Logic and Games: an Introduction

This special issue of *Studia Logica* is dedicated to *Logic and Games*.

Formal logic and game theory can meet in many ways. While the use of *games for logic*, e.g., to define semantics of quantifiers or to compare logical models, goes back a long time, *logic for games* is a more recent and currently very active research direction which has been precipitated by the introduction of the notion of (multi-)agency in logic. If agents are assumed to be self-interested and to act rationally, then reasoning about action in a multi-agent setting requires reasoning about game theoretic concepts. Furthermore, logics capturing, e.g., the principles of action, belief, knowledge, time, preference and so on, can help explaining the foundations of game theoretic solutions, algorithms, and so on. Active research topics in the intersection between formal logic and game theory include but are not limited to logical formalisations of game properties; logics for reasoning about strategic interaction; the use of logic to characterise or explain concepts, solutions, algorithms, etc., relevant for games; logical-epistemic foundations of solution concepts; reasoning about preferences; logical aspects of social choice; judgment aggregation; game semantics; model comparison games characterising the expressiveness and succinctness of different logics.

The goal of the special issue of *Studia Logica* on Logic and Games is to illustrate current trends and present recent advances in this field. We invited submissions on all topics in the intersection between formal logic and game theory.

The special issue has articles by two invited authors, as well as five contributions selected from submissions after a call for papers. All invited and contributed articles have undergone extensive peer review. Together, these articles form a strong special issue on logic and games, representing both the depth and the breadth of this thriving research area.

The first invited author is Johan van Benthem. The topic of his article *Modeling Reasoning in a Social Setting* is logical modeling of interactive behaviour. The paper surveys some of the many logical models of games and

social interaction, and it is also a position paper, identifying some connecting themes and pointing out a range of interesting open problems. In particular, it identifies uses of logic in three phases of game-like activities: before, during and after play. The paper demonstrates that a great variety of logical operators are useful for reasoning about interaction in games, including operators for knowledge, belief and plausibility, time, action, preferences, rationality and public announcements.

The second invited author is Michael Wooldridge. One area in which logic and games meet is the recently active area of *Boolean games* [5]. These are (typically normal form) games where each player controls a number of atomic propositions, and a strategy corresponds to an assignment of truth values to those atomic propositions. Each player has a goal formula, a (typically) propositional logic formula that potentially involves atomic propositions controlled by other agents. In their paper *Manipulating Games by Sharing Information*, Michael Wooldridge and colleagues study a variant of Boolean games where some atomic propositions are controlled by the environment and cannot be accessed by the players, and where the players have private beliefs about these propositions. Players can manipulate the game through communication, in particular by announcing the value of some of the environment variables. It is shown how such communication can turn a game from having no stable states to being a stable game.

Another area in which modern logic and game theory meets is in formalising knowledge and belief. Most modal epistemic (or doxastic) logics [3] are based on Kripke structures, which again can be seen as equivalent to Aumann structures in game theory. Another approach to modeling knowledge and belief in epistemic game theory is Harsanyi type spaces. Recent work in epistemic logic has focused on modeling information change, resulting in different types of *dynamic epistemic logics* [8] where the *product update* operation on Kripke structures play a key role. In the contribution *Changing Types: Information Dynamics for Qualitative Type Spaces*, Dominik Klein and Eric Pacuit apply ideas from dynamic epistemic logic based on Kripke structures, to so-called knowledge structures, non-probabilistic variants of Harsanyi type spaces, in order to develop a theory of information change for knowledge structures. In particular, they generalise the product update operation from Kripke structures to knowledge structures.

The logic of knowledge and belief is also considered in the contribution *Logical-Epistemic Foundations of General Game Descriptions* by Ji Ruan and Michael Thielscher. The general *Game Description Language* (GDL) [6] is a formal language developed in the area of Artificial Intelligence in order to completely describe the rules of an arbitrary game to a (human or

machine) player, in order to enable that player to play the game. Games might involve incomplete information. Ruan and Thielscher analyse the epistemic structure and expressive power of GDL, by translations to and from epistemic logic.

As amply illustrated in the invited paper by van Benthem, *modal* logic is a natural framework for reasoning about games, among other reasons due to the intuitively close resemblance between Kripke models and game structures [2], and modal logics interpreted in game-like structures have been an active area of research in recent years [7, 1]. While many of the logics developed in this area can be used to express solution concepts such as the existence of equilibria, from a game-theoretic viewpoint most of them capture only very basic concepts. In particular, most existing modal game logics can only be used to reason about pure strategies, and, curiously, logics taking mixed strategies into account have been studied comparatively very little. In the contribution *A Modal Logic for Mixed Strategies*, Joshua Sack and Wiebe van der Hoek takes a step in that direction, by developing a modal logic interpreted in strategic form games, which can be used to reason about mixed strategies.

In the contribution *Subgames within Large Games and the Heuristic of Imitation*, Soumya Paul and R. Ramanujam use modal logic to describe *types* of players, corresponding to their strategic abilities and/or behaviour. The setting is repeated normal form games with a large number of players. It is argued that it is interesting to divide such games into subgames, called neighbourhoods. Players can choose, and switch between, strategies in a neighbourhood, or move to another neighbourhood. One type of strategy that is of particular interest is *imitation*, and it is shown how the strategy of imitation can be specified in logic.

For the final contributed paper we move on to “games for logic”. In *Studia Logica* 40 years ago Robin Giles [4] characterised infinite-valued Łukasiewicz logic using a game that combines Lorentzen-style dialogue rules for logical connectives with a “betting” scheme for atomic propositions. In the contribution *From Games to Truth Functions: A Generalization of Giles’s Game*, Christian G. Fermüller and Christoph Roschger generalise Giles’s game to other many-valued logics. In particular, they define conditions on Giles-style games that enables the extraction of a truth-functional semantics, and show how a number of well-known logics can be characterised using such games.

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