure of confirmation would have led one to expect.

Keywords Probability · Confirmation · Justification

1 Introduction

Many functions have been proposed as measures of the confirmation that an evidential proposition gives to a hypothetical proposition. They are functions of the conditional probability that the hypothesis is true, given the evidence, as well as of the unconditional probabilities accorded to the hypothesis and to the evidence. The plethora of confirmation measures is a serious impediment to progress, since the various functions are not ordinally equivalent to one another. According to one measure, hypothesis h_1 might be more highly confirmed by evidence e_1 than h_2 is confirmed by e_2, whilst another measure might reverse the ordering.

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If a confirmation function is to serve as a measure of *justification*, it must have some special properties. In order to say that a hypothesis is justified by some evidence, it is not enough that a particular confirmation function produces a number that is positive: the number must be greater than some threshold of acceptance. Moreover, not any confirmation function is acceptable, for a minimal requirement is surely that, if two or more independent hypotheses are each confirmed to a degree more (less) than the threshold level of acceptance, then their conjunction must also be confirmed more (less) than that same threshold level. Shogenji calls this criterion of acceptability the General Conjunction Requirement (GCR), and he proposes a particular measure that satisfies it Shogenji (2009).

In this paper I prove that, while Shogenji's new measure is by no means the only function that satisfies GCR, it is ordinally equivalent to any function that does so. We may therefore say that Shogenji's measure is, *up to ordinal equivalence,* the unique measure that respects the requirement. If hypothesis h_1 is more highly confirmed by evidence e_1 than h_2 is confirmed by e_2, according to one measure that satisfies GCR, this remains true for any other measure that does so.

I consider also the problem of the irrelevant conjunction: while its Bayesian resolution is not robust with respect to all the measures of confirmation on the market, it is resolved by all measures of justification that respect GCR. I also give a new class of examples of what may be called the Conjunction Effect, in which the degree of justification of the conjunction of two hypotheses is greater than the degrees of confirmation of either of the hypotheses separately.

2 Confirmation and justification

A measure of the confirmation of a hypothesis, h, by some evidence, e, may in general be a function of the three double probabilities $P(h \wedge e)$, $P(h \wedge \neg e)$ and $P(\neg h \wedge e)$, which are independent, except that their sum cannot be greater than one. More conveniently, one replaces these variables by $P(h|e) = P(h \wedge e)/P(e)$, $P(h)$ and $P(e)$,which are also independent. Such a measure is then some function $F[P(h|e), P(h), P(e)]$. Following Shogenji, we shall first argue that the last argument should be dropped in any measure of the *justification* of h by e. For if it were not so, one would be able to change the value of F simply by appending some arbitrary, probabilistically independent and irrelevant extra 'evidence' to e, say e'. The new measure would have the form $F[P(h|e \wedge e'), P(h), P(e \wedge e')]$. Since e' is assumed to be irrelevant both to the hypothesis and to the evidence, it is supposed that

$$P(e \wedge e') = P(e)P(e') \qquad P(h \wedge e \wedge e') = P(h \wedge e)P(e'),$$

so $P(h|e \wedge e') = P(h|e)$. We require that the measure of justification be unaltered by the adjunction of e', so

$$F[P(h|e), P(h), P(e)] = F[P(h|e), P(h), P(e)P(e')].$$

Now since $P(e')$ could in principle be any number in [0, 1], F cannot depend nontrivially on the third argument. Such a measure of justification should in fact have the following three properties:

(i) It should be a function of $P(h|e)$ and $P(h)$ only.
(ii) At constant $P(h|e)$, it should be a decreasing function of $P(h)$.
(iii) At constant $P(h)$, it should be an increasing function of $P(h|e)$.

We write, as the general form of a measure of justification,

$$J(h, e) = F[P(h|e), P(h)]. \tag{1}$$

Many candidate measures of *confirmation* have been suggested in the course of the years. A popular one, due to Carnap (1962), is

$$D(h, e) = P(h|e) - P(h).$$

Another one, invented by Keynes (1921), is

$$R(h, e) = \log \left[\frac{P(h|e)}{P(h)} \right], \tag{2}$$

and it is clear that both of these satisfy (i), (ii) and (iii). Two other measures, favoured by Kemeney and Oppenheim, and by Good, respectively, are

$$K(h, e) = \frac{P(e|h) - P(e|\neg h)}{P(e|h) + P(e|\neg h)}$$
$$L(h, e) = \log \left[\frac{P(e|h)}{P(e|\neg h)} \right].$$

They appear at first sight to have a different structure; but a little algebra serves to show that they too can be expressed as functions of $P(h|e)$ and $P(h)$ only, and also that requirements (ii) and (iii) are respected. On the other hand, the confirmation measures

$$C(h, e) = P(h \wedge e) - P(h)P(e)$$
$$S(h, e) = P(h|e) - P(h|\neg e)$$
$$N(h, e) = P(e|h) - P(e|\neg h)$$

depend nontrivially on $P(e)$, as well as $P(h|e)$ and $P(h)$, and therefore cannot be regarded as candidate measures of justification.

3 Justification and the conjunction requirements

What is the 'correct' justification function, F? Evidently conditions (i), (ii) and (iii) do not determine it uniquely. Although the measures of confirmation C, S and N are ruled out of court by (i), D, R, K and L survive as possibilities, and more can be conjured up. The worry is not merely that D, R, K and L are different measures, but rather that they are ordinally inequivalent. To illustrate this, consider the set of examples

$$P(h|e) = 2^{-n} \quad P(h) = 2^{-m} \quad \text{with} \quad m > n. \tag{3}$$

Using subscripts to indicate the values of m and n, we find

$$\begin{aligned} D_{53} < D_{52} &< D_{21} \\ R_{21} < R_{53} &< R_{52} \\ K_{21} < K_{53} &< K_{52} \\ L_{21} < L_{53} &< L_{52}. \end{aligned}$$

Evidently Keynes (R), Kemeny and Oppenheim (K) and Good (L) agree about the relative ordering of the degrees of confirmation that e accords to h in these examples, but they disagree with Carnap (D). But should we have more confidence in Keynes' ordering than in Carnap's on the basis of a majority vote? Evidently we need some other criterion to pick out a unique ordering.

According to Shogenji (2009), one should add to (i), (ii) and (iii) an additional requirement of the following kind:

(iv) Suppose that h_1 and h_2 are two hypotheses, unconditionally independent and also independent conditionally on evidence e. If both h_1 and h_2 have measures of justification greater (less) than t, then the conjunction of these independent hypotheses must also have a measure of justification greater (less) than t.

Symbolically, if

$$P(h_1 \wedge h_2) = P(h_1)P(h_2) \quad \text{and} \quad P(h_1 \wedge h_2|e) = P(h_1|e)P(h_2|e)\,,$$

then it is required that

$$J(h_1, e) < / > t \quad \text{and} \quad J(h_2, e) < / > t \quad \text{entails} \quad J(h_1 \wedge h_2, e) < / > t.$$

Shogenji calls this the General Conjunction Requirement (GCR). Moreover, GCR is required to hold, not merely for two independent hypotheses, but for any finite number of them. It should be noted that the conditions (i), (ii) and (iii) require that a measure of justification does not to depend on t, the choice made for the threshold of acceptance, but only on the conditional and unconditional probabilities assigned to the hypothesis, in light of the evidence.

All the popular measures of confirmation fail to meet the General Conjunction Requirement [for a discussion of the nine measures of confirmation that we specified in the previous section, see Atkinson et al. (2009)].

Condition (iv) implies

(iv′) If both h_1 and h_2 have measures of justification *equal to* t, the conjunction of these independent hypotheses must also have a measure of justification *equal to* t.

This is so, because, as Shogenji shows, if $J(h_1, e) = J(h_2, e) = a$ and $J(h_1 \wedge h_2, e) = b$, then if a were not equal to b one could choose t intermediate between a and b and deduce that GCR would have to be violated at t, contrary to hypothesis. Symbolically,

$$J(h_1, e) = t \quad \text{and} \quad J(h_2, e) = t \quad \text{entails} \quad J(h_1 \wedge h_1, e) = t.$$

Shogenji calls this the Special Conjunction Requirement (SCR). Thus GCR entails SCR.

In the Appendix we show that the most general measure of justification that satisfies SCR, and the conditions (i), (ii) and (iii), can be written in the form

$$J(h, e) = f\left[\frac{\log P(h|e)}{\log P(h)}\right],$$

where $f[x]$ is a continuous, monotonically decreasing function of x. Moreover, we show that such a measure satisfies GCR too, so it is in fact the most general form that is allowed by the General Conjunction Requirement. It is also shown in the Appendix that all such measures are ordinally equivalent to one another.

The simplest example is

$$J(h, e) = 1 - \frac{\log P(h|e)}{\log P(h)}. \tag{4}$$

The latter is identical to the function that Shogenji proposes, namely

$$J(h, e) = \frac{\log_2 P(h|e) - \log_2 P(h)}{-\log_2 P(h)}.$$

Note that the value of this expression does not change if one replaces the base 2 by any other positive number. Another possibility is

$$J'(h, e) = 2^{J(h,e)} - 1,$$

Of course, J and J' are ordinally equivalent to one another, the first having the infinite range $(-\infty, 1]$, the second having the finite range $[-1, 1]$.

The measure J' can also be rewritten in the form

$$J'(h, e) = 2\,[P(h|e)]^{\gamma} - 1,$$

where the exponent is defined by

$$\gamma = -\frac{1}{\log_2 P(h)}.$$

Note that γ is positive.

Although justification measures are far from unique—the general form involves after all an arbitrary monotonic function—the ordering that they all induce *is* unique, and this is usually the important consideration. Returning to the examples (3), we find the ordering

$$J_{53} \; < \; J_{21} \; < \; J_{52}.$$

It is interesting that the Shogenji ordering of these examples differs from those given by Carnap, by Keynes, by Good, or by Kemeny and Oppenheim.

4 Irrelevant conjunction

According to the hypothetico-deductive method, evidence e is deemed to confirm hypothesis h relative to some background b when $h \wedge b$ logically entails e, i.e. $e \wedge h \wedge b = h \wedge b$. However, it follows ineluctably that, for any other hypothesis h' whatsoever, $e \wedge h \wedge h' \wedge b = h \wedge h' \wedge b$. This means that e also confirms the conjunction of h' and h, relative to the same background b, even if h' is independent of e, h and $e \wedge h$. This is so counter-intuitive, nay so unacceptable, that this fact has seemed to many to sound the death knell for the hypothetico-deductive method itself.

Bayesians distance themselves from this unpleasantness as follows. Hawthorne and Fitelson (2004) have shown that, if h' is irrelevant in the sense that the likelihood of e is not affected by the conjunction of h' with h and b, so $P(e|h \wedge h' \wedge b) = P(e|h \wedge b)$, then the Carnap confirmation that e accords to h, in the presence of b, is greater than that accorded to the conjunction of h' and h. This follows because the condition of irrelevance means that

$$P(e \wedge h \wedge h') = P(h \wedge h')P(h \wedge e)/P(h),$$

where for brevity we have suppressed explicit mention of the background b. Consider in this and subsequent formulae that the probability space has been reduced in such a way that $P(b) = 1$. Then

$$\begin{aligned} D(h \wedge h', e) &= \frac{P(e \wedge h \wedge h')}{P(e)} - P(h \wedge h') \\ &= \frac{P(h \wedge h')P(e \wedge h)}{P(e)P(h)} - P(h \wedge h') \\ &= D(h, e)\frac{P(h \wedge h')}{P(h)}. \end{aligned}$$

It follows that, if h is Carnap-confirmed by e, then

$$0 \leq D(h \wedge h', e) \leq D(h, e).$$

The conjunction of h and h' is also Carnap-confirmed by e, but generally to a lesser degree [unless $P(h \wedge h') = P(h)$, which is so only if h logically entails h' (in the presence of b)].

This result is satisfactory, for an irrelevant h' that has little overlap with h will produce a conjunction that is Carnap-confirmed only to a slight extent. But there is a snake in the grass. This result does not hold for all measures of confirmation. It does work with Good's measure L, and with Kemeny and Oppenheim's measure K, but not with Keynes' measure R. In fact

$$\begin{aligned} R(h \wedge h', e) &= \log \frac{P(e \wedge h \wedge h')}{P(e)P(h \wedge h')} \\ &= \log \frac{P(e \wedge h)}{P(e)P(h)} = R(h, e). \end{aligned}$$

So with Keynes' measure the irrelevant conjunction problem is back in full force: the conjunction of a hypothesis with an irrelevant hypothesis (remember, always relative to some background) is Keynes-confirmed to precisely the same degree as is the original hypothesis alone.

So Hawthorne and Fitelson's result is not robust, in the sense that it fails for one commonly used measure of confirmation (as these authors realized full well). But Shogenji has now shown us that a solution is at hand! We should not be using measures of confirmation, but rather measures of justification. In fact the measure (4) of justification can be written

$$J(h, e) = -\frac{R(h, e)}{\log P(h)},$$

and therefore

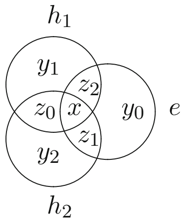
$$J(h, e) = -\frac{R(h \wedge h', e)}{\log P(h)} = \frac{\log P(h \wedge h')}{\log P(h)} J(h \wedge h', e) \geq J(h \wedge h', e).$$ [1]

Because all measures of justification are ordinally equivalent, this resolution of the irrelevant conjunction problem applies to them all.

5 Conjunction effects

In this section we shall be looking for situations in which two hypotheses h_1 and h_2 are both confirmed by some evidence e, and in which the conjunction $h_1 \wedge h_2$ is even more highly confirmed. From the Venn diagram of Fig. 1 we read off

[1] Since $\log P(h \wedge h') \leq \log P(h)$ and $\log P(h)$ is negative, it follows that $\log P(h \wedge h')/\log P(h) \geq 1$.



$$\begin{aligned} P(h_1 \wedge h_2 \wedge e) &= x \\ P(h_1 \wedge h_2) &= x + z_0 \\ P(h_1 \wedge e) &= x + z_2 \\ P(h_2 \wedge e) &= x + z_1 \\ P(h_1) &= x + y_1 + z_0 + z_2 \\ P(h_2) &= x + y_2 + z_0 + z_1 \\ P(e) &= x + y_0 + z_1 + z_2 \end{aligned}$$

Fig. 1 Venn diagram for hypotheses and evidence

We consider first the Carnap measure of confirmation,

$$D(h, e) = P(h|e) - P(h),$$

before returning to Shogenji's measure of justification. The strategy will be to reduce the effective number of dimensions from seven to two. Consider the case in which six of the triple probabilities are equal, $x = y_0 = y_1 = y_2 = z_1 = z_2$, but where the seventh, z_0, has in general a different value. The global constraint is that the sum of all seven triples cannot exceed unity. We find

$$\begin{aligned} D(h_1, e) &= \tfrac{1}{2} - 3x - z_0 \\ D(h_1 \wedge h_2, e) &= \tfrac{1}{4} - x - z_0 \\ D(h_1, e) - D(h_1 \wedge h_2, e) &= \tfrac{1}{4} - 2x. \end{aligned}$$

The following domains can thus be distinguished:

$$\begin{aligned} \text{global constraint} \quad &\longrightarrow 6x + z_0 \leq 1 \\ D(h_1, e) > 0 &\longleftrightarrow 3x + z_0 < \tfrac{1}{2} \\ D(h_1 \wedge h_2, e) > 0 &\longleftrightarrow x + z_0 < \tfrac{1}{4} \\ D(h_1, e) > D(h_1 \wedge h_2, e) &\longleftrightarrow \quad x < \tfrac{1}{8} \end{aligned} \tag{5}$$

What happens if we use Shogenji's new measure of justification instead of Carnap's measure of confirmation? The first three inequalities of (5) are unaffected. This is clear, for $D(h_1, e) > 0$ is equivalent to $P(h_1|e) > P(h_1)$, and that is equivalent to $J(h_1, e) > 0$, and similarly $D(h_1 \wedge h_2, e) > 0$ is equivalent to $J(h_1 \wedge h_2, e) > 0$.

The matter is different for the last inequality of (5). When are the hypotheses h_1 or h_2 *less justified* by e than their conjunction is justified by e? From the definition (4) we calculate

$$J(h_1, e) - J(h_1 \wedge h_2, e) = \frac{\log(x + z_0) - 2\log(3x + z_0)}{\log(x + z_0)\log(3x + z_0)}.$$

So there is a conjunction effect, namely $J(h_1, e) < J(h_1 \wedge h_2, e)$ and $J(h_2, e) < J(h_1 \wedge h_2, e)$, whenever $(3x + z_0)^2 > (x + z_0)$.

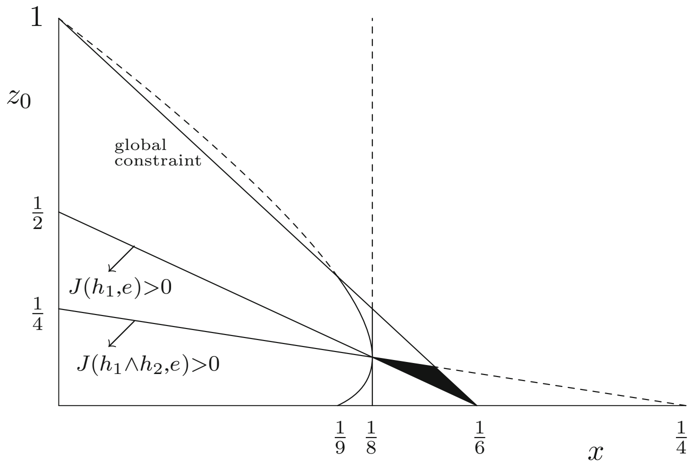


Fig. 2 Allowed region in $x - z_0$ plane. *Dashed lines* are outside the allowed region

Figure 2 displays various regions of the x–z_0 plane. In the triangular region bounded by the horizontal x-axis, the vertical line at $x = \frac{1}{8}$, and the sloping line of the global constaint, $h_1 \wedge h_2$ is more highly Carnap-confirmed by e than are h_1 or h_2 separately; but in fact the domain in which this conjunction has a higher measure of Shogenji justification is slightly larger, namely to the right of the parabola indicated by the curved line in Fig. 2. Below the line from $\left(0, \frac{1}{4}\right)$ to $\left(\frac{1}{4}, 0\right)$ the measure $J(h_1 \wedge h_2, e)$ is positive, but only the part of this line that is to the left of the global line from (0, 1) to $\left(\frac{1}{6}, 0\right)$ is allowed. Below the line from $\left(0, \frac{1}{2}\right)$ to $\left(\frac{1}{6}, 0\right)$ the measure $J(h_1, e)$ is positive. Thus the region in which a conjunction effect occurs is bounded by the parabola on its left and the global constraint line on its right.

The black triangle within the above triangular region indicates where there is a conjunction effect (according to both D and J), and where moreover h_1 and h_2 are disconfirmed by e, while $h_1 \wedge h_2$ is confirmed. The conjunction is not merely confirmed, but is in fact *justified* by e, while the conjuncts are each unjustified. This yields a new example of what was called the Alan Author effect in Atkinson et al. (2009).

In the paper just cited, it was proved that the conjunction effect occurs whenever $z_1 = 0 = z_2$, irrespective of the values of the remaining five triple probabilities. This was demonstrated in that paper to be true for ten different interpretations of what confirmation means. The result was on that account called *robust*. With the new measure of justification, we may ask whether $J(h_1 \wedge h_2, e) > J(h_1, e)$ whenever $z_1 = 0 = z_2$. We find

$$J(h_1 \wedge h_2, e) - J(h_1, e) = \frac{\log\left(1 + \frac{y_0}{x}\right) \log\left(1 + \frac{y_1}{x+z_0}\right)}{\log(x + z_0) \log(x + y_1 + z_0)},$$

and this is strictly positive if none of the remaining five triples vanish, since the two factors in the numerator are positive, while the two factors in the denominator are negative. Similarly $J(h_1 \wedge h_2, e) > J(h_2, e)$. Thus $z_1 = 0 = z_2$ is a sufficient condition for a justificatory conjunction effect.

Acknowledgements I would like to thank Tomoji Shogenji for his most stimulating paper and for fruitful email correspondence. I am also indebted to Theo Kuipers for pointing out that the proof of ordinal uniqueness depends crucially on the requirement that the measure of justification was not allowed to depend on the choice of the threshold of justification.

Appendix

$J(h, e)$ is a function of $x = P(h|e)$ and $y = P(h)$ only,

$$J(h, e) = F(x, y), \tag{6}$$

where $F(x, y)$ is a continuous function for $x \in [0, 1]$ and $y \in (0, 1)$. Discontinuities or divergences are allowed if $P(h)$ is extremal (0 or 1), but continuity with respect to the conditional probability, $P(h|e)$, is required at both end points.

Let h_1, h_2 and e be propositions such that

$$P(h_1|e) = P(h_2|e) = x$$
$$P(h_1) = P(h_2) = y$$

and let h_1 and h_2 be independent of one another, conditionally with respect to e, and also unconditionally:

$$P(h_1 \wedge h_2|e) = P(h_1|e)P(h_2|e) = x^2$$
$$P(h_1 \wedge h_2) = P(h_1)P(h_2) = y^2.$$

If t is the threshold of acceptance, SCR requires that, if $J(h_1, e) = t$ and $J(h_2, e) = t$, then $J(h_1 \wedge h_2, e) = t$. Thus $J(h_1 \wedge h_2, e) = J(h_1, e)$, and so, from Eq. 6,

$$F(x, y) = F(x^2, y^2). \tag{7}$$

Change the variables and the function from $F(x, y)$ to $G(x, u)$, where

$$u = \frac{\log x}{\log y} \qquad G(x, u) = F(x, y).$$

Condition (7) becomes

$$G(x, u) = G(x^2, u).$$

For any $x \in (0, 1)$, we can iterate this equation to obtain

$$G(x, u) = G(x^2, u) = G(x^4, u) = \ldots = G(x^{2^n}, u).$$

Since the function $G(x, u)$ is required to be continuous at $x = 0$, we can take the limit $n \to \infty$ and conclude that $G(x, u) = G(0, u) \equiv f[u]$ is an arbitrary continuous function of u. Hence

$$J(h, e) = f\left[\frac{\log P(h|e)}{\log P(h)}\right]. \tag{8}$$

$J(h, e)$ is an increasing function of $P(h|e)$ and a decreasing function of $P(h)$, so it follows that $f[u]$ must be a decreasing function of u (since $\log P(h|e)$ and $\log P(h)$ are both negative). The most general function of justification that satisfies SCR has the form (8), subject to the constraint that $f[u]$ is a continuous, monotonically decreasing function of u.

We shall show that the general form (8) respects GCR as well as SCR. Let h_1 and h_2 be independent of one another, conditionally with respect to e, and also unconditionally. Suppose that a realization, J, of Eq. 8 is such that $J(h_1, e) > t$ and $J(h_2, e) > t$, where t is some threshold of acceptance. So

$$\frac{\log P(h_1|e)}{\log P(h_1)} < f^{-1}(t) \quad \text{and} \quad \frac{\log P(h_2|e)}{\log P(h_2)} < f^{-1}(t)\,,$$

where the inverse function, f^{-1}, is guaranteed to exist, given the monotonicity of f. Then

$$\begin{aligned}
\log[P(h_1 \wedge h_2|e)] &= \log[P(h_1|e)P(h_2|e)] \\
&= \log P(h_1|e) + \log P(h_2|e) \\
&> f^{-1}(t)[\log P(h_1) + \log P(h_2)] \\
&= f^{-1}(t) \log[P(h_1)P(h_2)] \\
&= f^{-1}(t) \log[P(h_1 \wedge h_2)].
\end{aligned}$$

Therefore

$$\frac{\log P(h_1 \wedge h_2|e)}{\log P(h_1 \wedge h_2)} < f^{-1}(t)\,,$$

and so $J(h_1 \wedge h_2, e) > t$. A similar proof works with the inequalities working in the opposite direction, i.e. if $J(h_1, e) < t$ and $J(h_2, e) < t$ then $J(h_1 \wedge h_2, e) < t$. Moreover, the method extends straightforwardly to an arbitrary finite number of independent hypotheses $h_1, h_2, \ldots, h_n$, instead of two. This concludes the demonstration that Eq. 8 encapsulates the most general measure of justification.

All measures that satisfy the above conditions are ordinally equivalent to one another. For consider two different measures:

$$J_1(h, e) = f_1\left[\frac{\log P(h|e)}{\log P(h)}\right] \quad \text{and} \quad J_2(h, e) = f_2\left[\frac{\log P(h|e)}{\log P(h)}\right].$$

Because f_1 is a monotonically decreasing function, a necessary and sufficient condition that $J_1(h_1, e_1) > J_1(h_2, e_2)$, is

$$\frac{\log P(h_1|e_1)}{\log P(h_1)} < \frac{\log P(h_2|e_2)}{\log P(h_2)},$$

and because of the monotonicity of f_2, this is a necessary and sufficient condition that $J_2(h_1, e_1) > J_2(h_2, e_2)$. Analogous reasoning holds if the sign > is replaced by < or by =. Thus all measures of justification are ordinally equivalent to one another.

If h and e are such that $P(h|e) = P(h)$, then $J(h, e) = f[1]$, irrespective of the value of $P(h) \in (0, 1)$. Shogenji calls this the condition of equineutrality, and we conventionally set $f[1] = 0$. If, on the other hand, h and e are such that $P(h|e) = 1$, then $J(h, e) = f[0]$, irrespective of the value of $P(h) \in (0, 1)$. Shogenji calls this the condition of equimaximality, and we conventionally set $f[0] = 1$.

The simplest realization of the above constraints is $f[u] = 1 - u$, which leads to

$$J(h, e) = 1 - \frac{\log P(h|e)}{\log P(h)} = \frac{\log P(h|e) - \log P(h)}{-\log P(h)}. \tag{9}$$

It is easy to see that this realization is a continuous function of $P(h|e)$ [for fixed $P(h) \in (0, 1)$], when $P(h|e) \in (0, 1]$, but not at $P(h|e) = 0$, for at this point $J(h, e)$ diverges to minus infinity. So $J(h, e)$ is not acceptable if we insist on continuity at $P(h|e) = 0$. A realization that achieves continuity at both ends is

$$f[u] = 2^{1-u} - 1\,,$$

which gives the measure

$$J'(h, e) = 2^{J(h,e)} - 1\,, \tag{10}$$

and this takes on the value -1 when $P(h|e) = 0$, the value 0 when $P(h|e) = P(h)$, and 1 when $P(h|e) = 1$.

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