RĂZVAN DIACONESCU, **Institution-independent Model Theory**, Studies in Universal Logic, Birkhäuser Basel, 2008, pp. 370. ISBN 978-3-7643-8707-5 (soft-cover) EURO 96,25; ISBN 978-3-7643-8708-2 (eBook) EURO 74,96.

I should start with a warning and apologies: in no way I can write an objective review of this monograph, nor of any other text on this topic. I fell in love with the theory of institutions some 30 years ago when I read one of the early version of [7] (yes, it was some 10 years before the final journal version of this seminal paper appeared) — and institutions have remained a fascinating major topic and tool in my own research ever since then. I might have been the first to explicitly formulate the programme of developing a version of abstract model theory within the institutional framework, with [13] opening the area by offering a few basic concepts and easy facts. I recall talking about this on numerous occasions at the time, and then being surprised that for quite a while not much was done to develop this promising area. That is, until Răzvan Diaconescu and his students in Bucharest took up the torch and started cracking the classical concepts and results of standard model theory one after another, providing a beautiful abstract treatment of subsequent topics in institution-independent model theory — with most of this work collected in the monograph under review.

Here is the overall idea. To begin with, we take a *model-theoretic* view of logic, putting aside the issues of proof for the time being. Classical model theory [6] was developed for first-order logic. One of important departures, referred to as *abstract* (or *soft*) *model theory* [2, 3], abstracts away from the form of logical sentences concentrating entirely on their expressive power. Institutions take this quite a bit further, abstracting away also from the exact nature of semantic structures involved. We are left then with two abstract collections of objects we think of as *models* (actually, models come with morphisms between them and form a category) and as *sentences*, respectively, linked by a relation we think of as *satisfaction relation*. These are uniformly defined for an “arbitrary but fixed” *signature*. Again, we abstract away from the exact nature of signatures and instead consider them with *signature morphisms*, forming a category. A crucial extra twist is to realise that models and sentences classified by signatures are uniformly translated when we move from one signature to another: sentences are translated in the same direction and models are translated (“reduced”) contravariantly, in the opposite direction, so that the satisfaction relation is preserved in a “twisted way”. This last requirement, referred to as the *satisfaction condition*, captures the fact that *satisfaction is preserved under change of notation and context*.

The original motivation for such an extremely abstract and general view of a logical system came from an attempt to unify various theories of software specification, at the time (and still now) often based on various choices of the concepts of “algebra” and “axiom”. The categorical structure of the collection of signatures was exploited to develop means of structuring specifications in an arbitrary institution [4, 5, 11]. Indeed, by now it is the standard practice to develop specification formalisms in a structured way, separating basic specifications (given, modulo syntactic sugar, as a signature and a set of sentences over this signature) from the means of building

and using more complex specifications, in essence introduced independently of the institution used for basic specifications. The design of CASL [9] provides a mature example here.

Back to model theory though: *institution-independent model theory* aims at developing, as much as possible, the basic concepts and results of the classical first-order model theory in the context of an arbitrary institution. This is not always possible, some crucial bits of information about the underlying structures may be missing, so “the game” now is to identify those missing assumptions in a way as abstract and general as possible. The expected gain is that concepts and results will turn out re-usable for many logical systems other than first-order logic, and perhaps more importantly, that we would thus be able to put our hands on the “real essence” of the matter, by showing which aspects of what one knows about first-order logic are really relevant for the given development. Indeed, the monograph shows this on quite a few occasions, resulting in beautifully abstract presentation of a considerable body of model-theoretic concepts and results.

The crucial tool for the developments is the categorical structure of the collection of signatures and the translations of sentences and models induced by signature morphisms. To take a very simple example, consider the notion of a free variable in first-order logic. Although in an institution we deal with closed sentences only, free variables are there! Namely, a set of variables for a signature may be identified with a signature extension (along a signature inclusion that adds these variables as new constants) and then valuation of the variables in a model is just a model expansion along this inclusion. Given this, we can work further and for instance deal with quantification over so introduced variables, etc. The idea has been used in the institutional context for a while by now [14, 10], but this monograph adds an important twist by characterising signature morphisms that add a finite set of constants, so that quantification over those captures “first-order” quantification as expected.

The monograph starts by recalling some basics of category theory in Chap. 2. These are sketched rather than properly presented, I guess nobody would learn much here without knowing the material beforehand — but this chapter is necessary, at least to fix the notation and identify few less obvious facts used later.

This is followed by a chapter that introduces institutions. As expected, first-order logic is used to guide the definition in a rather standard way. I like numerous examples that follow, presented without unnecessary formality, rightly imposing on the reader the broad spectrum of logical systems that the notion of institution captures. Already here, the author equips institutions with categorical structure. Actually, two concepts are introduced: institution morphisms and comorphisms, thus defining two distinct categories of institutions. This is standard in the area, where the three layers present in any institution (signatures, models, sentences) multiply the number of reasonable concepts of an institution map [15, 8]. The known results on completeness and cocompleteness of these categories are given here.

Chapter 4 offers a number of ideas and results known from the earlier institutional literature, propped up with a few useful improvements. We have here the notion of a theory, with the classical result that the category of theories inherits colimits from the category of signatures [7] and numerous instructive examples of

theoroidal institution comorphisms (comorphisms to the institution of theories). Then amalgamation of models over signature pushouts is discussed (generalised to commuting squares of signature morphisms). This prepares the ground for fundamental model-theoretic technique: the method of *elementary diagrams*, elegantly improving the crude predecessor in [14]. Of course, not every logic offers elementary diagrams — so this is a beautiful example where a strong classical technique is extracted from first-order model theory, formulated as a property of institutions, and used for further developments and results concerning institutions that *admit the method of diagrams* — that is, have this property. The analysis of important results on the existence of free models starts here, using the method of diagrams to link freeness and initiality.

The author moves then (Chap. 5) to the standard ways of building logical formulae, but within an arbitrary institution. This is quite obvious when logical connectives are concerned, but becomes interesting when quantification is introduced, with classification of the morphisms used to introduce variables becoming of crucial importance. In fact, one may build an institution of “first-order logic” over an arbitrary institution of atomic sentences — an important construction, especially when coupled with later (Chap. 13) development of proof calculi for the logics so built. Incidentally, the proof systems here turn out to be perhaps nicer and cleaner than the standard proof calculus for first-order logic in at least one aspect: the explicit account of signatures over which proofs are considered makes the treatment and role of *eigenvariables* clear. Also in Chap. 5, following [1], the model-theoretic toolkit is further expanded by introducing injectivity w.r.t. model morphisms to capture satisfaction of logical formulae.

Chapters 6–10 form the hard model-theoretic core of the monograph. A number of standard non-trivial constructs and results known from classical first-order model theory are introduced here by extracting the essential requirements on the underlying logic that make them work. No room here for a detailed account of what has been achieved, let me just mention tools like ultraproducts and saturation, with their standard model-theoretic applications carried out, results on the existence of initial (and free) models, Beth-style definability results, and interpolation results. Interpolation as studied here is perhaps my favourite: as in earlier institutional literature, interpolation is considered over an arbitrary pushout of signatures (rather than just w.r.t. their intersection and union, as in the standard formulation), allows for a set of interpolants (rather than a single interpolant sentence), and permits extra parameter sentences to be considered. This frees one from implicit assumptions linked with first-order logic (like conjunction and implication being freely available) and, perhaps more crucially, makes many applications realistic, notably in the context of computer science and specification theory there (some illustration may be found in Chap. 14).

To continue the overview of the contents of the monograph: modal logics over an arbitrary institution are discussed in Chap. 11, a Grothendieck-style construction for families of institutions linked by institution (co)morphisms is considered in Chap. 12, and proof-theoretic aspects are added to institutions in Chap. 13. I view these chapters as somewhat peripheral to the main achievements presented in this

monograph, even though some important and interesting concepts and results are presented here, notably concerning proof calculi, and the institutional Grothendieck construction used in the development of foundations for *heterogeneous specifications*. Much the same applies to the final two chapters dealing with specifications and, respectively, logic programming in an arbitrary institution. Both deserve separate monographs (I cannot resist to mention [12] in this context) so what is given here may be viewed just as a teaser for the interested reader who may want to put more effort into further study and perhaps development of these areas.

Praising highly the contents of the monograph, I would be more hesitant to repeat the high complements when commenting on the details of presentation. Unfortunately, I do not like the style in which some of the results are formulated, with some visible sloppiness and lack of precision here and there. Worse, the author failed to avoid some evident technical mistakes. So, for instance looking through Chap. 2, I hasten to ensure the reader that:

- the forgetful functor considered in Prop. 2.3 does not create arbitrary colimits (I believe it does create colimits of non-empty connected diagrams, as well as arbitrary limits);
- the only weak coproduct of two empty sets is the empty set, so not every superset of a disjoint union of two sets is their weak coproduct, contrary to a claim at p. 16;
- the brief definition of a skeletal category at p. 17 departs from the standard one; when auto-isomorphisms are not allowed, as here, the expected property that any category is equivalent to some skeleton must fail.

Such problems may also be found in later chapters (take for instance Cor. 8.18, where the “only if” part is clearly false) but overall they are easy to fix and do not really affect the main line of developments. So, they should be taken as a warning, but most emphatically not as a discouragement to the reader. Minor problems with presentations and occasional technical slips must not hide the overall interest and depth of the developments this monograph opens.

To sum up: I love and very highly value this style of study and research in model theory and I love the contents of this book. The research presented offers a new perspective on classical model-theoretic developments, making it possible to identify their essence and carry them over to many, often not-so-standard contexts. In fact, it opens new area for further exciting research, with plenty of open questions and problems, both inherited from classical model theory, as well as entirely new. At the same time, it is mature by now, with a number of concepts and results that are of direct interest and use already. I heartily advertise the monograph to anyone interested in model theory in its abstract version, both to the students and newcomers to model-theory, as well as to those who would take the developments here as a starting point for their own research in the area.

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