Editorial introduction

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This special issue focuses on a stochastic network described by a multidimensional stochastic process with reflecting boundaries. This is a widely used model in queueing theory and operations research. We are particularly interested in its stationary characteristics. Although a variety of models have been studied for stationary properties, exact results are very limited due to the complexity of multidimensional models. Thus, much effort has been dedicated to their asymptotic behaviors. We distinguish two types of them, rough and exact. The rough (logarithmic) asymptotic is given by decay rate while the exact asymptotic is given by a function which is asymptotically proportional to the tail of the distribution.

The tail asymptotic of the stationary distribution is one of the most important asymptotics, which often not only leads to performance bounds and approximations but also allows for performance comparison. This considerably widens the door to applications of a stochastic network. However, this subject has not yet been fully studied for a wide class of the stochastic networks because of technical difficulties. In particular, it is hard to get analytically tractable or intuitively comprehensive results in terms of modeling primitives or system parameters.

The aim of this special issue is to challenge these problems. We have eight papers, which could be categorized into three groups. The first three papers discuss tail asymptotic behaviors of two-dimensional reflecting random walks including Markov modulation or Brownian motion. The tail asymptotics are derived under general assumptions including stability. The next three papers discuss specific models of two-dimensional random walks with and without reflections. More detailed asymptotics are obtained.

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The last two papers discuss a general dimensional reflecting process and a large scale system for telecommunications, respectively. We detail those three groups of papers below.

Ozawa considers a Markov modulated two-dimensional reflecting skip-free random walk in the quarter plane. The decay rates of its stationary probabilities in coordinate directions are obtained. This generalizes the corresponding results when there is no Markov modulation. This paper considerably widens the class of models for which the tail asymptotics can be obtained.

Li, Tavakoli, and Zhao consider a two-dimensional reflecting skip-free random walk in the quarter plane under the assumption that the random walk is singular. By singularity it is meant that the random walk in the interior of the quarter plane can be reduced to having one-dimensional dynamics. It considerably simplifies the derivation of the tail asymptotics of the stationary distribution. However, there are many different cases depending on how the singularity occurs. This paper completely determines their exact tail asymptotics in the coordinate directions.

Dai and Miyazawa consider a two-dimensional semi-martingale reflecting Brownian motion on the nonnegative quadrant. Three topics are studied. First, the large deviations rate function concerning the stationary tail probabilities is reproduced using geometric objects. Second, a new geometric characterization is derived for the stationary distribution to have product form. Finally, exact asymptotic functions are obtained for the stationary measures on the two boundary faces.

Kurkova and Raschel consider a special class of a two-dimensional skip-free random walk, which is motivated by a locally influenced voter behavior. A major interest is in the hitting times at the coordinate axes and at either one of them when the random walk starts from a given interior point in the quarter plane. Their generating functions are obtained in closed form, and the tail asymptotics of their distributions are derived. Complex analysis and boundary value technique are the main tools in the analysis.

Guillemin, Knessl, and van Leeuwaarden consider a certain class of two-dimensional reflecting skip-free random walks in the quarter plane, motivated by wireless 3-hop networks with stealing. The generating function of the stationary distribution is obtained in a closed form. Using this result, exact tail asymptotics are obtained for the stationary marginal distribution in the coordinate directions. Some new challenges for the boundary value technique are presented.

Adan, Kapodistria, and Leeuwaarden consider a system of two parallel queues with Erlang arrivals joining the shorter queue, which is formulated as a multilayered (a special type of Markov modulation) two-dimensional reflecting skip-free random walk in the quarter plane. The compensation method is extended for this multilayered process, and its stationary distribution is obtained as an infinite series of product forms. As a corollary of this result, a finer tail asymptotic is obtained for the stationary distribution of the minimum queue length.

Kobayashi, Sakuma, and Miyazawa consider a system of k parallel queues with Poisson arrivals joining the shortest queue for an arbitrary positive integer k , where service times are exponentially distributed but servers may have different service rates. This model is formulated in two ways, as a quasi-birth-and-death process and a $k + 1$ dimensional reflecting skip-free random walk on the orthant. The exact tail asymptotic is obtained for the stationary distribution of the minimum queue length.

Haddad and Mazumdar consider a large stochastic fluid system, single link or a certain tree network, that operates under a balanced fair bandwidth allocation policy. This model has a closed form stationary distribution which is insensitive with respect to input file size distributions. However, it is still challenging to compute performance measures due to the scale of the system. They are approximately obtained as the scaling of the arrival and maximum service rates is increased in such a way that their ratio is fixed.

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